Q1)

Solution:~

Coordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3K$

or $OQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 1K) - (4i + 1j + 3K)$

$= -3i + 1j + 1K$ —— ①

Now distance between P & $Q = |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11}$ ⟶ ②

Let M be the point which divided PQ in ratio 1:3, then by ratio theorem Position vector of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3K) + (1)(i + 2j + 4K)}{1 + 3}$$

$$= \frac{12i + 3j + 9K + i + 2j + 4K}{4}$$

$$= \frac{13i + 5j + 13K}{4} \text{ —— ③}$$

Hense eq ①, ② & ③ are the required solution.

Q2)

solution:~

$$2x^2+x \,\overline{)\,4x^3+10x+4}$$

$$\underline{\pm 4x^3 \qquad\qquad \pm 2x^2}$$

$$-2x^2+10x+4$$

$$\underline{+2x^2 \mp x}$$

$$11x+4$$

So $2x-1+\dfrac{11x+4}{2x^2+x} - \dfrac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1 + \int \frac{11x+4}{2x^2+x} \quad - (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \quad - (2)$$

Now Find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad — (A)$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)}$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1)+Bx \quad — (3)$$

Put $x=0$ in eq (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in eq (3)

$$11(-1/2)+4 = B(-1/2)$$

$$\frac{-11}{2}+4 = \frac{-B}{2}$$

$$\frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$-3 = -B$

$\Rightarrow \boxed{B = 3}$

Putting values of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Putting these values in (2)

$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1|$ put value in (1)

$$\int \frac{4x^3+10x-4}{2x^2+x}\,dx = x^2 - x + 4\ln(x) + \frac{3}{2}\ln|2x+1| + C$$

Ans

Q3)

a) $\int_0^2 x^2 e^x \, dx$

Now ~~find~~ first find Integration

$= \int x^2 e^x \, dx$

$= x^2 \int e^x dx - \int \left(e^x dx \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) dx$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(e^x dx \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$

$= x^2 e^x - 2x e^x + 2 e^x$

Now put limits

$= \left| x^2 e^x - 2x e^x + 2 e^x \right|_0^2$

$= (2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0)$

$= (\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2) = \boxed{2e^2 - 2}$ Ans

(b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

First find Integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \rightarrow ①$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2y = \frac{1}{\sqrt{x}}\,dx}$$ Put in ①

$$\int \sin(y)\,(2\,dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

Put limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2\left(\cos\sqrt{2} - \cos 1\right)$$

$$= -2\cos\sqrt{2} + 2\cos(1)$$ Ans

Q4)

Solution:~

The laplace eq in 3 dis

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{— (A)}$$

So $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial^2 u}{\partial x^2} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \quad \text{— (1)}$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{du}{dy} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{d^2u}{dy^2} = -\left[y\left(-3/\cancel{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}\left(\cancel{2}y\right)+\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{d^2u}{dy^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2}\left(x^2+y^2+z^2\right)^{-3/2} \quad — ②$$

$$\frac{du}{dz} = -1/\cancel{2}\left(x^2+y^2+z^2\right)^{-3/2}\left(\cancel{2}z\right)$$

$$\frac{du}{dy} = -z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{d^2u}{dz^2} = 3z^2\left(x^2+y^2+z^2\right)^{-5/2}-\left(x^2+y^2+z^2\right)^{-3/2} \quad — ③$$

Putting eq ①, ② & ③ in eq ④

$$3x^2\left(x^2+y+z^2\right)^{-5/2}-\left(x^2+y^2+z^2\right)^{-3/2}+3y^2\left(x^2+y^2+z^2\right)^{-5/2}-\left(x^2+y^2+z^2\right)^{-3/2}$$

$$+3z^2\left(x^2+y^2+z^2\right)^{-5/2}-\left(x^2+y^2+z^2\right)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-x^2-y^2-z^2+3y^2-x^2-y^2-z^2+3z^2-x^2-y^2-z^2\right]$$

$$= (x^2+y^2+z)^{-5/2}(0) = 0$$

so the given $u(x,y,z)$ is solution of laplace equation.