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SUBJECT:. DIFFERENTIAL EQUATION.

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Differential Equation

Q1):- Solve the following objective type questions:-

i):- The order of matrix A is $m \times p$ and the order of B is $p \times n$. Then the order of matrix AB is $m \times n$.

ii):- The number of non-zero rows in an Echelon form is one.

iii):- If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a =$ 8.

iv):- If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$\boxed{i^2 = -1}$$

$$= -2i^2 - i^2$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$= 3$ Answer.

②

v):- The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is : Scalar Matrix.

A is a Scalar Matrix because Non-diagonal elements are zero and the diagonal elements are same. So, it is a Scalar Matrix.

vi):- Solution of $\frac{dy}{dx} + 2xy = y$?

Solution:-

$$\frac{dy}{dx} + 2xy = y.$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

Taking y common.

we get,

$$\frac{dy}{dx} = y(1-2x).$$

$$\frac{dy}{y} = (1-2x)\,dx.$$

Integration both sides.

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx.$$

$$\ln y = \int 1\,dx - \int 2x\,dx.$$

P.T.O

$$= \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$= \ln y = x - x^2 + c$$

$$= e^{\ln y} = e^{x - x^2 + c}$$

$$\boxed{y = e^{x(1-x)+c}}$$

vii):- The order and degree of differential equation,

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Solution:-

Degree = 3

Order = 2

viii):- The order and degree of differential equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

Degree = one

Order = 2.

(4)

ix):- The Differential equation $2\frac{dy}{dx} + x^2 y = 2x + 3$, $y(0) = 5$ is?

Solution:-

$$2y' + x^2 y = x^2 + 3,$$
$$y(0) = 5.$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

$$\mu = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6} y = \frac{1}{2} e^{x^3/6} (x^2+3).$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + c}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2}$$

$$y(0) = 3/2$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$$

Answer

P.T.O

x) :- $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Solution :-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by C_1

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$= 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$

$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b.$

$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$

$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2.$

$= a^2(c-b) + b^2(a-c) + c^2(b-a).$

Answer

Differential Equation.

QNo:-2):-i):- Express the Determinant.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2).$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^2cb^3 - a^3b^2c$$

Common is abc.

$$= abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b).$$

$$= abc[bc(c-b) - ac(c-a) + ab(b-a)]$$

Answer

Q2) ii):. Find the Eigen Value. $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$.

Solution:.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic Equation $\longrightarrow |A-\lambda I| = 0 \longrightarrow (A)$.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now Take Determinant, $|A-\lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand it by R_1,

$$= 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow (B)$$

$\because$ Again,
$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1,

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big)+1\big((-1)(2-\lambda)-(-1)(-1)\big)$$
$$-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \longrightarrow (a)$$

$$= +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$= -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$= -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$= \boxed{-\lambda^2+6\lambda-8} \rightarrow b$

$\rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$

Expand by C_1

$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$

$= -[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)]$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$= \boxed{-\lambda^2+6\lambda-8} \rightarrow c$

Put a, b & c in B

$(2-\lambda)[-\lambda^3+8\lambda^2-18\lambda+8]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8$

$-\lambda^2+16\lambda-8$

$\because = \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda$
$+ 6\lambda + 16 - 16$

$= \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By Synthatic Division.

We Gets

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization method,

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)(\lambda-4)$

$\lambda = 4, \lambda = 4$

$\boxed{\lambda_1 = 0}, \boxed{\lambda_2 = 2}, \boxed{\lambda_3 = 4}, \boxed{\lambda_4 = 4}$

Answer

Differential Equation

Q.No.:3):- The Rate of change in the form of differential Equation is given by.

$(x^2+3y^2)\,dx - 2xy\,dy = 0$. Find the general solution at $x = 2$ and $y = 6$.

Solution:.

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$= (x^2+3y^2)\,dx = 2xy\,dy.$$

Dividing both sides by $2xy\,dx$

We Gets

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x\cancel{^2}}{2\cancel{x}y} + \frac{3y\cancel{^2}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow (*)$$

Let $y = vx$.

$dy = v\,dx + x\,dv.$

Dividing by dx.

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a)$$

Putting (a) in eqn (*).

$$v + x\frac{dv}{dx} = 1/2\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = 1/2\left[\frac{1}{v} + 3v\right]$$

X ing both sides by 2.

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v.$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v.$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v.$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

X ing both sides by dx/dv.

we gets

$$2x\,dv = \frac{1+v^2}{v}\,dx.$$

• X ing both sides by $\frac{v}{x(1+v^2)}$ we get,

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx.$$

Take "$\int$" on both sides.

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{dx}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Taking "e" on both sides.

$$e^{\ln(1+v^2)} = e^{\ln|xc|}$$

$$1 + v^2 = xc$$

Putting $v = \frac{y}{x}$.

$$1 + \left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2 + y^2 = x^3 c \longrightarrow \text{equ } (a').$$

Putting $x = 2$, $y = 6$ in equation (a').

$4 + 36 = 8c$

$c = \frac{40}{8}$

$\boxed{c = 5}$ → Putting in equ (a')

we get,

$x^2 + y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking $\sqrt{\ }$ on both sides.

$\boxed{y = +x\sqrt{5x-1}, \ y = -x\sqrt{5x-1}}$

or.

$\boxed{y = \pm x\sqrt{5x-1}}$

Answer.