Question No 1: Find the Solution of the following.

(a) $\int (x^2 e^x)\,dx$

<u>Sol:</u> $P(x) = x^2 =$ First Function

$e^x =$ Second Function

$\int x^2 e^x\,dx = x^2 e^x - \int 2x e^x\,dx$ — (1)

Now,

$\int x e^x\,dx = x e^x - \int 1 \cdot e^x\,dx$ — (2)

Substitute (2) in (1) to get

$\int x^2 e^x\,dx = x^2 e^x - 2\int x e^x\,dx$

$= x^2 e^x - 2(x e^x - e^x)$

$= x^2 e^x - 2x e^x + 2 e^x + c$ Ans.

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(b) $\int (1+3t) \cdot t^3 \, dt$

Sol: Given

$$\int (1+3t) \cdot t^3 \, dt$$

$$= \int (t^3 + 3t^4) \, dt$$

$$= \int t^3 dt + 3 \int t^4 dt$$

Integrate w-r-to t

$$= \frac{t^4}{4} + \frac{3t^5}{5} + c$$

$$= \int (1+3t) t^3 \cdot dt = \frac{1t^4}{4} + \frac{3t^5}{5} + c \quad \text{Ans}$$

(c) $\int (e^x - e^3) \, dx$

Sol: $= \int e^x dx - \int e^{3x} dx$

= Integrate w-r- to x

$$= e^x - \frac{e^{3x}}{\text{[illegible]}} + c \quad \text{Ans}$$

Question No 2: Find the derivative of the following.

(a)

$F(y) = x \cdot \sin x.$

Sol:

$F(y) = x \cdot \sin x$

$= \cancel{\frac{d}{dx} (x^2) \cdot \cos x}$

$F(y') = x' \cdot \sin x + x \cdot \sin x'$

$= \sin x + x \cos x$ Ans:

(b) $F(y) = x^2 \cdot \cos x.$

Sol: $F(y) = x^2 \cdot \cos x.$

$= \frac{d}{dx} x^2 \cdot \cos x + x^2 \cdot \frac{d}{dx} \cos x$

$= 2x \cdot \cos x + x^2 \cdot - \sin x$

$= 2x \cos x - x^2 \cdot \sin x$ Ans:

(c) $F(t) = z\cdot(2z-2)^2$

Sol:

$$z\cdot(2z-2)^2$$

$$= z\left[(2z)^2 + (2)^2 (z2)(2)\right]$$

$$= z\left\{4z^2 + 4 - 8z\right\}$$

$$= 4z^3 + 4z - 8z^2$$

By using power Rule.

$$= 4z^3 + 4z - 8z^2$$

$$= 3\cdot 4z^2 + 4 - (2\cdot 8z^1)$$

$$= 12z^2 + 4 - 16z$$

$$= 12z^2 - 16z + 4 \text{ Ans}$$

Question No 2:

Find the Taylor Series for $f(x) = e^{-6x}$ about $x = -4$.

Solution: Given

$$f(x) = e^{-6x} \text{ about } x = -4$$

As we know that from Taylor Series i.e

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) ---- + \frac{(x-a)^n}{n!} f'(n) \text{ --- } (1)$$

Here we will expand up to 3 terms than the Taylor series becomes.

$$f(x) = (f)(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} \text{ --- } (1)$$

$$f'(x) = -6e^{-6x}$$

$$f''(x) = 36e^{-6x}$$

$$f'''(x) = -216 e^{-6x}.$$

If $a = x = -4$ then

$f(-4) = e^{24} = 26.489 \times 10^{9}$

$f'(-4) = -6e^{24} = -158.9 \times 10^{9}$

$f''(-4) = 36e^{24} = 953.6 \times 10^{9}$

$f'''(-4) = -216e^{24} = -5721.6 \times 10^{9}$

From equation 1

$$f(4) = f(-4) + (x+4)(f')(-4) + \frac{(x+4)^2}{2!} f''(-4) \frac{(x+4)^3}{3!}$$

$$f'''(-4) + ----$$

we get.

$$e^{-6x} = 26.489 \times 10^{9} + (x+4)(-158.9 \times 10^{9}) + \frac{(x+4)}{2!}(953.6 \times 10^{9}) + \frac{(x+4)^3}{3!}(-5721.6 \times 10^{9})$$

$$e^{-6x} = \left\{ 26.489 - (x-4)(158.9) + \frac{(x+4)^2}{2!} \cdot (1953.6) - \frac{(x+4)^3}{3!}(5721.6) \right\} 10^{9}$$ Ans