Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 20

Student Details

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Q1.	(a)	Determine the response $y(n)$, $n \geq 0$, of the system described by the second order difference equation	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation.	
Q2.	(a)	Determine the causal signal $x(n)$ having the z-transform (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function	
Q. 3	(a)	A two- pole low pass filter has the system response Determine the values of b_0 and p such that the frequency response H(ω) *12.* Determine the values of b_0 and p such that the frequency response H(ω) *12.*	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its at $\omega = 4\pi/9$.	
Q 4	(a)	A finite duration sequence of Length L is given as Determine the N- point DFT of this sequence for $N \geq L$	Marks 4
	(b)	Compute the DFT of the four-point sequence	

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Q1 (a)

Sol:-

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \quad \text{— (a)}$$

The characteristic equation are

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$
$$\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

Therefore, the roots are $\lambda = -1, 4$, and the general form of the solution to the homogeneous equation is

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n$$
$$= C_1(-1)^n + C_2(4)^n \quad \text{— (b)}$$

The particular solution to equation (a)

$$y_p(n) = K(4)^n u(n)$$

or

$$y_p(n) = Kn(4)^n u(n) \quad \text{— (c)}$$

upon substitution of (c) into (a), we obtain

$$Kn(4)^n u(n) - 3K(n-1)(4)^{n-1}u(n-1) - 4K(n-2)(4)^{n-2}u(n-2)$$
$$= (4)^n u(n) + 2(4)^{n-1}u(n-1)$$

To determine K, we evaluate this equation for $n \geq 2$. We select $n=2$ from which we obtain $K = 6/5$. Therefore,

$$y_p(n) = \frac{6}{5}n(4)^n u(n) \quad \text{— (d)}$$

The total solution to difference equation is obtained by adding (b) to (d). Thus

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$$y(n) = c_1(-1)^n + c_2(4)^n + \frac{6}{5}n(4)^n \quad n \geq 0$$ ⓐ

where the constant c_1 and c_2 are determined such that the initial conditions are satisfied

$$y(0) = 3y(-1) + 4y(-2) + 1$$
$$y(1) = 3y(0) + 4y(-1) + 6$$
$$= 13y(-1) + 12y(-2) + 9$$

on other hand, ⓐ evaluated at $n=0$ & $n=1$

$$y(0) = c_1 + c_2$$
$$y(1) = -c_1 + 4c_2 + \frac{24}{5}$$

we can now equate these two sets of relations to obtain c_1 & c_2. we can simplyfy the computations above by setting $y(-1) = y(-2) = 0$.

$$c_1 + c_2 = 1$$
$$-c_1 + 4c_2 + \frac{24}{5} = 9$$

Hence $c_1 = -\frac{1}{25}$ & $c_2 = \frac{26}{25}$. Finally, we have the zero-state response to the forcing function $x(n) = (4)^n u(n)$ in the form

$$y(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n \quad n \geq 0$$

The total response of the system.

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Q1 (b)

Sol:

$$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$

$$y(n) - 0.6y(n-1) + 0.8y(n-2) = x(n)$$

To obtain the homogenous equation set input $x(n) = 0$

$$y(n) - 0.6y(n-1) + 0.8y(n-2) = 0$$

$$y_h(n) = \lambda^n$$

subtitue the solution to the homogenous equation

$$\lambda^n - 0.6\lambda^{n-1} + 0.8\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.8) = 0$$

Thus the roots are $\lambda_1 = 0.2$ & $\lambda_2 = 0.4$

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n$$
$$= c_1(0.2)^n + c_2(0.4)^n \quad — ①$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

$$K(-1)^n u(n) - 4K(-1)^{n-1}u(n-1) + 4K(-1)^{n-2}u(n-1) = 0$$

for $n = 2$, $K(1+4+4) = 2$

$$K = \frac{2}{9}$$

The total solution is

$$y(n) = \left[c_1 \cdot 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n).$$

For intial condition we obtained

$$y(-1) = y(-2) = 0$$

$$c_1 + \frac{2}{9} = 0$$

$$\boxed{c_1 = -\frac{2}{9}}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 0$$

$$\boxed{c_2 = \frac{1}{3}}$$

$$\boxed{y(n) = \left[\frac{-2}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n\right] u(n)}$$

Q 2(a)

Sol:- First we express $x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$ in terms of positive powers of z, in the form

$$\frac{x(z)}{z} = \frac{z^2}{(2z-1)(z-1)^2}$$

$$\frac{x(z)}{z} = \frac{A_1}{2z-1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}.$$

Find A, B & C values

$A = 4$

$B = -3$

$C = -1$

Hence $\boxed{x(n) = \left[4(2)^n - 3 - n\right] u(n)}$

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Q2(b)
Sol:- First we eliminate the negative powers, by multiplying both numerator & denominator by z^2. Thus,

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

The poles of $X(z)$ are $P_1 = 1$ & $P_2 = 0.5$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

A very simple method to determine A_1 & A_2 is to multiply the equation by the denominator term $(z-1)(z-0.5)$. Thus we obtain.

$$z = (z-0.5)A_1 + (z-1)A_2$$

Now if we set $z = P_1 = 1$ in above equation, we eliminate the term involving A_2. Hence

$$1 = (1-0.5)A_1$$

So $\boxed{A_1 = 2}$

Now set $z = P_2 = 0.5$ and eliminates A_1 term.

$$0.5 = (0.5-1)A_2$$

$$\boxed{A_2 = -1}$$

Therefore the result of the partial-fraction expression is

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

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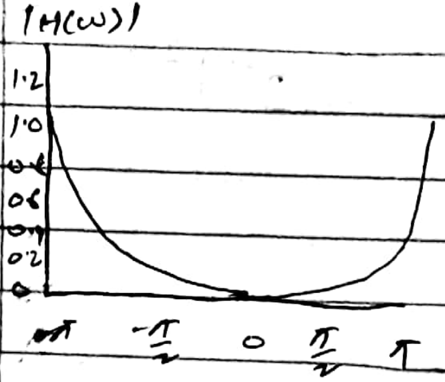
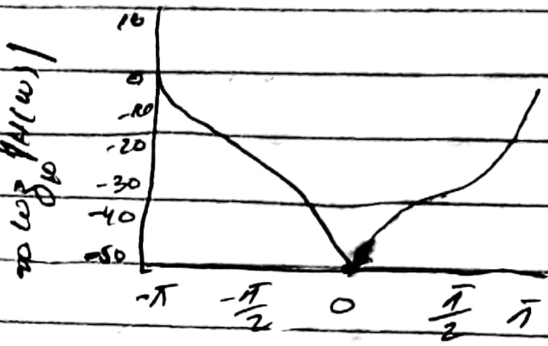
Q3
(a)
sol:

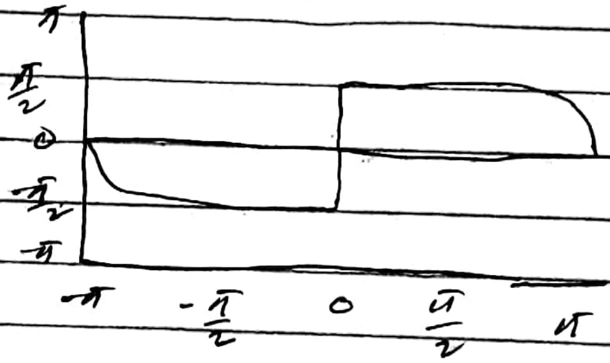
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

$$b_0 = (1-P)^2$$







At $\omega = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^4}{[(1-P/\sqrt{2})^2 + P^2/2]^2} = \frac{1}{2}$$

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or, equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$ satisfies this equation. Consequently the system function for desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

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Q3 (b)

Sol:- Clearly, the filter must have pole at

$$p_{1,2} = re^{j\pi/2}$$

and zeros at $z = 1$ and $z = -1$. consequently, the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\left(\frac{z^2-1}{z^2+r^2}\right)$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$= G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}.$$

or, equivalently.

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4.$$

The value of $r^2 = 0.7$ satisfisfies this equation. Therefore, The System function for the desired filter is

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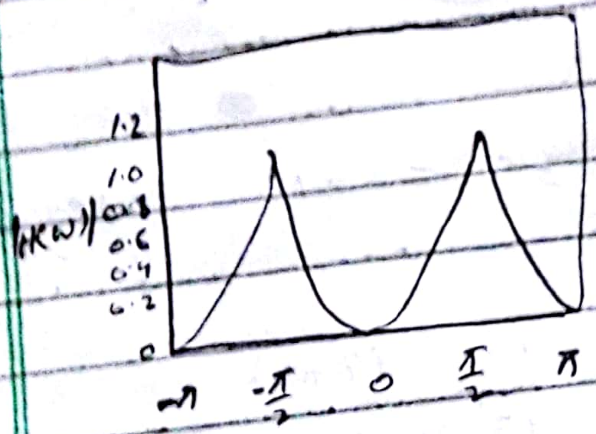
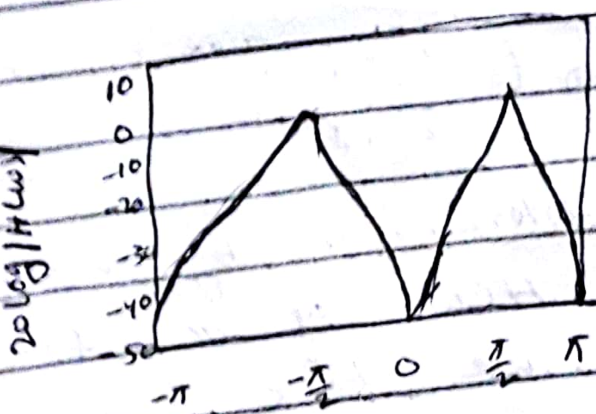
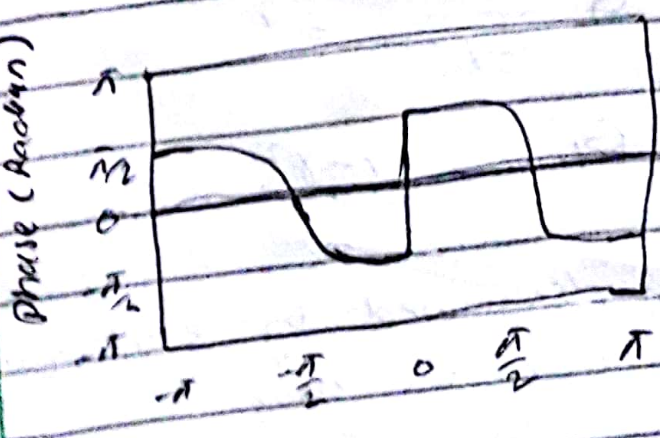
$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}$$

Its frequency response is illustrated in fig.







Q4
(a)
Sol:- The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated in fig for $L=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies $\omega_k = 2\pi k/N$, $k=0,1,\dots,N-1$. Hence

$$X(K) = \frac{1-e^{-j2\pi KL/N}}{1-e^{-j2\pi K/N}}$$

$$K = 0,1,2,\dots,N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$

If N is selected such that $N=L$ then the DFT becomes

$$X(K) = \begin{cases} L, & K=0 \\ 0, & K=1,2,\dots L-1 \end{cases}$$

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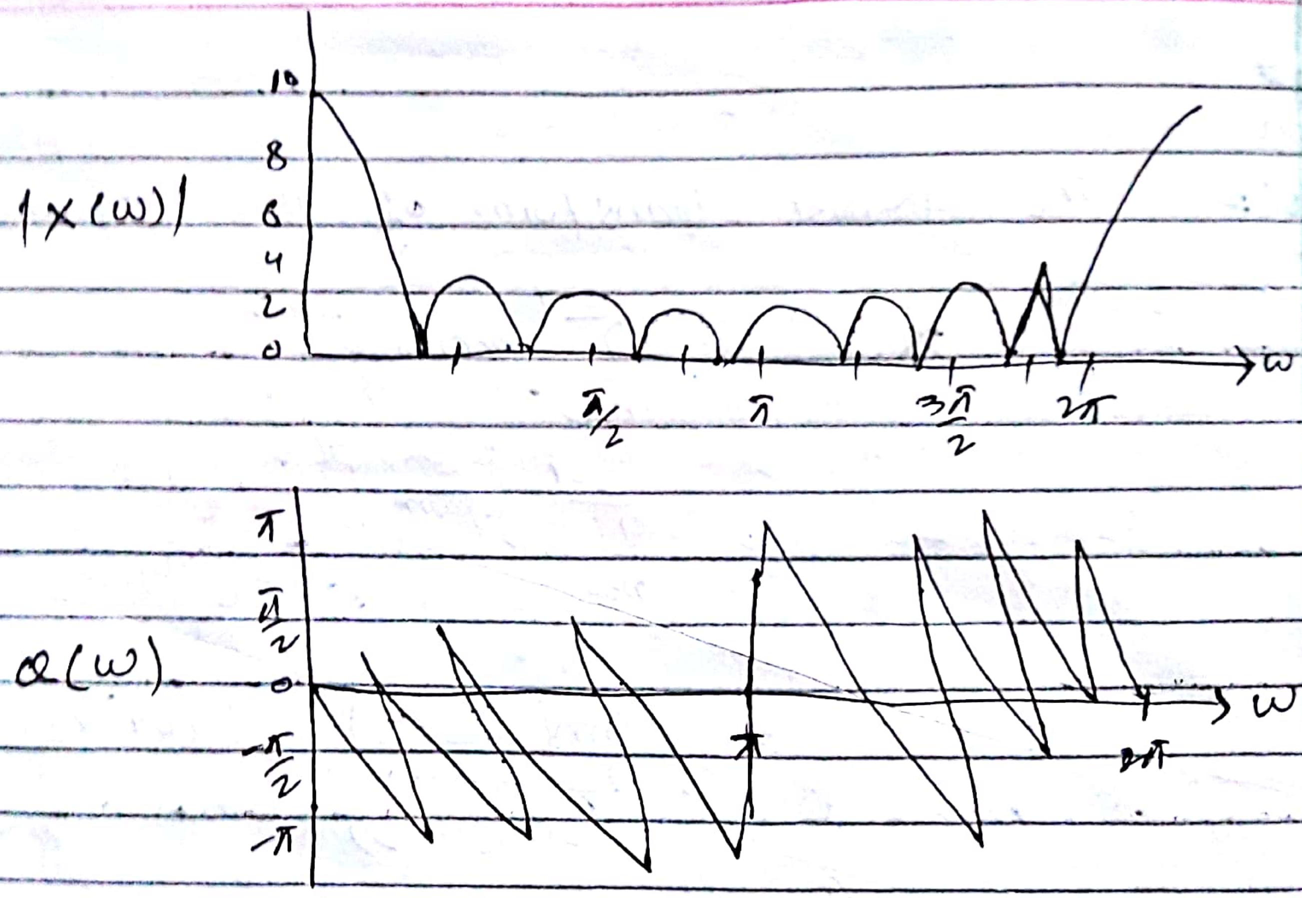

magnitude & phase characteristic

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Q4 (b)

Sol:- The first step is to determine the matrix W_4. By exploting the periodicity property of W_4 and the symmetry property.

$$W_N^{k+N/2} = -W_N^k$$

The matrix W_4 may be expressed as

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The IDFT of X_4 may be determined by conjugating the elements in W_4 to obtain W_4^* and then applying the formula $x_N = \frac{1}{N} W_N^* X_N$

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