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Paper = Linear Algebra

Program = Software Engineering

Semester = 2nd
Section = "B"

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Q.No(1)

Answer

Solution: ID = 16453 $ID_3 = 4$

Putting in equation

$$x_1 - 4x_2 \quad x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 \quad -5x_3 = 10$$

Now ~~you~~ we changing in Matrix form

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

Now we ~~multiply~~ divided row two by 2

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -10 & 5 \end{array}\right] \begin{array}{l} \frac{1}{2}R_2 \\ -5R_1 \end{array}$$

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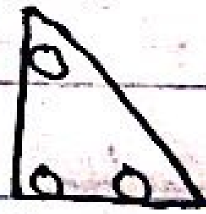
Consisten ⇝ At least one solution.

$$\begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -10 & 5 \end{bmatrix}$$

Lower triangle like 
all zero(0)
This is perfect triangle.
there is unique & consistent
b/c $x_1 = 1$

$x_2 = 1$

$x_3 = -10$

[illegible]

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Question No(2)

Answer

Solution = ID = 16453

4th ID = 5 Putting.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} \qquad |A| \neq 0$$

$$= 3\begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 3(-7+10) - 4(14-25) + 5(-4+5)$$

$$= 3(3) - 4(-11) + 5(1)$$

$$= 9 + 44 + 5$$

$$= \boxed{58}$$ Hence inverse exists.

Now we find Adjoint.

In adjoint first find ~~co-ffector~~ cofactor

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then transfoser

Adj ~~A~~

Cofactor of first element.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{11} = (-1)^2 (-7+10)$$
$$a_{11} = 3$$

$$a_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} \Rightarrow -1(14-25) = +11$$

$$a_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} \Rightarrow 1(-4+5)$$
$$= 1$$

$$a_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} \Rightarrow -1(-7+10) \Rightarrow -3$$

$$a_{22} = (-1)^{4} \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} \Rightarrow 1(21-25) = -4$$

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$$a_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} \Rightarrow -1(-6-20) = 26$$

$$a_{31} = (-1)^{4} \begin{vmatrix} 4 & 5 \\ -1 & 5 \end{vmatrix} \Rightarrow 1(20+5) = 25$$

$$a_{32} = (-1)^{5} \begin{vmatrix} 3 & 5 \\ 2 & 5 \end{vmatrix} \Rightarrow -1(15-10) = -5$$

$$a_{33} = (-1)^{6} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} \Rightarrow 1(-3-8) = -11$$

Now

$$\text{Adj}A = \begin{bmatrix} 3 & 11 & 1 \\ -3 & -4 & 26 \\ 25 & -5 & -11 \end{bmatrix}$$

Now take transpose

$$\begin{bmatrix} 3 & -3 & 25 \\ 11 & -4 & -5 \\ 1 & 26 & -11 \end{bmatrix}$$

Putting in this formula $A^{-1} = \frac{\text{Adj}A}{|A|}$

$$A^{-1} = \frac{1}{58} \begin{bmatrix} 3 & -3 & 25 \\ 11 & -4 & -5 \\ 1 & 26 & -11 \end{bmatrix}$$ Ans.

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Q No 3

Answer

Solution =>

$$2x + 2y + 4z = 18$$
$$x + 3xy + 2z = 13$$
$$3x + 2y - 3z = 14$$

Now changing into Matrix form.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 2 & 2 & 4 & 18 \\ 3 & 2 & -3 & 14 \end{array}\right]$$ interchanging R_1 into Row_2

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 1 & 1 & 2 & 9 \\ 3 & 2 & -3 & 14 \end{array}\right] -3R_3$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & 1 & 2 & 16 \\ 0 & 0 & -3 & 10 \end{array}\right] \begin{array}{l} \frac{1}{2}R_2 + 3R_3 \\ R_1 - \frac{1}{2}R \end{array}$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 11 \end{array}\right]$$

Hence ~~Now~~ $x = 9$

$y = 6$

$z = 11$

Q No(4)

Answer

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix} = 0$$

First we find eigan value

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$(4-\lambda)(3-\lambda)(1-\lambda)$

$4-\lambda \Rightarrow \lambda = 4$
$3-\lambda \Rightarrow \lambda = 3$
$\lambda = 1$

$\cancel{\lambda = \lambda}$
$\lambda = 4, 3, 1$

Now we find eigen vector when $\lambda = 4$

Let $x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be ibi eigen vector.

$\cancel{A = 4}$

$[A - 4I]\, x_1 = 0$

"I" mean identity.

$$\begin{bmatrix} 4-4 & 2 & -2 \\ -5 & 3-4 & 2 \\ -2 & 4 & 1-4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -2 \\ 5 & -1 & 2 \\ -2 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 & 2 \\ 0 & 2 & -2 \\ -2 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ Change R_1 to R

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$$\begin{bmatrix} 1 & -\frac{1}{5} & \frac{2}{5} \\ 0 & 2 & -2 \\ -2 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{5} & \frac{2}{5} \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{matrix} \frac{1}{5}R_1 + R_3 \\ \frac{1}{2}R_2 + 3R_2 \end{matrix}$$

Hence Rank of the matrix is 2

A deteriminate = 2

Matrix ⇒ 3 X 3

Linear independed = 3 - 2

$\boxed{L.I = 1}$

Hence Linear ~~ide~~ independing value & eigen vector is equal (1)

$x + y - 3z = 0$

$-4y + 8z = 0$

$y = 2z$ Putting in above equation

$x + 2z - 3z = 0$

$\boxed{x = 2}$

eigen. $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

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$\lambda_1 = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 - 2R_1$, $R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ vector $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Rank = 1
Matrix = 3
L.I = 3 - 1 = 2

$x + y + z = 0$
let $z = 0$, $y = 1$, $x = -1$

vector $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

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$|P| = 4$

$a_{11} = -1 \quad a_{12} = +1 \,, \; a_{13} = 3$

$a_{21} = 3 \,, \; a_{22} = 5 \,, \; a_{23} = 2$

$a_{31} = 1 \,, \; a_{32} = -2 \,, \; a_{33} = 3$

$$\begin{bmatrix} -1 & 1 & 3 \\ 3 & 5 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

Now take ~~transpose~~ transpose

$$\begin{bmatrix} -1 & 3 & 1 \\ 1 & 5 & -2 \\ 3 & 2 & 3 \end{bmatrix}$$

$$P^{-1} = \frac{1}{4} \begin{bmatrix} -1 & 3 & 1 \\ 1 & 5 & -2 \\ 3 & 2 & 3 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{4} \begin{bmatrix} -1 & 3 & 1 \\ 1 & 5 & -2 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 3 & 1 \\ 1 & 5 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Hence they are diagonalizable matrix.

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Question No (5)

Solution =>

$3x_1 + 5x_2 - 4x_3 = 0$
$-3x_1 - 25x_2 + 4x_3 = 0$
$6x_1 + x_2 - 8x_3 = 0$

First we change into an augmented Matrix.

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \frac{1}{3}R_1$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & -22 & 7 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] +3R_2$$

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$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -13 & 0 \end{array}\right] \begin{array}{l} \frac{1}{4}R_2 + 3R_3 \\ R_3 + 3R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -4 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] +13R_3$$

$\rightarrow$ Free variable.

$x_1 + \frac{5}{3}x_2 - 4x_3 = 0$

$0x \quad x_2 \quad 4x_3 = 0$

$0x_3 = 0$

$x_1 - 4x_3 = 0$

$x_2 + 4x_3 = 0$

$x_3 = 0x_3$

$x_1 = 4x_3$

$x_2 = 4x_3$

$x_3 = 0x_3$

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$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_3 \\ 4x_3 \\ x_3 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

Homogeneous equacton does not unique Solution.

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Question No (5)(6)

Answer

Solution :-)

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 6 & 9 & 0 \\ 1 & 3 & 4 & 0 \end{bmatrix} \quad -3R_2$$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 6 & 9 & 0 \\ 0 & 2 & 3 & 0 \end{bmatrix} \quad -1R_3$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 2 & 3 & 0 \\ 0 & 6 & 9 & 0 \end{bmatrix} \quad \text{Changing } R_2 \text{ to } R_3$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 6 & 9 & 0 \end{bmatrix} \quad \cdot \frac{1}{2} R_2$$

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$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} -6R_3$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} \frac{1}{2}R_1 + 3R_3 \\ R_1 + 3R_2 \\ \frac{1}{3}R_3 \end{matrix}$$

So

$$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$ normal form

Hence Rank of this matrix is 2 ~~b/c~~ b/c 2 is manimium matrix which value are non zero.

The End