Que:no#1

I) Order of Matrix A is $m \times P$
Order of Matrix B is $P \times n$

=> Order of Matrix AB = $m \times n$

II) The number of non-Zero rows in Echelon form is called Rank of Matrix.

III If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

$|B| = 0$ i.e Singular

$a = ?$

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$'' = 1 \times a - 2 \times 4$$

$$|B| = a - 8$$

=> $a - 8 = 0$

$a = 8$ Ans

#2

v) $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$, $|A| = ?$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

" $= 2i \times (-i) - i \times i$

" $= -2i^2 - i^2$

" $= -2(-1)^2 - (-1)^2$

" $= +2+1$

$A = \cancel{\#}3$

v) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

It is called Scalar Matrix.

#3

vii) $dy/dx + 2xy = y$?

Solution,

$$dy/dx = y - 2xy$$

$$" \quad = y(1-2x)$$

$$dy/y = (1-2x)\,dx$$

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln|y| = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln|y| = x - x^2 + c$$ Ans:*

$$e^{\ln y} = e^{x-x^2+c}$$

$$y = e^{x-x^2+c}.$$

viii) $\left(\frac{dy}{dx}\right)^3 - \sqrt{1-\left(\frac{dy}{dx}\right)^2}$

Order = 1 OR Order = 1

Degree = 6 Degree = 3

viii) $\left(\frac{d^2y}{dx^2}\right) - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$

Order = 2

Degree = 1

#5

ix) The differential Equation

$$2\frac{dy}{dx} + x^2 y = 2x + 3 \qquad y(0) = 5$$

Homogeneous differential Equation

OR

linear differential Equation.

Solution:-

$$2\frac{dy}{dx} + x^2 y = 2x + 3$$

$$2\frac{dy}{dx} = 2x + 3 - x^2 y$$

$$\int 2\,dy = \int (2x + 3 - x^2 y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - \frac{x^3}{3}y + c$$

$$2y + \frac{x^3 y}{3} = \frac{2x^2}{2} + 3x + c$$

$$y\left(2 + \frac{x^3}{3}\right) = x^2 + 3x + c$$

$$y\left(\frac{6 + x^3}{3}\right) = x^2 + 3x + c$$

$$y = \frac{3(u^2 + 3u + C)}{6+u^3}$$

$$y = \frac{3u^2 + 9u + C}{6+u^3} \quad ------ *$$

put $u=0$, $y=5$

$$5 = \frac{3(0)^2 + 9(0) + c}{6+0^3}$$

$$5 = \frac{0+0+c}{6} \Rightarrow C = 30$$

put in eq *

$$y = \frac{3u^2 + 9u + 30}{6+u^3}$$

#7

x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = ?$

Solution:→

$$\begin{vmatrix} 1 & a & b^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad \begin{matrix} \text{by} \\ R_3 - R_1 \\ R_2 - R_1 \end{matrix}$$

$$\begin{vmatrix} 1 & a & a^2 \\ 0 & (b-a) & (b-a)(b+a) \\ 0 & (c-a) & (c-a)(c+a) \end{vmatrix}$$

$$\begin{matrix} (b-a) \\ (c-a) \end{matrix} \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$(b-a)(c-a)\left[1 \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} - 0 + 0 \right]$$

$$(b-a)(c-a)\left[1(c+a)-(b+a)\right]$$

$$=(b-a)(c-a)\left[c+\cancel{a}-b-\cancel{a}\right]$$

$$=(b-a)(c-a)(c-b)$$ **Ans:-**

Que:no#3

$$(x^2+3y^2)dx-2xy\,dy=0$$

Solution:- *

$$(x^2+3y^2)dx-2xy\,dy=0$$

$$(x^2+3y^2)dx=2xy\,dy$$

$$\frac{x^2+3y^2}{2xy}=dy/dx$$

$$dy/dx=\frac{x^{\cancel{2}}}{2\cancel{x}y}+\frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$dy/dx=\frac{x}{2y}+\frac{3}{2}\cdot\frac{y}{x}$$

Let $v = y/x$ or $y = vx$ #9.

Dff w.r.x on b/s

$$dy/dx = v + x\frac{dv}{dx}$$ put these values

$$\frac{1}{2}\cdot\frac{1}{v} + \frac{3}{2}v = v + x\frac{dv}{dx}$$

$$\frac{1}{2v} + \frac{3v}{2} - v = x\frac{dv}{dx}$$

$$\frac{1+3v^2-2v^2}{2v} = x\frac{dv}{dx}$$

$$\frac{1+v^2}{2v} = x\frac{dv}{dx}$$

$$\int\frac{dx}{x} = \int\frac{2v}{1+v^2}$$

$$\ln x = \ln 1+v^2 + \ln c$$

$$\cancel{\ln} x = \cancel{\ln}(1+v^2)c$$

$$x = (1+v^2)c$$

$x = 2$, $y = 6$

#10

$v = y/x$

$x = \left(1 + \frac{y^2}{x^2}\right) c$ ----- *

$2 = \left(1 + \frac{6^2}{2^2}\right) c$

$2 = \left(1 + \frac{\cancel{36}^{\,9}}{\cancel{4}}\right) c$

$2 = 10c \Rightarrow c = \frac{\cancel{10}}{2} = \cancel{5}\ \ 1/$

$c = \frac{1}{5}$

Put in eq. *

$x = \left(1 + (y/x)^2\right) \frac{1}{5}$

$5x = \left(1 + \frac{y^2}{x^2}\right)$ **Ans:**

Que:no # 03

2nd Method:* $(x^2+3y^2)dx = 2xy\,dy = 0$

Find the General solution at $x=2,\ y=6$

Solution:*

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

$\div\ 2xy$ on b/s

$$\frac{x^2+3y^2}{2xy} = dy/dx$$

$$\frac{x^2}{2xy} + \frac{3y^2}{2xy} = dy/dx$$

$$\frac{x}{2y} + \frac{3y}{2x} = \frac{dy}{dx}$$

$$dy/dx = \frac{1}{2}\left(\frac{x}{y} + \frac{3y}{x}\right) \text{ - - - - *}$$

$\boxed{\text{let } v = y/x}$, $y = vx$

OR

$$\boxed{\frac{1}{v} = \frac{x}{y}}$$

Diff w.r.t x

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

put these eq in *

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v \Rightarrow 2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx$$

$$\ln|1+v^2| = \ln|x| + \ln C$$

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$\ln|1+v^2| = \ln x + \ln C$ $\quad \therefore \ln m + \ln n = \ln mn$

$\cancel{\ln}|1+v^2| = \cancel{\ln} xC$

$1+v^2 = xC$ ------ *1

$\boxed{\text{But } v = y/x}$

$1 + y^2/x^2 = xC$

$$\frac{x^2+y^2}{x^2} = xC$$

$x^2+y^2 = x^3C$ ------ *2

As $x = 2$, $y = 6$

$2^2+6^2 = 2^3C$

$4+36 = 8C$

$C = 5$

put in eq *2

$x^2+y^2 = x^3 5$

$y^2 = x^3 5 - x^2$

$y^2 = x^2(5x-1)$

$\sqrt{y^2} = \sqrt{x^2(5x-1)} \Rightarrow y = \pm x\sqrt{5x-1}$

Ans.

Que:no#2

#16

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Que:no#2

(a)

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Solution:→

$$= abc \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & (b^2-a^2) & c^2-a^2 \end{vmatrix}$$

$$= (abc)(b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

$$= (abc)(b-a)(c-a)\left[1\begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix} - 0 + 0 \right]$$

$$= (abc)(b-a)(c-a)\left[1(c+a) - 1(b+a) \right]$$

$$= (abc)(b-a)(c-a)(c+\cancel{a}-b-\cancel{a})$$

$$= (abc)(b-a)(c-a)(c-b)$$ Ans: *

Que:no#2

part(a) ; 2nd Method:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Solution:*

let $A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Expanding By R_1

$|A| = a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$

$'' = a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$

$'' = ab^2c^3 - ab^3c^2 - ba^2c^3 + ba^3c^2 + ca^2b^3 - ca^3b^2$

$'' = abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$ Ans.

Que:no#2

(B) Find the Eigen Value,

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:*

$$\text{let } A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Expand By R_1

$$2-\lambda \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} - (-1) \begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

$$+ (-1) \begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} + 0 = 0 \text{------} (B)$$

Now:*,
$$\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

Expanding By R_1

$$3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + (-1) \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix} = 0$$

$$3-\lambda \left\{ (3-\lambda)(2-\lambda) - (-1)(-1) \right\} + 1 \left\{ (-1)(2-\lambda) - (-1)(-1) \right\}$$

$$-1 \left\{ -1 \times (-1) - (3-\lambda)(-1) \right\} = 0$$

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$$(3-\lambda)(6-3\lambda-2\lambda+\lambda^2+1)+1(-2+\lambda-1)$$
$$-1(1+3-\lambda)=0$$

$$\rightarrow (3-\lambda)(5-5\lambda+\lambda^2)+(-3+\lambda)-(4-\lambda)=0$$

$$15-15\lambda+3\lambda^2-5\lambda+5\lambda^2-\lambda^3-3+\lambda-4+\lambda=0$$

$$-\lambda^3+8\lambda^2-18\lambda+8=0 \quad — \quad *$$

$$\rightarrow \quad +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} =$$

Expanding by R_1

$$-1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ 0 & 2-\lambda \end{vmatrix} +(-1)\begin{vmatrix} -1 & 3-\lambda \\ 0 & -1 \end{vmatrix}=0$$

$$-1\left\{(3-\lambda)(2-\lambda)-(-1)(-1)\right\}+1\left\{-1(2-\lambda)-0\times-1\right\}-1\left\{-1\times-1-0\times(3-\lambda)\right.$$

$$-6+3\lambda+2\lambda-\lambda^2+1-2+\lambda-1=0.$$

$(3-\lambda)(6-3\lambda)$

$$-\lambda^2+6\lambda-8=0 \text{ - - - - - } *_1$$

Now:

$$-1\begin{bmatrix}-1 & 3-\lambda & -1\\ -1 & -1 & -1\\ 0 & -1 & 2-\lambda\end{bmatrix}$$

Expanding by C_1

$$\rightarrow -1\left[-1\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix}+0\right]=0$$

$$-1\left[-1\left(-1(2-\lambda)-(-1)(-1)\right)+1\left((3-\lambda)(2-\lambda)-(-1)(-1)\right)\right]$$

$$-1\left[-1(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]=0$$

$$\rightarrow -1(3-\lambda+5-5\lambda+\lambda^2)=0$$

$$-\lambda^2+6\lambda-8=0 \text{ - - - - - - - } *_2$$

Putting equation $*1, *2, *$ in eq (B).

#21

$$(2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By Synthethic Division:-*

	1	-10	32	-32
2		2	-16	32
	1	-8	+16	0

So, $(\lambda-2)(\lambda^3-8\lambda^2+16\lambda)$

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$\lambda-2=0$

$\lambda=0$ | $\lambda=2$

$\lambda^2-8\lambda+16=0$ By Factorization..

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$\lambda-4=0,$ $\lambda-4=0$

$\lambda=4$ $\lambda=4$

$\lambda=0$

$\lambda=2$

$\lambda=4$

$\lambda=4$ ∴ Sol.