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Mid – Term Examination summer2020
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Course Code: MTH 203 **Course Title:** Differential equation
Prerequisite: CALCULUS **Instructor:** HIMAYATULLAH
Module: 3 **Program:** BEE **Total Marks:** 30 **Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	. Estimate the general solution of $y^{/} = (x+2)y^2$.	Marks 5 PLO1 C2
	(b)	. Estimate_the general solution of $y^{/} = (y+9x)^2$.	Marks 5 PLO1 C2
Q2	(a)	. Estimate the general solution of $x^3dx + y^3dy = 0$	Marks 10 PLO1 C2
Q 3	(a)	Find the general solution $4y'' - 20y' + 25y = 0$	Marks 5 PLO1 C2
	(b)	Estimate general solution of $4y'' - 6y' - 7y = 0$.	Marks 5 PLO1 C2

①

Name: Syed M. Zahon

Differential Equation. ID: 12595

Q1) Estimate The general Solution of

$$y' = (x+2)y^2$$

Solution

$$y' = (x+2)y^2$$

$$\frac{dy}{dx} = -(x+2)y^2$$

$$\int \frac{1}{y^2} dy = -\int (x+2) dx$$

$$\int y^{-2} dy = -\int (x+2) dx$$

$$\frac{y^{-1}}{-1} = \left[\frac{x^2}{2} + 2x\right] + C_1$$

$$y^{-1} = \frac{x^2}{2} + 2x + C_1 \quad \text{ungbls } -1$$

$$y = \frac{x^2}{2} + 2x + C$$

$$y = \frac{1}{\frac{x^2}{x} + 2x + C} \quad \underline{\text{Ans}}$$

Q1

(B) Estimate the general Solution of

$$y' = (y + 9x)^2$$

<u>Solution</u>

$$y' = (y + 9x)^2 \quad \text{— (a)}$$

Now

$$y + 9x = u \quad \text{— (1)}$$

$$\frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} - 9$$

Then equation 1 will become

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9$$

$$\int dx = \int \frac{1}{u^2 + 3^2}\, du$$

$$x + c = \frac{1}{3} \tan^{-1}(u/3)$$

$$3x + 3c = \tan^{-1}(u/3)$$

$$u/3 = \tan(3x + c)$$

$$u = 3\tan(3x + c)$$

$$y + 9x = 3\tan(3x + c)$$
$$y = -9x + 3\tan(x + c)$$

Q2

(a) Estimate the general solution of

$$x^3 dx + y^3 dy = 0$$

Solution

$$x^3 dx + y^3 dy = 0$$
$$M dx + N dy = 0$$

$$M = x^3, \quad N = y^3$$

$$\frac{\partial M}{\partial y} = \frac{\partial (x^3)}{\partial y}, \quad \frac{\partial N}{\partial x} = \frac{\partial (y^2)}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \text{So exact}$$

$$u = \int M dx + K(y)$$

$$u = \int x^3 dx + K(y)$$

$$u = \frac{x^4}{4} + K(y) \rightarrow \text{eq } 1$$

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{\partial y} K(y)$$

$$\frac{\partial y}{\partial y} = \frac{\partial}{\partial y} K(y)$$

④

Since

$$\frac{\partial u}{\partial y} = N \Rightarrow y^3$$

$$y^3 = \frac{\partial}{\partial y} k(y)$$

$$\int d\, k(y) = \int y^3 dy$$

$$k(y) = \frac{y^4}{4} + C_2 \rightarrow \text{put in eq ①}$$

$$u = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$\frac{x^4}{4} + \frac{y^4}{4} = C_2 - C_1$$

$$\frac{x^4}{4} + \frac{y^4}{4} = C$$

Q(3)

(A)

$$4y'' - 20y' + 25y = 0$$

Solution

$$ay'' + by' + cy = 0$$

and the solution to This is $y = e^{\lambda}$ — (1)

General Solution:

$$y = c_1 e^{\lambda} + c_2 + e^{\lambda}$$

Now

$$4\frac{d^2}{dx^2}(y) - 20\frac{d}{dx}(y) + 25(y) = 0 \quad (A)$$

put eq (1) in eq (A)

$$= 4\frac{d^2}{dx^2}(e^{\lambda x}) - 20\frac{d}{dx}(e^{\lambda x}) + 25e^{\lambda x} = 0$$

$$\Rightarrow \frac{d^2}{dx^2}e^{\lambda x} = \lambda^2 e^{\lambda x} \quad — (B)$$

Now eq (B) and eq (1) in eq (A)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 20 e^{\lambda x} + 25e^{\lambda x} = 0$$

$$\Rightarrow e^{\lambda x}(4\lambda^2 - 20\lambda + 25) = 0$$

$e^{\lambda x} \neq 0$

$\Rightarrow 4\lambda^2 - 20\lambda + 25 = 0$

$(2\lambda - 5)^2 = 0$

$\lambda = 5/2$ or $\lambda = 5/2$

$y(x) = y_1(x) + y_2(x)$

$\Rightarrow y(x) = C_1 e^{5/2x} + C_2 x e^{5/2x}$

(8) (7)

Q3

(B) Estimate general solution of

$$4y'' - 6y' - 7y = 0$$

Solution:

Assume $y(x) = e^{\lambda x}$

put in eq

$$4 \cdot \frac{d^2 y(x)}{dx^2} - 6\frac{d}{dx} y(x) - 7y(x) = 0$$

$$= 4 \cdot \frac{d^2}{dx^2} - 6\frac{d}{dx} y(x) - 7y(x) = 0$$

$$\Rightarrow 4 \cdot \frac{d^2}{dx^2}(e^{\lambda x}) - 6\frac{d}{dx}(e^{\lambda x}) - 7e^{\lambda x} = 0 \quad (1)$$

$$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \text{ — (A)}$$

$$\Rightarrow \frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \text{ — (B)}$$

put (A) and (B) in eq (1)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$$

$$\Rightarrow (4\lambda^2 - 6\lambda - 7)e^{\lambda x} = 0$$

$$\Rightarrow \lambda = 3/4 - \sqrt{37}/4$$

$$\Rightarrow \lambda = 3/4 + \sqrt{37}/4$$

$$y(x) = y_1(x) + y_2(x)$$

$$y(x) = C_1 e^{3/4 - \sqrt{37}/4)x} + C_2 e^{(3/4 + \sqrt{37}/4)x}$$

<u>Ans</u>.