Name = M. Shayan

ID = 15085

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Instructor = Himayat ullah sir

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Q1 (a) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

Identify the $(3,2)$ entry of AB.

Ans **Solution:**

$\text{row}_3(A) \cdot \text{column}_2(B)$

$= \begin{bmatrix} 0, 1, -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

$= (0)(4) + (1)(-1) + (-2)(2)$

$= 0 - 1 - 4$

$\boxed{A.B = -5 \text{ Ans}}$

Q1
(b)

Solution so

(A)

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$
$$a_3 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now

$$(x_1, y_1) = (1, 3)$$
$$(x_2, y_3) = (2, 4)$$
$$(x_3, y_1) = (3, 4)$$

put in

$$a_2 + a_1 + a_0 = 3$$
$$4a_2 + 2a_1 + a_0 = 4$$
$$9a_2 + 3a_1 + a_0 = 4$$

$$A_3 = \begin{bmatrix} 1 & 1 & 1, & 3 \\ 4 & 2 & 1, & 4 \\ 9 & 3 & 1, & 4 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 1 & 1, & 3 \\ 0 & -2 & -3, & -8 \\ 0 & -6 & -8, & -20 \end{bmatrix} \begin{matrix} R_2 - 4R_1 \\ R_3 - 9R_1 \end{matrix}$$

$$\overset{R}{\rightarrow}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 - 3R_2$$

So

$a_2 + a_1 + a_0 = 3$ —— ①

$2a_1 - 3a_0 = -8$ —— ②

$\boxed{a_0 = 4}$ put in 2.

$-2a_1 - 12 = -8 \Rightarrow \boxed{a_2 = 1}$

So $y = x^2 - 2x + 4$

Q2 (a) If A and B are $n \times n$ matrices where $|A| = 2$ and $|B| = -3$, Calculate $|A^{-1} B^{T}|$.

Ans Solution:

since $|A^{-1} B^{t}| = |A^{-1}| |B^{t}|$

$= \frac{1}{|A|} \cdot |B|$ because: $|B^{t}| = |B|$

So $|A^{-1} B^{t}| = \frac{1}{|A|} |B|$

$= \frac{1}{2} \cdot 3 = \frac{3}{2}$ Ans

Q2 (b) Estimate the linear System of equation

$$x + y + 2z = -1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

Solution:-

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ 3 \end{bmatrix}$$

$$A.B = \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & 2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\sim R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & -5 & 6 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\sim R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 3 & 0 \\ 0 & -2 & -5 & 6 \end{array}\right] R_2 + 4$$

$$\sim R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 6 \end{array}\right] R_3 + 2R_1$$

So

$$x + y + 2z = -1 \quad \text{——} \ (1)$$

$$y + 3z = 0 \quad \text{——} \ (2)$$

$$z = 6 \quad \text{——} \ (3)$$

put $z = 6$ in eq (2)

$$y + 3(6) = 6$$

$$y + 18 = 6$$

$$y = -18$$

put $y = -18$, $z = 6$

$$x - 18 + 2(6) = -1$$

$$x - 18 + 12 = -1$$

$$x - 6 = -1$$

$$x = -1 + 6$$

$$x = 5$$

verification

$$5 - 18 + 12 = -1$$

$$17 - 18 = -1$$

$$\boxed{-1 = -1}$$ proof.

Q3 Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Ans **Solution:**

$$|A| = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

$$= 3 \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2 \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-4-6) + 2(-15-2) + 1(0-6)$$

$$|A| = \boxed{-94}$$

Now

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = \boxed{-18}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = -15-2 = -17 = (-1)(-17) = \boxed{17}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6 = (-1)(6) = \boxed{-6}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = 6 = (-1)(6) = \boxed{-6}$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -9-1 = -10 = (1)(-10) = \boxed{-10}$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 0+2 = (-)(2) = \boxed{-2}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -4-6 = -10 = (1)(-10) = \boxed{-10}$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = 6-5 = 1 = (-1)(1) = \boxed{-1}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix} = 18+10 = 28 = (1)(28) = \boxed{28}$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$