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Subject = Linear Algebra

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Q1  
(a)

Let  $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

Identify the  $(?, 2)$  entry of  $AB$ .

Ans

Solution so

$$\text{row}_3(A) \cdot \text{column}_2(B)$$

$$= [0, 1, -2] \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$= (0)(4) + (1)(-1) + (-2)(2)$$

$$= 0 - 1 - 4$$

$$\boxed{A \cdot B = -5 \text{ Ans}}$$

Q 1

(b)

Solution go

$$\begin{aligned} \text{(A)} \quad a_2 x_1^2 + a_1 x_1 + a_0 &= y_1 \\ a_2 x_2^2 + a_1 x_2 + a_0 &= y_2 \\ a_2 x_3^2 + a_1 x_3 + a_0 &= y_3 \end{aligned}$$

Now

$$(x_1, y_1) = (1, 3)$$

$$(x_2, y_2) = (2, 4)$$

$$(x_3, y_3) = (3, 4)$$

Put in

$$a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 4$$

$$A_3 = \begin{bmatrix} 1 & 1 & 1 & , & 3 \\ 4 & 2 & 1 & , & 4 \\ 9 & 3 & 1 & , & 4 \end{bmatrix}$$

$$\xrightarrow{R} \begin{bmatrix} 1 & 1 & 1 & , & 3 \\ 0 & -2 & -3 & , & -8 \\ 0 & -6 & -8 & , & -20 \end{bmatrix} \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$R) \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array} \right] R_1 - 3R_2$$

So

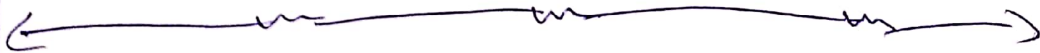
$$a_2 + a_1 + a_0 = 3 \quad \text{--- (1)}$$

$$2a_1 - 3a_0 = -8 \quad \text{--- (2)}$$

$$\boxed{a_0 = 4} \quad \text{put in 2.}$$

$$-2a_1 - 12 = -8 \Rightarrow \boxed{a_2 = 1}$$

$$\text{So } y = x^2 - 2x + 4$$



Q2  
(a)

If  $A$  and  $B$  are  $n \times n$  matrices where  $|A| = 2$  and  $|B| = -3$ , calculate  $|A^{-1} B^T|$ .

Ans

Solution so

$$\text{since } |A^{-1} B^T| = |A^{-1}| |B^T|$$

$$= \frac{1}{|A|} |B| \quad \text{because: } |B^T| = |B|$$

$$\text{So } |A^{-1} B^T| = \frac{1}{|A|} |B|$$

$$= \frac{1}{2} \cdot (-3) = -\frac{3}{2} \quad \underline{\underline{\text{Ans}}}$$





Q2  
(b)

Estimate the linear system of equation

$$x + y + 2z = -1$$

$$x - 2y + z = -5$$

$$3x + y + z = 3$$

Solution go

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ 3 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 & 1 & 2 & : & -1 \\ 1 & 2 & 1 & : & -5 \\ 3 & 1 & 1 & : & 3 \end{bmatrix}$$

$$\begin{matrix} \text{R}_2 - \text{R}_1 \\ \text{R}_3 - 3\text{R}_1 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & : & -1 \\ 0 & -3 & -1 & : & -4 \\ 0 & -2 & -5 & : & 6 \end{bmatrix}$$

$$\begin{matrix} \text{R}_2 + 4 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & : & -1 \\ 0 & 1 & 3 & : & 0 \\ 0 & -2 & -5 & : & 6 \end{bmatrix}$$

$$\begin{matrix} \text{R}_3 + 2\text{R}_1 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & : & -1 \\ 0 & 1 & 3 & : & 0 \\ 0 & 0 & 1 & : & 6 \end{bmatrix}$$

So

$$x + y + 2z = -1 \quad \text{--- (1)}$$

$$y + 3z = 0 \quad \text{--- (2)}$$

$$z = 6 \quad \text{--- (3)}$$

Put  $z = 6$  in eq (2)

$$y + 3(6) = 0$$

$$y + 18 = 0$$

$$y = -18$$

Put  $y = -18, z = 6$ 

$$x - 18 + 2(6) = -1$$

$$x - 18 + 12 = -1$$

$$x - 6 = -1$$

$$x = -1 + 6$$

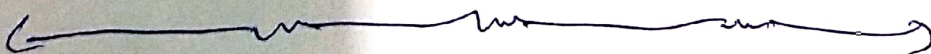
$$x = 5$$

Verification

$$5 - 18 + 12 = -1$$

$$17 - 18 = -1$$

$$\boxed{-1 = -1} \text{ proof.}$$



Q3

Find  $A^{-1}$  where  $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Ans

Solution :-

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$= 3 \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2 \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-4-6) + 2(-15-2) + 1(0-6)$$

$$|A| = \boxed{-94}$$

Now

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = \boxed{-18}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = \cancel{-15-2} = -17 = (-1)(-17) = \boxed{17}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6 = (-1)(6) = \boxed{-6}$$



$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = 6 = (-1)(6) = \boxed{-6}$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -9 - 1 = -10 = (1)(-10) = \boxed{-10}$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 0 + 2 = (-1)(2) = \boxed{-2}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -4 - 6 = -10 = (1)(-10) = \boxed{-10}$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = 6 - 5 = 1 = (-1)(1) = \boxed{-1}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix} = 18 + 10 = 28 = (1)(28) = \boxed{28}$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$

