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SUBJECT: DIFFERENTIAL EQUATIONS:

INSTRUCTOR:

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# Question No 1:

$$f(t) = 1+t \quad -\pi \le t \le \pi$$

Sol:

Here we use formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \longrightarrow eq \ 1$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}(1+t)\,dt$$

$$a_0 = \frac{1}{2\pi}\left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{\pi}\right)\right)$$

$$a_0 = \frac{1}{2\pi}\left(2\pi + \frac{2\pi^2}{2}\right)$$

$$a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)$$

$$\rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left(\left.(1+t)\frac{\sin nt}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

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$$a_n = -\frac{1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = -\frac{1}{n^2\pi}\left(-1-(-1)\right)$$

$$a_n = 0$$

$$\rightarrow b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt \frac{d}{dt}(1+t)\,dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left((1+t)(-\cos nt)\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\sin nx}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$$

$$b_n = -\frac{1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+(\pi))(\cos n(\pi))\right)$$

$$b_n = -\frac{1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos\pi - \cancel{\cos n\pi} + \pi\cos\pi\right)$$

$$b_n = -\frac{1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

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$b_n = \frac{2}{n}(-1)^{n+1}$

So equation becomes

$$f(x) = \frac{1}{2\pi}(2\pi + \pi^2) + 0 + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin t$$

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Question No 2:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen Values = ?

Sol:

Step 1:

We have

$(A - \lambda I)x = 0$   A = Given Matrix

Step 2:

The characteristic equation is given by

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

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Step 3 :

$$\lambda^3 - \left|\text{Sum of diagonal element}\right| \lambda^2 + \left|\text{Sum of Diagonal minors}\right| \lambda - |A| = 0 \quad \text{---(B)}$$

Sum of diagonal elements $= 1+1+2 = 4$

Sum of diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{---(C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= 0$

By putting values in B

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

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$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Using Quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 1 \\ b &= -4 \\ c &= -3 \end{aligned}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigenvalues

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$

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Question No 3:

Sol:

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \underline{R_4 \; R_2}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+4/5 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad \underline{-1/5 \times R_3}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5 \times R_3 \text{ and } 5 \times R_4$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad \underline{5R_3} \text{ and } \underline{5R_4}$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & 7 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/6 & 2/5 & 3/5 \\ 0 & 7 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad C_2 \times -5$$

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$$\left[\begin{array}{cccc|c} 1 & p & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 26/21 \\ 0 & 0 & 1 & 1 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & 0 & -\frac{31}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right.$$

$$(x, y, z, m) = \left(\frac{3}{4}, \frac{31}{21}, \frac{-11}{21}, \frac{1}{3}\right)$$

$x = \frac{3}{4}$

$y = \frac{31}{21}$

$z = -\frac{11}{21}$

$m = \frac{1}{3}$

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Question No 4:

$$u(x,t) = \sin(x+2t)$$

Sol:

$$u(x,t) = \sin(x+2t)$$

Differential w.r.t $x$ partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x+2t)$$

$$\partial u/\partial x = \cos(x+2t)\ \partial/\partial x\,(x+2t$$

$$\partial u/\partial x = \cos(x+2t)(1+0)$$

$$\partial u/\partial x = \cos(x+2t)$$

$$\partial^2 u/\partial x^2 = \partial/\partial x\ \cos(x+2t)$$

$$\partial^2 u/\partial x^2 = -\sin(x+2t)\cdot \partial/\partial x\,(x+2t)$$

$$\partial^2 u/\partial x^2 = -\sin(x+2t)(1+0)$$

$$\partial^2 u/\partial x^2 = -\sin(x+2t)$$

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and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t 't'

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)\,(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t)\,(0+2)$$

$$\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)$$

We know that one dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2\left[-\sin(x+2t)\right]$$

$$-4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

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For the orbitary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the orbitary constant

$$c = 2$$

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