Iqra National University, Peshawar
Department of Electrical Engineering

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Final – Term Examination summer2020
Date:24/9/2020

Course Code: MTH 101 **Course Title**Linear Algebra
Prerequisite: NA **Instructor:** HIMAYTULLAH
Module: 1 **Program:** BEE **Total Marks:** 50 **Time Allowed:** -

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1. (a) . **Express** the equation of plane passing through the points A(2,-2,1) , B(-1,0,3), C(5,-3,4) Marks 5
PLO2 C2

(b) Marks 5
Express a pair of planes whose intersection is the given line,

PLO2
C2

Q2 . **illustrate** that L is linear transformation ? Marks 10

PLO1
C3

Q3 Using the matrix then **interpret** to decode the message 77 54 38 71 49 29 68 51 33 76 48 40 86 53 52 Marks 10
PLO1
C3

Q4 Find an equation of the plane passing through the point (-1, 3, 2) and perpendicular to the vector $n = (0, 1, -3)$ Marks 10
C3 PLO1

Q5 Find an Eigen values and Eigen vectors of matrix . Mark10
c3 plo1

Name : Fayaz ullah
ID : 13854
subject : Linear Algebra
Final term

Question 1

Part (a)

A(2, -2, 1), B(-1, 0, 3), C(5, -3, 4)

$\overrightarrow{P_1P_2} = (-3, 2, 2)$

$(-1, 0, 3) - (2, -2, 1)$

$\overrightarrow{P_1P_3} = (3, -1, 3)$

$(-1, 0, 3) - 2, +2, -1)$

$(-3, +2, 2)$

the perpendicular vector is

$m = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$, $\overrightarrow{P_1P_2} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\overrightarrow{P_1P_2} = \sqrt{(-1-2)^2}$

$$m = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$m = i\,(6+2) - j\,(-9-6) + k\,(3-6)$

$$m = 8i + 15j - 3k$$

$$m = (8, 15, -3)$$

Now,

$$P_1(x_0, y_0, z_0) = (2, -3, 1)$$

$$m(a, b, c) = (8, 15, -3)$$

So, equation of plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$8(x - 2) + 15(y + 2) - 3(z - 1) = 0$$

$$8x + 15y - 3z - 16 + 30 + 3 = 0$$

$$\boxed{8x + 15y - 3z + 17 = 0}$$

Part (b)

$$x = 2 - 3t \,, \quad y = 3 + t \,, \quad z = 2 - 4t$$

Sol:

$$x - 2 = -3t \,, \quad y - 3 = t \,, \quad z - 2 = -4t$$

$$t = \frac{x-2}{-3} \,, \quad t = \frac{y-3}{1} \,, \quad t = \frac{z-2}{-4}$$

So,

$$\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

~~the~~ given line intersection,

$$\frac{x-2}{-3} = \frac{y-3}{1} \quad \& \quad \frac{x-2}{-3} = \frac{z-2}{-4}$$

$$x - 2 = -3(y-3)$$

$$x - 2 = -3y + 9$$

$$\boxed{x + 3y - 11 = 0}$$

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$$\frac{u-2}{-3} = \frac{z-2}{-4}$$

$$-4(u+8) = -3z + 6$$

$$-4u + 3z + 8 - 6 = 0$$

$$\boxed{-4u + 3z + 2 = 0}$$

These are two planes.

Question 2

$L(x,y) = (x+1, y, x+y)$

illustrate if it is linear transformation

$L(x,y) = (x+1, y, x+y)$

It is linear if it satisfies,

(1): $L((x_1, y_1) + (x_2, y_2)) = L(x_1, y_1) + L(x_2, y_2)$

(2): $L(c(x_1, y_1)) = c[L(x_1, y_1)]$

$L((x_1, y_1) + (x_2, y_2)) = L((x_1 + x_2, y_1 + y_2))$

$L((x_1, y_1) + (x_2, y_2)) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

$\hookrightarrow$ eq (1)

Now finding

$L(x_1, y_1) = (x_1 + 1, y_1, x_1 + y_1)$

$L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$

$L(x_1, y_1) + L(x_2, y_2) = x_1 + x_2 + 1$

$L(x_1, y_1) + L(x_2, y_2) = (x_1 + 1, y_1, x_1 + y_1)$
$+ (x_2 + 1, y_2, x_2 + y_2)$

$L(x_1, y_1) + L(x_2, y_2) = (x_1 + x_2 + 2, y_1 + y_2,$
$x_1 + x_2 + y_1 + y_2)$

$\hookrightarrow$ eq 2

eq 1 is not equal to eq 2

eq 1 $\neq$ eq 2

so first property is not satisfies, therefore, $L(x, y)$ is not a linear transformation.

Question No 3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

message = 77 54 38 71 49 29
68 51 33 76 48
40 86 53 52

Break the message into five vectors

$$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}, \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}, \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix}, \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

Solve the equation

$$L(u_1) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax_1$$

for u_1

$$X_1 = A^{-1} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

Similarly

$$u_2 = A^{-1} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$u_3 = A^{-1}\begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$u_4 = A^{-1}\begin{bmatrix} 76 \\ 48 \\ 33 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$u_5 = A^{-1}\begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

Using our correspondence between letters and numbers, the following message is :

$$\boxed{\text{PHOTOGRAPHPLANS}}$$

Question No 4

(9)

Point $(-1, 3, 2)$

$n = (0, 1, -3)$

Sol:

$P_0(x_0, y_0, z_0) = (-1, 3, 2)$

$n(a, b, c) = (0, 1, -3)$

equation of plane

$$a(x-x_0) + b(y-y_0) + c(z-z_0)$$

$$0(x-(-1)) + 1(y-3) + (-3)(z-2)$$

$$0(x(1)) + 1(y-3) - 3(z-2)$$

$$0 + y - 3 - 3z + 6 = 0$$

$$y - 3z + 6 - 3 = 0$$

$$\boxed{y - 3z + 3 = 0}$$

Question No 5

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

Since we know that $AX = \lambda X$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \lambda \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 + u_2 \\ -2u_1 + 4u_2 \end{bmatrix} = \begin{bmatrix} \lambda u_1 \\ \lambda u_2 \end{bmatrix}$$

then

$$u_1 + u_2 = \lambda u_1 \quad \text{— (i)}$$

$$-2u_1 + 4u_2 = \lambda u_2 \quad \text{— (ii)}$$

So,

$$u_1 - \lambda u_1 + u_2 = 0$$

$$= (1-\lambda)u_1 + u_2 = 0$$

&

$$-2u_1 + 4u_2 - \lambda u_2 = 0$$

$$-2u_1 + (4-\lambda)u_2 = 0$$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

characteristic equation

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda)+2 = 0$$

$$4-\lambda-4\lambda+\lambda^2+2 = 0$$

$$\lambda^2-5\lambda+6 = 0$$

$$\lambda^2-3\lambda-2\lambda+6 = 0$$

$$\lambda(\lambda-3)-2(\lambda-3) = 0$$

$$(\lambda-3)(\lambda-2) = 0$$

$$\lambda-3 = 0 \quad , \quad \lambda-2 = 0$$

$$\boxed{\lambda = 3} \quad , \quad \boxed{\lambda = 2}$$

These are eigen values

Now to find eigen vectors of $\lambda_1 = 3$ put in (i) & (ii)

$$u_1 + u_2 = 3u_1 \quad \text{—— (1)}$$

$$\Rightarrow -2u_1 + u_2 = 0$$

$$-2u_1 + 4u_2 = 3u_2 \quad \text{— (ii)}$$

$$-2u_1 + u_2 = 0$$

$$2u_1 - u_2 = 0$$

$$u_1 = \frac{1}{2} u_2$$

let $u_2 = r$

where $r \neq 0$

So,

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} r \\ r \end{bmatrix}$$

eigen vector for $\lambda_2 = 2$ put in (i) & (ii)

$$u_1 + u_2 = 2u_1 \quad \text{— (i)}$$
$$-2u_1 + 4u_2 = 3u_2 \quad \text{— (ii)}$$

$$\Rightarrow -u_1 + u_2 = 0 \quad \text{— (i)}$$

$$\Rightarrow u_1 - u_2 = 0$$

$$\Rightarrow u_1 = u_2$$

$\Rightarrow -2u_1 + 4u_2 = 2u_1$ — (ii)

$\Rightarrow -2u_1 + 2u_2 = 0$

$\Rightarrow u_1 - u_2 = 0$

$u_1 = u_2$

$u_1 = r$, then $u_2 = r$

So,

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$$