Name: Sajid Ahmad

ID: 12671

Programe: BE(E)

Subject: DSP

Module: 10th Semester.

1

1 #1 A

Sol:- $y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$

The Characteristic equation is

$$\lambda^2-4\lambda+4=0$$

$$\lambda=2,2.$$ Hence

$$y_h(n)=C_1 2^n+C_2 n 2^n$$

The particular Solution is

$$y_p(n)=K(-1)^n u(n).$$

Substituting this solution into the difference equation we obtain

$$K(-1)^n u(n)-4K(-1)^{n-1}u(n-1)+4K(-1)^{n-2}u(n-2)=(-1)^n u(n)-(-1)^{n-1}u(n-1)$$

For $n=2, K(1+4+4)=2 \Rightarrow K=2/9$. The Total Solution is

$$y(n)=\left[C_1 2^n+C_2 n 2^n+2/9(-1)^n\right]u(n)$$

From the initial Condition, we obtain $y(0)=1, y(1)=2$ then

$$C_1+2/9=1$$

$$\Rightarrow C_1=7/9$$

$$2C_1+2C_2-2/9=2$$

$$\Rightarrow C_2=1/3$$

So put in equation:

$$y(n)=\left[\frac{7}{9}2^n+\frac{1}{3}n2^n+2/9(-1)^n\right]u(n).$$

Ans.

Name : Sajid Ahmad Program : BE(E)

ID : [redacted] 12671 ② Subject : DSP

Q#1 B

$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - 2(n-2)$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ hence,

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence, $c_1 + c_2 = 2$ and

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}$$

Those equations yield

$$c_1 = \frac{10}{3}, \; c_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$$

The Step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n (2^{n+1} - 1)\, u(n) - \frac{1}{3}\left(\frac{1}{5}\right)^n (5^{n+1} - 1)\, u(n)$$

Name: 8' Sajid Ahmad
ID: 12671

Programme: BE(E)
Subject: DSP

Q2: A

Determine the causal signal $x(n)$ having Z-transform
$$x(z) = 1/(1-2z^{-1})(1-z^{-1})^2$$

Sol:-
$$x(z) = 1/(1-2z^{-1})(1-z^{-1})^2$$

by partial function methods.

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad \text{--- (1)}$$

put $z = 1$

$$1 = A(1-0)^2 + B(1-2)(1-1) + C(1)(1-2)$$

$$1 = 0 + 0 - C$$

$$\boxed{C = -1}$$

Put $z = 2$ in equation (1)

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$$

$$1 = \frac{A}{4} + 0 + 0 \qquad \boxed{A = 4}$$

Name : Sajid Ahmad　　　　　　　　　　　　　　　　　　　　Porogrome: BE(E)

ID : 12671　　　　　　　　　　4　　　　　　　　　　　　Subject: DSP

Put $z = 3$ in equation 1

$$1 = A\left(1-\tfrac{1}{2}\right)^2 + B\left(1-\tfrac{2}{3}\right)\left(1-\tfrac{1}{3}\right) + C\left(\tfrac{1}{3}\right)\left(1-\tfrac{2}{3}\right)$$

$$1 = A\left(\frac{4}{4}\right) + B\left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right) + C\left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right)$$

$$1 = \frac{4A}{9} + \frac{2B}{9} + \frac{1C}{9}$$

$$1 = \tfrac{4}{9}(4) + \frac{2}{9}(B) - \tfrac{1}{9}$$

$$1 = \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$= \frac{4}{9} \times \frac{9}{2} = B$$

$$\boxed{-3 = B}$$

Hence $x(n) = [4(2)^n - 3 - n]\, u(n)$ **Ans.**

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Name : Sajid Ahmad

ID : 12671

Programme: BE(E)

Subject: DSP

Module: 10th Semester

5

Q#2 (b)

$$X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

Sol:

Using the complex inversion integral we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

where C is a circle at a radius greater than $|a|$. we shall evalute this integral with $f(z) = z^n$. we distingush two cases.

① If $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. the only pole inside C is $z = a$. Hence.

② If $n < 0$, $f(z) = z^n$ has an nth-order pole at $z = 0$ which is also inside C. thus there are ~~contributio~~ contributions from both poles for $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

Name: Sajid Ahmad
ID: 12671

Program: BE(E)
Subject: DSP
Module: 10th Semester

If $n = -2$, we have

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$. Thus

$$x(n) = a^n u(n).$$

Name: Sajid Ahmad

ID: 12671

Programe: BE(E)

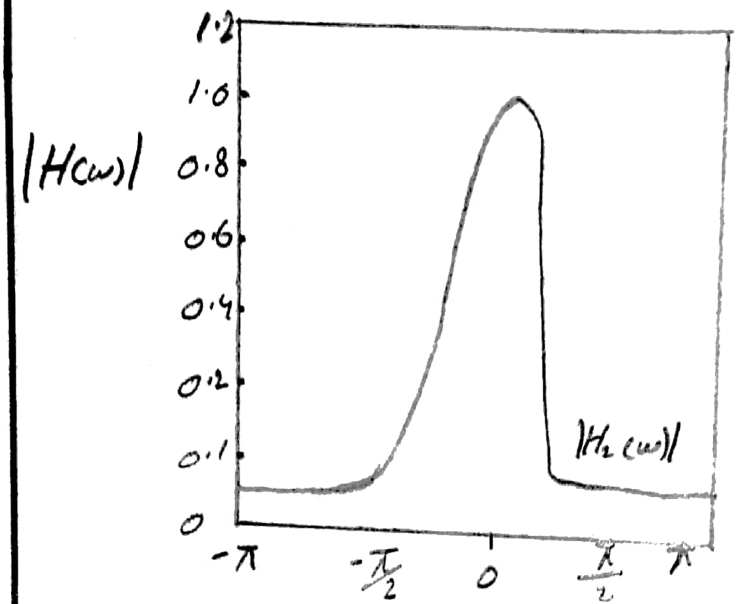
Subject: DSP

Module: 10th Semester

7

Q# 3 (a) A two pole low pass filter has the system response.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$



Sol:

At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$

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Name : Sajid Ahmad

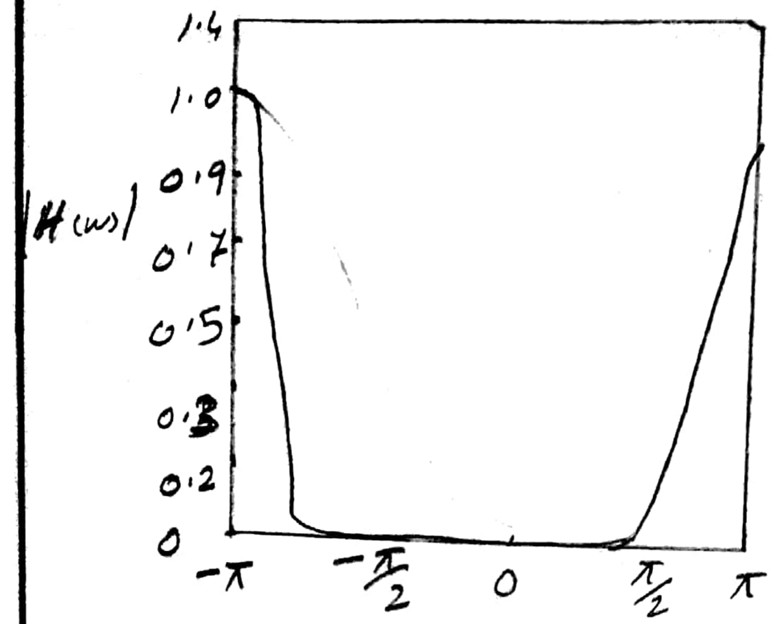
ID : 12671

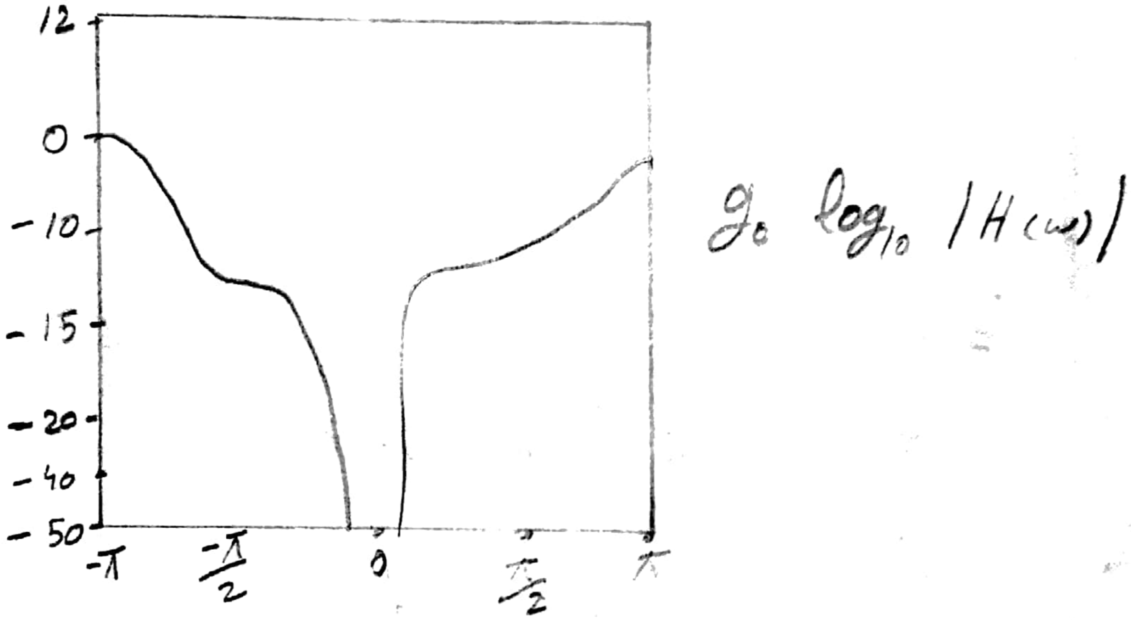
8

Programe: BE(E)

Subject : DSP

Module : 10th Semester





At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

Name: Sajid Ahmad
ID: 12671

Programme: BE(E)
Subject: DSP
Module: 10th semester

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^4}{[(1-P/\sqrt{2})^2 + P^2/2]^2} = \frac{1}{2}$$

equivalently

$$\sqrt{2}\,(1-P)^2 = 1 + P^2 - \sqrt{2}\,P$$

the value of $P = 0.32$ satisfies this equation consequently the system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

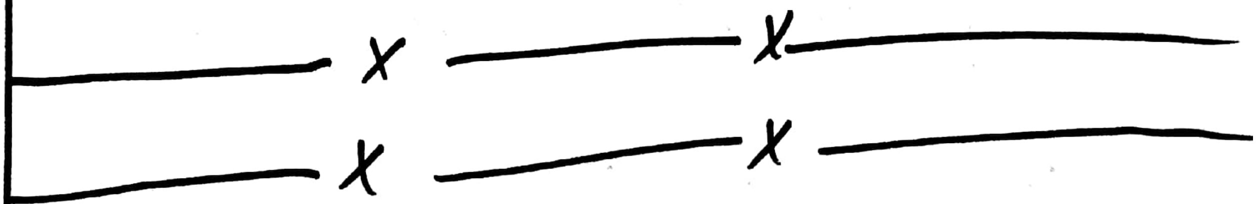
the same principles can applied for the design of bandpass filter Basically the band pass filter should contain one or more pairs of complex conjugate poles

Name: Sajid Ahmad
ID: 12671

Program: BE (E)
Subject: DSP
Module: 10th Semester

near the ~~poles~~ unit circle in the vicinity of the frequency band that constitues the passband of the filter.

~~the~~



Name: Sajid Ahmad
ID: 12671

Programme: BE(E)
Subject: DSP
Module: 10^{th} Semester

Q#3 (B) Design a two-pole bandpass filter that has the center of its passband at $w = \pi/2$ zero in its frequency response characteristics $w = 0$ and $w = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $w = \frac{4\pi}{9}$.

Sol:

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z = 1$ and $z = -1$

Consequently the system function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2+r^2}$$

frequency response $H(w)$ of the filter at $w = \pi/2$ thus we have

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

Name: Sajid Ahmad
ID: 12671

Programme: BE(E)
Subjet: DSP
Module: 10th Semster

$$G = \frac{1-r^2}{2}$$

the value of is determined by evaluating $H(w)$ at $w = \frac{4\pi}{9}$. then we have.

$$H\left(\frac{4\pi}{9}\right)\Big|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos\left(\frac{8\pi}{9}\right)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r)^2 = 1-1.88r^2+r^4$$

the value of $r^2 = 0.7$ Satifies the equation

for the desired filter

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

so $H(z) = 0.15[(1-z^{-2})/(1+0.7z^{-2})]$

Name: Sajid Ahmad
ID: 12671

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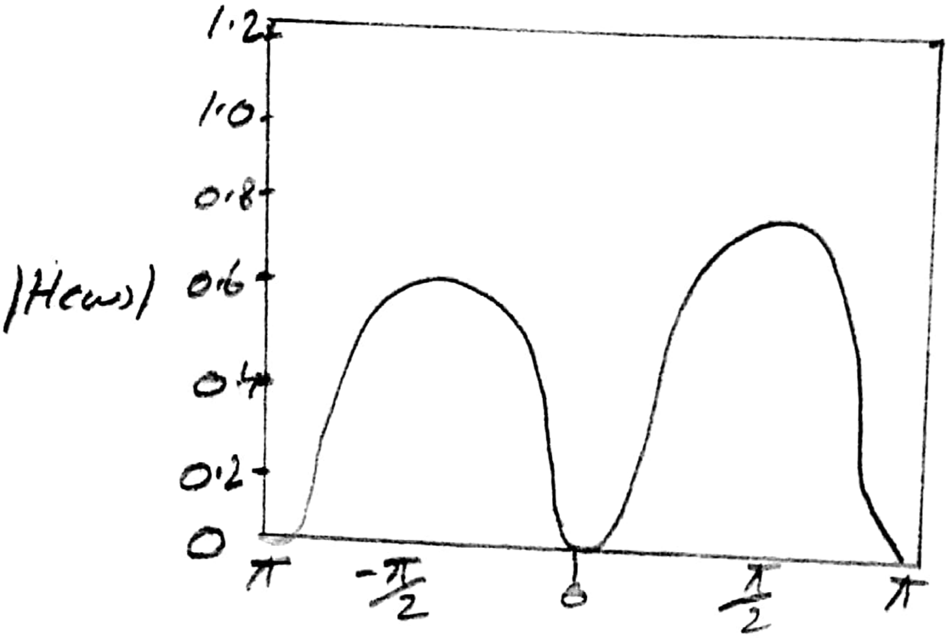
Subject: DSP
Module: 10th Semester

13



It should be emphased that the main purpose of foregoing methodlogy for designing simple digital filter by pole zero placement is to provide insight into the effect the poles and zeros have on the frequency response characteristic of system the medthodology is not intended as a good method for desiging digital filter with well-specified passband and stopband characteristics systematic method for the design.

x x
x x

Name: Sajid Ahmad
ID: 12671
Program: BE
Subject: DSP
Module: 10th Semester

(14)

Q#4 (A)

$$x(n)=\begin{cases}1, & 0\le n\le L-1\\ 0 & \text{otherwise}\end{cases}$$

Determine the N-point DFT of this sequence for $N \geq L$.

Solution:⇒ The fourier Transform of this sequence is:

$$X(w)=\sum_{n=0}^{L-1} x(n)\, e^{jwn}$$

$$=\sum_{n=0}^{L-1} e^{-jwN} = \frac{1-e^{-jwN}}{1-e^{-jw}}$$

$$=\frac{\sin(wL/2)}{\sin(w/2)}\, e^{-jw(L-1)/2}$$

The magnitude and phase of $X(w)$ are illustrated for $L=10$ the N-point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequency.

$wk = 2\pi K/N,\ K=0,1,\ldots,N-1$. Hence

$$X(K)=\frac{1-e^{-j2\pi K^2/N}}{1-e^{-j8\pi K/N}} \quad K=0,1\ldots N-1$$

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Name : Sajid Ahmad

ID: 12671

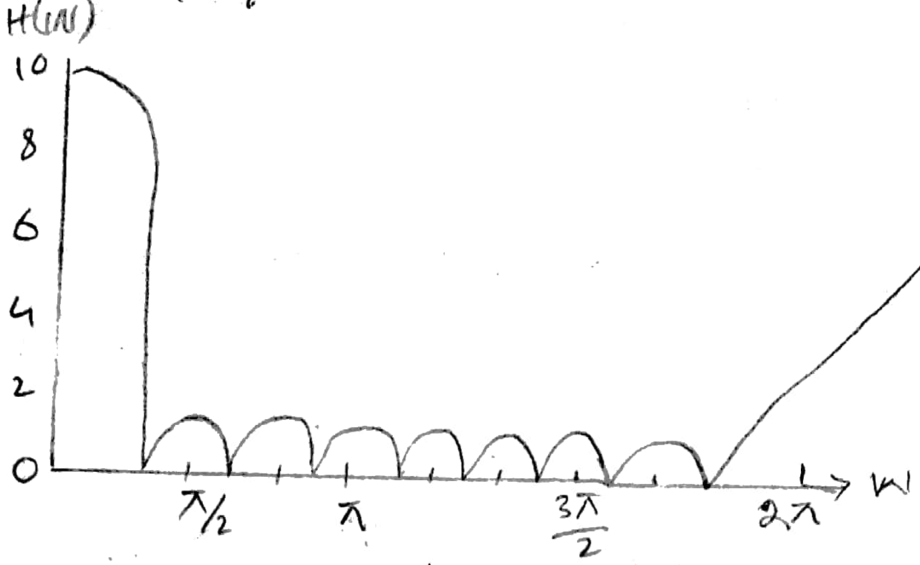
Programme : BE (E)

Subject : DSP

Module : 10th Semester

15

$$\frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$



If N is selected such that $N=L$ then the DFT becomes.

$$X(K) = \begin{cases} L & K=0 \\ 0 & K=1,2 \cdots L-1 \end{cases}$$

Thus There is only one non zero value in the DFT.

Since $X(w)=0$ at the reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT. In magnitude and phase for $L=10$, $N=50$ & $N=100$ as one will conclude by comparing these spectra with the continious spectrum $X(w)$.

Name : Sajid Ahmad

ID : 12671

Program : BE(E)

Subject : DSP

Q4:
(B)

Perform the circular convolution of the following two sequences. Solve the problem step by step.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Sol: Sequence consist of four non zero points

m = 0 we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$$x_3(0) = 14$$

for m=1 we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

$$x_3(1) = 16$$

for m=2 we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

$$x_3(2) = 14$$

For m=3 we have

$$x_3(3) = \sum_{n=0}^{3} x_1(3)\, x_2((3-n))_4$$

$$x_3(3) = 16$$

$$x_3(n) = \{14, 16, 14, 16\}$$

$$x_3(m) = \sum_{n=0}^{N-1} x_2(n)\, x_1((m-n))_N \quad m = 0, 1, \ldots N-1.$$

The following example series to illustrate the computation of $x_3(n)$ by means of the DFT & IDFT

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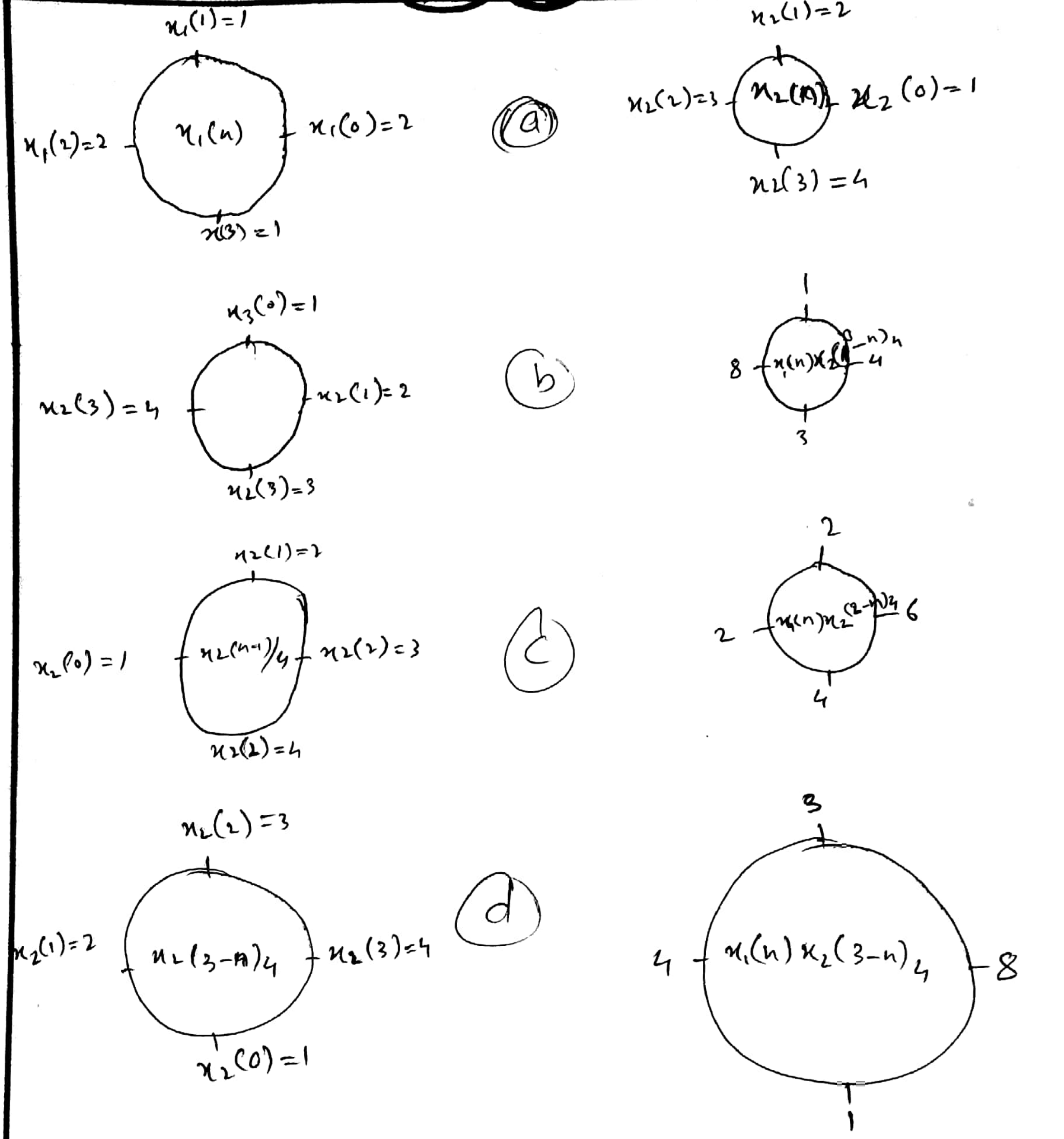
Name : Sajid Ahmad Programme : BE(CE)
ID : 12671 Subject : DSP
Module 10th Semester



Folded Sequence

product Sequence

The end