Calculus

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Matrices And Determinants

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Exercise 9.1

Q. No. 1 Write The following matrics in tabular form.

[redacted] (i) $A = [a_{ij}]$ where $i = 1, 2, 3$ and $j = 1, 2, 3, 4$

Ans

(i) $\begin{bmatrix} 1, 2, 3 \end{bmatrix}$ (j) $\begin{bmatrix} 1, 2, 3, 4 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

(ii) $B = [b_{ij}]$, where $i = 1$ and $j = 1, 2, 3, 4$

Ans $\begin{bmatrix} 1 \end{bmatrix}$ $\begin{bmatrix} 1, & 2 & 3 & 4 \end{bmatrix}$

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \end{bmatrix}$$

(iii) $C = [c_{jk}]$, where $j = 1, 2, 3$

and $k = 1$

Ans $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 \end{bmatrix}$

$$\begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix}$$

Q No 2 write each sum as a single matrix:

(I) $\begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 3 & 0 \\ -2 & 1 & 0 \end{bmatrix}$

Ans $\begin{bmatrix} 2+6 & 1+3 & 4+0 \\ 3-2 & -1+1 & 0+0 \end{bmatrix}$

$\begin{bmatrix} 8 & 4 & 4 \\ 1 & 0 & 0 \end{bmatrix}$ $\boxed{\text{Ans}}$

(II) $\begin{bmatrix} 1 & 3 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 1 & 3 \end{bmatrix}$

Ans $\begin{bmatrix} 1+0 & 3-2 & 5+1 & 6+3 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 6 & 9 \end{bmatrix}$ Ans

Sol:-
$$\begin{bmatrix} 12 & 2 \\ 0 & -6 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -12 & -6 \\ 0 & -3 \\ 15 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -4 \\ 0 & -9 \\ 13 & 7 \end{bmatrix}$$

QH03:- show That $\begin{bmatrix} b_{11}-a_{11} & b_{12}-a_{12} \\ b_{21}-a_{21} & b_{22}-a_{22} \end{bmatrix}$ is a sol of The matrix equation $X+A=B$, where

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

Sol:- Let $B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

subtract 'B' to 'A'

$$B-A=\begin{bmatrix} b_{11}-a_{11} & b_{12}-a_{12} \\ b_{21}-a_{21} & b_{22}-a_{22} \end{bmatrix}$$

$X+A=B$

Let $X=\begin{bmatrix} b_{11}-a_{11} & b_{12}-a_{12} \\ b_{21}-a_{21} & b_{22}-a_{22} \end{bmatrix}$ $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$(X+A)=\begin{bmatrix} b_{11}-\cancel{a_{11}}+\cancel{a_{11}} & b_{11}-\cancel{a_{12}}+\cancel{a_{12}} \\ b_{21}-\cancel{a_{21}}+\cancel{a_{21}} & b_{22}-\cancel{a_{22}}+\cancel{a_{22}} \end{bmatrix}$$

$$B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Q. 4 Solve each of The following matrix equations:

(i)

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

Sol:. Let $X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

(ii)

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

Q Prob:- if $A=\begin{bmatrix}1 & 4\\2 & 1\end{bmatrix}$, $B=\begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix}$ $C=\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$

Find A^2+BC

sol:- $A^2 = A \cdot A + B \cdot C$

$$\begin{bmatrix}1 & 4\\2 & 1\end{bmatrix} \cdot \begin{bmatrix}1 & 4\\2 & 1\end{bmatrix} + \begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1+8 & 4+4\\2+2 & 8+1\end{bmatrix} + \begin{bmatrix}-3+0 & 0+4\\4+0 & 0+2\end{bmatrix}$$

$$= \begin{bmatrix}9 & 8\\4 & 9\end{bmatrix} + \begin{bmatrix}-3 & 4\\4 & 2\end{bmatrix}$$

$$= \begin{bmatrix}9-3 & 8+4\\4+4 & 9+2\end{bmatrix}$$

$$= \begin{bmatrix}6 & 12\\8 & 11\end{bmatrix}$$ Ans

Q7:- show That $A=\begin{bmatrix}-1 & 2\\0 & 1\end{bmatrix}$ & $B=\begin{bmatrix}1 & 0\\-1 & 2\end{bmatrix}$

Then $(A+B)(A+B) \neq A^2+2AB+B^2$

sol:- Taking L-H-S

$$A+B = \begin{bmatrix}-1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\-1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}0 & 2\\-1 & 3\end{bmatrix}$$

Q no 5 $5/2015$ That

$$\begin{vmatrix} l & a & a \\ a & l & a \\ a & a & l \end{vmatrix} = (2a+l)(l-a)^2$$

sol:- $\begin{vmatrix} l & a & a \\ a & l & a \\ a & a & l \end{vmatrix} =$ Taking L-H-S

Expand by R_1

$l(l^2-a^2)-a(al-a^2)+a(a^2-al)$

$l^3-a^2l-a^2l-\cancel{a^3}+\cancel{a^3}-a^2l$

$l^3-a^2l-a^2l-a^2l$

$l(l^2-a^2)-2a^2l$

$l(l-a)^2-2a^2l$

$(2a+l)(l-a)^2$

so L-H-S = R-H-S

Q#10b Prov That

$$\begin{vmatrix} a & b+c & a+b \\ b & c+a & b+c \\ c & a+b & c+a \end{vmatrix} = a^3+b^3+c^3-3abc$$

Sol:- Expend by $R_1 C_1$.

$$\begin{vmatrix} a & b+c & c+b \\ b & c+a & b+c \\ c & a+b & c+a \end{vmatrix}$$

$= \cancel{a(c^2+a^2-b^2+c^2)-b+c((bc+ba)-(bc+c^2))+c+b((ab+b^2)-(c+ac))}$

$\cancel{a(c^2+a^2-b^2+c^2)-(b+c)(bc+ba)}$

$= a((c^2+ac+ac+a^2)-(ba+b^2+ca+cb))-b((bc+ba+c^2+ca)-$

$(ca+cb+ba+b^2))+c((b^2+bc+cb+c^2)-(ac+a^2+bc+ba))$

$= (ac^2+a^2c+a^2c+a^3-ba^2-ab^2-ca^2+cba-b^2c-b^2a-bc^2-bca$

$+cb^2+b^2a+b^3+b^2c+bc^2+c^2b+c^3-ac^2-a^2c-bc^2-bac)$

$= a^3+b^3+c^3-3abc(\cancel{ac^2}+\cancel{a^2c}+\cancel{a^2c}-\cancel{ba^2}-\cancel{ab^2}-\cancel{ca^2}-\cancel{b^2c}-\cancel{b^2a}$
$-\cancel{bc^2}+\cancel{cb^2}+\cancel{b^2a}+\cancel{b^2c}+\cancel{bc^2}+\cancel{c^2b}-\cancel{ac^2}-\cancel{a^2c}-\cancel{bc^2})$

$= a^3+b^3+c^3-3abc$

so $L\text{-}H\text{-}S = R\text{-}H\text{-}S$

Q 7:- Find The value of x.

$$\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$$

sol:- Expand by R_3

$$x(4-3x) - 1(12+x) + 0(9+1) = -30$$

$$(4x - 3x^2) - (12+x) + 0$$

$$4x - 3x^2 - 12 \mp x$$

$$-3x^2 + 3x - 12 = 0 - 30$$

$$-3(x^2 - x) = -30 + 12$$

or

$$-3x(x-1) = -18$$

$-3x = -18$ or $x - 1 = -18$

$x = \dfrac{-18}{-3}$ $\qquad$ $x = -18 + 1$

$x = 6$ $\qquad$ $x = -17$

Ans

Hence $x = \frac{|A_x|}{|A|}$

Putting the values

$$\boxed{x = \frac{13}{5}}$$

$$y = \frac{|A_y|}{|A|} = \frac{3}{5}$$

(iii)
$$x - 2y + z = -1$$
$$3x + y - 2z = 4$$
$$y - z = 1$$

Sol:- Hence the determinant of the coefficient is:

$$|A| = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix}$$

Expend by C_1

$$1(-1+2) - 3(-2-1) + 0(4-1)$$
$$1 + 9 + 0$$
$$|A| = 10$$

For $|A_x|$ replace the column of $|A|$

with the corresponding constant $-1, 4, 1$
we have

$$|A_x| = \begin{vmatrix} -1 & -2 & 1 \\ 4 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$$

Expend by c_1

$-1(-1+2) - 4(2-1) + 1(4-1)$

$-1(1) - 4(1) + 1(3)$

$-1 - 4 + 3$

$|A_x| = -2$

similarly

$$|A_y| = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 4 & -2 \\ 0 & 1 & -1 \end{vmatrix}$$

Expend by c_1;

$1(-4+2) - 3(1-1) + 0(2-4)$

$-2 - 0 + 0$

$|A_y| = -2$

similarly

$$|A_z| = \begin{vmatrix} 1 & -2 & -1 \\ 3 & 1 & 4 \\ 0 & 1 & 1 \end{vmatrix}$$

Expend by c_1

$1(1-4) - 3(-1+2) + 0(-1+2)$

$-3 - 3 + 0$

$|A_z| = -6$

Hence

$$x = \frac{|A_x|}{|A|} = \frac{-2}{10} = -\frac{1}{5}$$

$$y = \frac{|A_y|}{|A|} = \frac{-2}{10} = -\frac{1}{5}$$

$$z = \frac{|A_z|}{|A|} = \frac{-6}{10} = -\frac{3}{5}$$

(v) $x+y+z=0$
$2x-y-4z=15$
$x-2y-z=7$

sol:- The determinant of The cofficient as

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -4 \\ 1 & -2 & 1 \end{vmatrix}$$

Expend by R_1

$1(-1-8)-1(2+4)+1(-4+1)$
$-9-6-3$
$|A| = -18$

We can Find $|A_x|$ determinant and Put Firs column as 0,15,7 we have

$$|A_x| = \begin{vmatrix} 0 & 1 & 1 \\ 15 & -1 & -4 \\ 7 & -2 & 1 \end{vmatrix}$$

Expend by R_1

$= 0(-1-4)-1(15+28)+1(-30+7)$
$0-43-23$
$|A_x| = -66$

Similarly

$$|A_y| = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 15 & -4 \\ 1 & 7 & 1 \end{vmatrix}$$ Expend by R_1

$1(15+28)-0(2+4)+1(14-15)$

$43-0-1$

$|Ay| = 42$

similarly

$$|Az| = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 15 \\ 1 & -2 & 7 \end{vmatrix}$$

Expand by R_1

$1(-7+30)-1(14-15)+0(-4+1)$

$23+1+0$

$|Az| = 24$

Hence

$$x = \frac{|Ax|}{|A|} = \frac{-66}{-18} = \frac{11}{3}$$

$$x = 11/3$$

$$Y = \frac{|Ay|}{|A|} = \frac{42}{-18} = -\frac{7}{3}$$

$$Y = -7/3$$

$$Z = \frac{|Az|}{|A|} = \frac{24}{-18} = -\frac{4}{3}$$

$$Z = -4/3$$

The solution set as:

Q 2 Which of The following matrix is symmetrix and skey-symmetrix.

(i) $\begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$

(i) sol:- if $A^t = A$ The A is symetrix ~~otherwise~~ matrix.

$$A = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$$

So $A = A^t$ so A is symmetrix.

(ii) sol:- if $A = -A^t$ Then A is called skey symetrix

$$A = \begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 0 & -3 & 5 \\ 3 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$$

Q3:- Find K such That The following matrices are singular.

(i) $\begin{vmatrix} K & 6 \\ 4 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & K \\ -4 & 2 & 6 \end{vmatrix}$

(i) sol :-

$\begin{vmatrix} K & 6 \\ 4 & 3 \end{vmatrix}$

$3K - 24 = 0$

$3K = 24$

$K = \frac{\cancel{24}^{8}}{\cancel{3}}$

$K = 8$

(ii) sol:-

$\begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & K \\ -4 & 2 & 6 \end{vmatrix}$

Expand by R_1

$1(24-2K) - 2(-18+4K) - 1(-6+16)$

$24 - 2K + 36 - 8K + 6 - 16 = 0$

$-10K + 50 = 0$

$-10K = 50$

$K = \frac{50}{10}$

$K = 5$ Ans

Where $(A+A^T)$ is symmetric matrix and $(A-A^T)$ is skew-symmetric metrix.

Q: 4 Difine diagonal matrix.

Ans: A square matrix is called a diagonal matrix if nondiagonal entries are all Zero The main diagonal can be constants or Zero. A diagnol matrix must fit The Following.

$$D \begin{bmatrix} d_{11} & 0 & 0 & \cdots & 0 \\ 0 & d_{22} & 0 & \cdots & 0 \\ 0 & 0 & d_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_{nn} \end{bmatrix}$$

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Objective type Exercies

Q1 Each question has four possible answer. chose The correct ans and encircle it.

(1) The order of The matrix $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ is:

{Ans} (c) 3×1

(2) The order of matrix $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is:

Ans {a} 1×3

(3) The matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called: Ans Null

(4) Two matrics A and B are conformabe for multiplication

(III) $\begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$

Sol $\begin{bmatrix} 4+6 \\ 3+0 \\ -1-1 \end{bmatrix}$

$= \begin{bmatrix} 10 \\ 3 \\ -3 \end{bmatrix}$

(IV) $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Sol $\begin{bmatrix} 2+0 & 3+0 & 4+0 \\ -1+0 & 6+0 & 2+0 \\ 1+0 & 0+0 & 3+0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 4 \\ -1 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix}$

(V) : $2 \begin{bmatrix} 6 & 1 \\ 0 & -3 \\ -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 4 & 2 \\ 0 & 1 \\ -5 & -1 \end{bmatrix}$

Sol

R-H-S

$$= \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

L-H-S

$$= \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0-2 \\ 0-1 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

Q. 4 (iii)

$$3x + \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

$$3\begin{bmatrix} \quad \end{bmatrix} + \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Q. 4 (iv)

$$x + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$x + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

let $x = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

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Q 5(i) $\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 2 \end{bmatrix}$

Sol:-

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3+0+(-1) & -3+2-2 \\ 0+(-1)+2 & 0-2+4 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$$

(ii) $\begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

sol:- $[3-4-4]$

$= [-5]$

(iv) $\begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

sol:- $\begin{bmatrix} -2-2+5 & 2-2+0 & 1+4-5 \\ -2-1+3 & 2-1+0 & 1+2-3 \\ -2-2+4 & 2-2+0 & 1+4-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(A+B)(A+B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cancel{+0}-2 & 0+6 \\ 0-3 & -2+9 \end{bmatrix}$$

$$(A+B)(A+B) = \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix} \longrightarrow \text{(1)}$$

Taking R-H-S

$$A^2 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+0 & -2+2 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A^2$$

$$2AB = \begin{bmatrix} -2 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2-4 & 0+8 \\ 0-2 & 0+4 \end{bmatrix} = \begin{bmatrix} -6 & 8 \\ -2 & 4 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ -1-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$A^2+2AB+B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -6 & 8 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+1 & 0+8+0 \\ 0-2-3 & 1+4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ -5 & 9 \end{bmatrix}$$

since $L-H-S \neq R-H-S$

$$\begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix} \neq \begin{bmatrix} 6 & 8 \\ -5 & 9 \end{bmatrix}$$

Q8 show That

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$$

sol:- $$= \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$= \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$$

$$\begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix} = \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$$

$L-H-S = R-H-S$

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EXERCISE 9.2

Q No1:- Expend The determinants

(i) $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix}$

sol:- Expend by R_1

$$= \begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix}$$

$$= 1(-3-4)-2(9+8)+0$$

$$= -7-2(17)$$

$$= -7-34$$

$$-41$$

(ii) $\begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$

sol:- Expend by R_1

$$\begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

$$x(\cancel{0}x^2-0)-0(0-0)+0(0-0)$$

$$x^3-0-0+0$$

$$= x^3$$

Ans

Q 2:- without Expanding verify That

(i) $\begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix}$

sol:- $\begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix}$

$(2)R_3 \begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -2 & 1 & 0 \end{vmatrix} = 0$

because R_1 & R_3 are same
by The properties of Determinant.

Q3:- show That

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1x+d_1 & c_2x+d_2 & c_3x+d_3 \end{vmatrix} = x \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

Sol:- Taking L-H-S

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1x & c_2x & c_3x \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

$$x \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

L-H-S = R·H-S

Q No 8:- use cramer rules to solve The system of equation.

(i) $x-y=2$
$x+4y=5$

sol:- Hence The determinant of The coefficients is:

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 4 \end{vmatrix}$$

$= 4+1 = 5$

$|A| = 5$

For $|A_x|$ replace The first column of $|A|$ with The corresponding constant 2, 5, we have

$$|A_x| = \begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix}$$

$8+5 = 13$

$|A_x| = 13$

similarly $|A_y| = \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix}$

$5-2 = 3$

$|A_y| = 3$

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EXERCISE 9.3

Q No1 which of the following matrix are singular or non singular.

(i) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix}$

(i)

sol:- so if $|A| = 0$ Then A is called singular

Hence

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

Expand by C_1

$1(-1+2) - 3(1+2) + 0(1+4)$

$= 1 - 9 + 0$

$= -8$

so A is non singuler

(ii) $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix}$

sol:- if $|A| = 0$ Then A is called singular otherwise A is non singular

so

$$|A| = \begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{vmatrix}$$

Expand by R_1

$1(24-10) - 2(-18+20) \cancel{+} -1(-6+16)$

$14 - 2(-2) - 1(10)$

$14 \mp 4 - 10$

$14 - 14$

$|A| = 0$

so A is singular matrix.

$$-A^t = -\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$A = -A^t$ so A is skew symmetric.

Q4:- Find the inverse if it exist of the following matrices.

(i) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

Sol:- (i) Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

So

$$A^{-1} = \frac{1}{|A|} \text{adj } A.$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$$

$$|A| = -1 - 6$$

$$|A| = -7$$

$$\text{adj of } A = \begin{bmatrix} -1 & -3 \\ 2 & 1 \end{bmatrix}$$

So

$$A^{-1} = \frac{1}{-7} \begin{bmatrix} -1 & -3 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} +\frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{1}{-7} \end{bmatrix}$$

Ans

So A^{-1} exist.

(iii) sol:- Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

We know That

$$A^{-1} = \frac{1}{|A|} \text{ adj of } A$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{vmatrix}$$

Expend by C_1

$$1(0-8)+1(4-6)+0(0-8)$$

$$-8-2+0$$

$$|A| = -10$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

adj $A =$

a_{11} $(0-8) = -8$

a_{12} $(-2-0) = 2$

a_{13} $3(-2-0) = -2$

a_{21} $-1(6-4) = -2$

a_{22} $+0-2 = 0-2$

a_{23} $4(3-2) = 4$

a_{31} $(0-8) = -8$

a_{32} $2(4+3) = -7$

a_{33} $(0+2) = 9$

$$adj\ A = \begin{bmatrix} -8 & 2 & -2 \\ -2 & -9 & -1 \\ -8 & -7 & 2 \end{bmatrix}$$

$$adj\ A^t = \begin{bmatrix} -8 & -2 & -8 \\ 2 & -9 & -7 \\ -2 & -1 & 2 \end{bmatrix}$$

so

$$A^{-1} = \frac{1}{|A|} \text{ adj of } A$$

putting the values

$$= \frac{1}{-10} \begin{bmatrix} -8 & -2 & -8 \\ 2 & -9 & -7 \\ -2 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4/5 & -\frac{1}{5} & 0 \\ 2/10 & 0 & 7/10 \\ 1/5 & 1/10 & -1/5 \end{bmatrix}$$

Ans

$$A^{-1} = \begin{bmatrix} 4/5 & -1/5 & -4/5 \\ -1/5 & -1/5 & 7/10 \\ 1/5 & 1/5 & -1/5 \end{bmatrix}$$

Ans

{Short Questions}

Q.1 Define raw and column vectors.

Ans In liner algebra, a column vector or column matrix is an m×1 That is a matrix consisting of a single column of m element.

$$x = \begin{bmatrix} x1 \\ x2 \\ \cdot \\ \cdot \\ \cdot \\ xm \end{bmatrix}$$

~~Simi~~ Similarly: a row vector or row matric is a 1×m matrix, consisting of a single row of m element

Throughout, boldface is used for the rows and column vectors. The transpose (indicated by T) of a row vector is a column vector

$$\begin{bmatrix} x_1 x_2 \cdots x_m \end{bmatrix}^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix},$$

~~and~~

Q. 2: Define identity matrix?

Ans <u>Identity matrix</u> I_n is a $n \times n$ square matrix with the main diagonal of 1's and all other elements are 0s.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

if A is a $m \times n$ matrix, Then

$$I_m A = A \text{ and } A I_n = A$$

if A is a $m \times n$ matrix, Then

$$AI_n = I_n A = A$$

Q.3 Define symmetric m?

Ans

A squre matrix A is called a symmetric matrix, if $A^T = A$

A squre matrix A is called a skew-symmetric matrix, if $A^T = -A$

Any square matrix cab be expressed as The sum of a symmetric and a skew-symmetric matrix.

$$A = \frac{A + A}{2} + \frac{A - A^T}{2},$$

if : Ans {A} No of columns in A = No of rows in B

(5) if The order of The matrix A is $p \times q$ and order of B is $q \times r$, Then order of AB will be: {c} $p \times r$

(6) In an identity matrix all The diagonal elements are:

{c] 1

(7) The value of determinant ~~are~~ ~~identical Then its value~~

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$ is :

(8) if two rows of a determinant are identical The its value is:

{b} Zero

(9) if $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ is a matrix, Then Cofactor of 4 is

{a} -2

(10) if all The elements of a row or a column are Zero, Then value of The determinant is:

{c} Zero

(11) value of m for which matrix $\begin{bmatrix} 2 & 3 \\ 6 & m \end{bmatrix}$ is singular.

{d} ~~equal~~ 9

(12) if [aij] and [bij] are of

{d} ~~equa~~

The sames order and $a_{ij} = b_{ij}$

Then matrix will be {d} equal

13. Matrix $[a_{ij}]_{m \times n}$ is a raw matrix if:

{c} $m = 1$

14. matrix $[c_{ij}]_{m \times n}$ is a rectanguler if:

{d} $m - n \neq 0$

(15) if $A = [a_{ij}]_{m \times n}$ is a scaler matrix

if: {d} ((a) and b) $a_{ij} = 0 \ \forall i \neq j$
$a_{ij} = k \ \forall i = j$

16 matrix $A = [a_{ij}]_{m \times n}$ is an edentity

matric if: {d} $\forall i = j, a_{ij} = 0$
$\forall i \neq j \ a_{ij} = 0$

17. Which matric can be tectangular

matric? {d}

18 if $A = [a_{ij}]$ Then order kA is:

* {a} ~~A+B=~~o $m \times n$

19 $(A-B)^2 = A^2 - 2AB + B^2$, if and only if: {b} $AB - BA = 0$

20 if A and B ARE symmetric, Then $AB =$ {b} $A^t B^t$