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Subjet = Differential
Equation.

Question No 1 (A) $y' = (x+2)\,y^2.$

Solution

$$y' = (x+2)\,y^2.$$

$$\frac{dy}{dx} = (x+2)\,y^2$$

$$\int \frac{1}{y^2}\,dy = \int (x+2)\,dx$$

$$\int y^{-2}\,dy = \int (x+2)\,dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + c$$

$$\frac{y^{-1}}{-1} = -\left[\frac{x^2}{2} + 2x\right] + c_r$$

Now

$$y^{-1} = \frac{x^2}{2} + 2x + c$$

$$\boxed{y = \frac{1}{\frac{x^2}{2} + 2x + c}}$$

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Question 1(b)

Sol:

let $y + 9x = u$ — ①

$dy/dx + 9 = du/dx$

$dy/dx = \frac{du}{dx} - 9$

So its become.

$\frac{du}{dx} - 9 = u^2$

$\frac{du}{dx} = u^2 + 9$

$\int \frac{1}{u^2+9} du = \int dx$

$= \int \frac{1}{(3)^2 + (u)^2} dx = \int dx.$

$\frac{1}{3} \tan^{-1} \left(\frac{u}{3} \right) = x + c_1.$

$\tan^{-1} \left(u/3 \right) \; 3x + 3c_1$

$\tan^{-1} \left(u/3 \right) = 3x + c.$

$u/3 = \tan (3x + c)$

$u = 3 \tan (3x + c)$

$y + 9x = 3 \tan (3x + c)$

$y = -9x + 3 \tan (x + c)$

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Question No # 2

$x^3 dx + y^3 dx = 0$

Sol:

$M = x^3 , \quad N = y^3$

$\frac{\partial M}{\partial y} = 0 , \quad \frac{\partial N}{\partial x} = 0$

Now equation will be exact.

$u = \int M dx + k(y)$

$u = \int x^3 dx + ky \rightarrow$ 1

Now

$\partial u / \partial y = 0 + d/dy \, ky$

But $\frac{\partial u}{\partial y} = N$

So

$N = d/dy \, ky$

$\int d \, k(y) = \int y^3 dy$

$k(y) = y^4/4 + c_1$

So eq 1 becomes.

$u = x^4/4 + y^4/4 + c_1$

$c_2 = x^4/4 + y^4/4 + c_1$

$c = x^4/4 + y^4/4$

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Question 3A

$4y'' - 20y' + 25y = 0$

dividing by 4

$\frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = 0$

$y'' - 5y' + \frac{25y}{4} = 0$

$a = -5$, $b = 25/4$

Now

$\lambda^2 - 5\lambda + 25/4 = 0$

formula

$\boxed{\lambda^2 - 2ab + b^2 = 0}$

$\lambda^2 - 5x + \left(5/2\right)^2 = 0$

$\left(\lambda - 5/2\right)^2 = 0$

$\left(\lambda - 5/2\right)\left(\lambda - 5/2\right) = 0$

$\lambda_1 = 5/4$, $\lambda^2 = 5/4$

same & real roots.

So $\boxed{y = (c_1 + c_2 x)e^{5/4 x}}$

Question 3b

$4y'' - 6y' - 7y = 0$

Assume $y(x) = e^{\lambda x}$ put in eq

$\Rightarrow 4 \frac{d^2}{dx^2} y(x) - 6 \frac{d}{dx} y(x) - 7y(x) = 0$

$\Rightarrow 4 \cdot \frac{d^2}{dx^2}(e^{\lambda x}) - 6 \frac{d}{dx}(e^{\lambda x}) - 7e^{\lambda x} = 0 \rightarrow (1)$

$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \rightarrow (A)$

$\frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \rightarrow (B)$

put (A) and (B) in eq (1)

$\Rightarrow 4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$

$\Rightarrow (4\lambda^2 - 6\lambda - 7) e^{\lambda x} = 0$

$\Rightarrow \lambda = 3/4 - \sqrt{37}/4$

$\Rightarrow \lambda = 3/4 + \sqrt{37}/4$

$\Rightarrow y(x) = y_1(x) + y_2(x)$

$\Rightarrow y(x) = C_1 e^{(3/4 - \sqrt{37}/4)x} + C_2 e^{(3/4 + \sqrt{37}/4)x}$

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