Date: ....

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Subject : Calculas and analytical Geo

Program : BS (CS)

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Date:

Q.1
Part (a)

Solution :

$$y = \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

Quotient rule

$$\frac{dy}{dx} = \frac{(x^3+1)\frac{d}{dx}(3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5)\frac{dy}{dx}(x^3+1)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(12x^3 - 6x^2) - (3x^4 - 2x^3 + 5)(3x^2)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{12x^6 - 6x^5 + 12x^3 - 6x^2 - 9x^6 + 6x^5 - 15x^2}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{12x^6 - 6x^5 + 12x^3 - 6x^2 - 9x^6 + 6x^5 - 15x^2}{x^6 + 2x^3 + 1}$$

$$\frac{dy}{dx} = \frac{3x^6 + 12x^3 - 21x^2}{x^6 + 2x^3 + 1}$$

$$\boxed{\frac{dy}{dx} = \frac{3x^6 + 12x^3 - 21x^2}{x^6 + 2x^3 + 1}}$$ Ans

Q1 Part (b)

Solution:

$$y = \frac{(x^3+1)^2}{x^3-1}$$

$$\frac{dy}{dx} = \frac{(x^3-1)\frac{d}{dx}\{(x^3+1)^2\} - (x^3+1)^2\frac{dy}{dx}(x^3-1)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3-1)\{2(x^3+1)\}\frac{d}{dx}(x^3+1) - (x^3+1)^2(3x^2)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3-1)(6x^3+2)(3x^2) - (x^6+2x^3+1)(3x^2)}{x^6-2x^3+1}$$

$$\frac{dy}{dx} = \frac{(6x^6+2x^3-6x^3-2)(3x^2) - (3x^8+6x^5+3x^2)}{x^6-2x^3+1}$$

$$\frac{dy}{dx} = \frac{18x^8+6x^5-18x^5-6x^2-3x^8-6x^5-3x^2}{x^6-2x^3+1}$$

$$\frac{dy}{dx} = \frac{18x^8+6x^5-18x^5-6x^2-3x^8-6x^5-3x^2}{x^6-2x^3+1}$$

$$\boxed{\frac{dy}{dx} = \frac{15x^8-18x^5-9x^2}{x^6-2x^3+1}}$$ Ans

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Q. 2

Part (a) Ans

Solution:-

$$y = \int \frac{1}{\sqrt{x^5}}\, dx$$

$$= \int \frac{1}{x^{5/2}}\, dx$$

$$= \int (x)^{-5/2}\, dx$$

$$= \frac{x^{-\frac{5}{2}+1}}{-\frac{2}{5}} + C$$

$$= \frac{x^{-4/2}}{-4/2} + C$$

$$= \frac{x^{-2}}{-2} + C$$

$$= -\frac{1}{2}\, x^{-2} + C$$

$$\boxed{= -\frac{1}{2} \quad \frac{1}{\sqrt{x^2}}}$$

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Q.2 Part (b)

Ans.

Solution :-

$$y = \int \frac{1}{(8x+7)^8} dx$$

$$= \int (8x+7)^8 dx$$

$$= \frac{(8x+7)^{-8+1}}{-8+1} \int (8x+7) dx$$

$$= -\frac{1}{7} (8x+7)^{-7} \left[\frac{8x^2}{2} + 7x\right] + c$$

$$= -\frac{1}{7(8x+7)^{-7}} (4x^2 + 7x) + C$$

$$= \boxed{-\frac{4x^2 + 7x}{7(8x+7)^7} + C}$$

Q.3

Part (a) Ans

Solutions

$$y = \int \frac{-x+9}{2x^2-8x+6}\,dx$$

$$\frac{-x+9}{2x^2-8x+6} = \frac{1}{2}\left[\frac{A}{x-3} + \frac{B}{(x-1)}\right]$$

$$= A(x-1) + B(x-3)$$

For A $x=3$

$$-3+9 = A(3-1) + B(3-3)$$

$$6 = A(2)$$

$$A = 3$$

For B $x=1$

$$-1+9 = A(1-1) + B(1-3)$$

$$8 = -2B$$

$$B = -4$$

$$= \frac{1}{2}\left[\frac{3}{x-3} - \frac{+4}{x-1}\right]$$

$$= \frac{1}{2}\int\left[\frac{3}{x-3} - \frac{4}{x-1}\right]dx$$

$$= \frac{3}{2}\ln(x-3) - 2\ln(x-1) + c$$

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Q3 Part (b)

Ans

Solution

$$\frac{4x^2+8x}{x^3(x^2+2x+3)}$$

Expand the numerator

$$\frac{4x(x+2)}{x^3(x^2+2x+3)}$$

$$= \frac{4x^2+8x}{x^3(x^2+2x+3}$$

The form of the partial fraction decomposition is

$$\frac{4x^2+8x}{x^3(x^2+2x+3)} = \frac{A}{x} + \frac{B}{x^2}$$

$$+ \frac{C}{x^3} + \frac{Dx+E}{x^2+2x+3}$$

Writ the right-hand side as a single fraction

$$\frac{4x^2+8x}{\cancel{x^3(x^2+2x+3)}} = x^3(Dx+E)$$

$$= \frac{\begin{array}{l} + x^2(x^2+2x+3)A \\ + x(x^2+2x+3)B \\ + (x^2+2x+3)C \end{array}}{\cancel{x^3(x^2+2x+3)}}$$

EMAN NOTES

$$4x^2+8x = x^3(Dx+E)$$
$$+x^2(x^2+2x+3)A$$
$$+x(x^2+2x+3)B$$
$$+(x^2+2x+3)C$$

Expand the right-hand side:

$$4x^2+8x = x^4A + x^4D$$
$$+2x^3A + x^3B + x^2E$$
$$+3x^2A + 2x^2B + x^2C$$
$$+3xB + 2xC + 3C$$

Collect up the like terms:

$$4x^2+8x = x^4(A+D)$$
$$+x^3(2A+B+E)$$
$$+x^2(3A+2B+C)$$
$$+x(3B+2C)+3C$$

The coefficients near the like terms should be equal, so the following system is obtained.

$$A+D=0$$
$$2A+B+E=0$$
$$3A+2B+C=4$$
$$3B+2C=8$$
$$3C=0$$

$$A=-\frac{4}{9},\ B=\frac{8}{3},\ C=0$$

$$D=\frac{4}{9},\ E=-\frac{16}{9}$$

Therefore,

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$$\frac{4x^2+8x}{x^3(x^2+2x+3)} = \frac{-\frac{4}{9}}{x}$$

$$+ \frac{\frac{8}{3}}{x^2} + \frac{0}{x^3} + \frac{\frac{4x}{9} - \frac{16}{9}}{x^2+2x+3}$$

$$= \frac{-\frac{4}{9}}{x} + \frac{\frac{8}{3}}{x^2} + \frac{\frac{4x}{9} - \frac{16}{9}}{x^2+2x+3}$$

$$\boxed{\frac{4x^2+8x}{x^3(x^2+2x+3)} = \frac{-\frac{4}{9}}{x} + \frac{\frac{8}{3}}{x^2} + \frac{\frac{4x}{9} - \frac{16}{9}}{x^2+2x+3}}$$

Answer

Q.4
Part (a)

Solution:

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} \rightarrow eq\ (1)$$

$$X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5-3 & 1+1 \\ -3-2 & 1-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} \text{ Ans.}$$

for Prove
put X in eq (1)

$$= \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 2-1 \\ -5+2 & -1+2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

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Q4 Part (b)

Solution:-

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5+0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} \rightarrow Eq(1)$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

for prove

Put X in Eq (1)

$$\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} -1 & -1 & -2+0 \\ -1 & +0 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

Q4 part (c)

Solution:-

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \rightarrow eq\ (1)$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

For prove
Put X in the eq (1)

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1+2 & -1+0 \\ 1+0 & 0+2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Q5
Ans

Solution :-

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

$$B+C = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$B+C = \begin{bmatrix} -3+1 & 2+0 \\ 4+0 & 0+2 \end{bmatrix}$$

$$B+C = \begin{bmatrix} -2 & 2 \\ 4 & 2 \end{bmatrix}$$

$$A^2+BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 4 & 2 \end{bmatrix}$$

$$A^2+BC = \begin{bmatrix} 9-2 & 8+2 \\ 4+4 & 9+2 \end{bmatrix}$$

$$A^2+BC = \begin{bmatrix} 7 & 10 \\ 8 & 11 \end{bmatrix}$$ Ans.