Name Waseem Khan

ID 7337

Program Be (civil)

instructor Madam shumaila

subject Diffrential equation

15 - June - 2020

Assignment # 3

Q1

$$x^3 y''' + 2x^2 y'' + 2y = 100x + 10/x \quad \text{— (1)}$$

solution Put $x = e^t$ then.

$dx/dt = e^t \Rightarrow \frac{dt}{dx} = e^{-t}$.

Now

$dy/dx = dy/dt \cdot dt/dx = dy/dt \cdot e^{-t}$

or $y' = dy/dx = e^{-t} Dy \quad \because d/t \to D$

$y''' = e^{-3t}[D(D-1)(D-2)]y$

using these value in eq (1)

$e^{3t} e^{-3t} [D[D-1](D-2)]y + 2\cancel{e^{2t}}\cancel{e^{-2t}}$

$[D(D-1)]y + 2y =$

$\Rightarrow (D^3 - 3D^2 + 2D)y + (2D^2 - 2D)y + 2y = 10e^t + 10e^{-t}$

$\Rightarrow D^3 y - D^2 y + 2y = 10e^t + 10e^{-t}$

or

$$\frac{d^3 y}{dt^3} - \frac{d^2 y}{dt^2} + 2y = 10e^t + 10e^{-t} \quad \text{— (2)}$$

The assosiced homogenous equation of 2

$$d^3y/dt^3 - \frac{d^2y}{dt^2} + 2y = 0$$

Page #

$d/dt^2 = K^2 \, , \, d^3/dy^3 = K^3$

$\Rightarrow (K^3y - K^2y + 2y) = 0$

$= (K^3 - K^2 + 2)\, y = 0$

for Non trivaral sol, $y \neq 0$

$K^3 - K^2 + 2 = 0$

Q3

$$x^2y^4 + 2xy' - 6y = 10x^2 \rightarrow ①$$

def $x = e^t$ i.e $t = \log x$ $y(1) = 1$

$y'(1) = -6$

Now $xy' = \Delta y \Rightarrow x^2 y'' = \Delta(\Delta - 1)y$

where $\Delta = d/dt$

Then eq $= [\Delta(\Delta-1) + 2\Delta - 6)y = 10e^{2t}$

$[\Delta^2 - \Delta + 2\Delta - 6]y = 10e^{2t}$

$(\Delta^2 + \Delta - 6]y = 10e^{2t}$

char from eq) $\Delta^2 + \Delta - 6 = 0$

$\Delta + 3\Delta - 2\Delta - 6 = 0$

$\Delta = -3, \Delta = 2$

Complementry function

$Cf = c\, e^{-3t} + c_2 e^{2t}$

Also P Integral

$$P.I = \frac{10}{\Delta^2 + \Delta - 6}$$

$$= \frac{10}{2+2-6} e^{2t}$$ replace by case of failure

$P.I = 10 t \frac{1}{2\Delta} + e^{2t} = 10t \cdot \frac{1}{2(2)+1} e^{2t}$

Hence required $y = Cf + P.I$

$y = c_1 e^{-3t} + c_2 e^{2t} + 2t\, e^{2t}$

$c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2$ (8)

Apply initial cond

$y(1) = 1$ we get

$1 = c_1 + c_2 + 0$ — A

2. $y'(1) = -6$

$y' = -3c_1x^4 + 2c_2x + 2x + 4x\log x$

$-6 = -3c_1 + 2c_2 = -8 \rightarrow$ B

eq A × 3 and add with B

$3 = 3c_1 + 3c_2$

$-8 = -3c_1 + 2c_2$

$5c_2 = -5$

$c_2 = -1$

eq A = $1 = c_1 - 1$

$c_1 = 2$

Thus,

$y = 3x^{-3} - x^2 + 2x^2\log x$

Scanned with CamScanner

Q4 ①

$x^2 y'' + 7xy' + 5y = x^5$ $\quad y(0) = 2$

$y'(1) = 2$

Let

$x = e^t \Rightarrow t = \ln x \;,\quad D = d/de$

Now

$xy = \Delta y \Rightarrow x^2 y' = \Delta(\Delta - 1) y$

Then

$(\Delta(\Delta-1) - 7\Delta + 5) y = e^{5t}$

$(\Delta^2 - \Delta + 7\Delta + 5) y = e^{5t}$

$(\Delta^2 + 6\Delta + 5) y = e^{5t}$

Char eq is $\Delta^2 + 6\Delta + 5 = 0$

$\Delta^2 + 5\Delta + \Delta + 5 = 0$

$\Delta = -5 \quad -1$

$C.F = c_1 e \, e^{-5t} + c_2 e^{-t}$

P integral $5t$

$$P.I = \frac{1}{\Delta^2 + 6\Delta + 5}$$

$$= \frac{1}{2} e^{5t} \text{ replacing } \Delta \text{ by}$$

$5^2 + 6(5) + 5$ $\quad 5$

$= 1/60 \; e^{5t}$

Thus $q = 5st$

$y = c_1 e^{-5t} + c_2 e^{t} + 1/60 \; e^{5t}$

$y' = -5c_1 x^{-6} = c_2 x^{-2} + 1/12 \, x^4$

$y(0) = 2 \qquad x = 1 \qquad y = 2$

$2 = -5c_1 - c_2 + 1/12$

$-5c_1 - c_2 = 23/12$ ⓑ

$A + B - 4c_1 = \frac{234}{60} \Rightarrow c_1 = \frac{-117}{120}$

Now

$y = \frac{-117}{120} x^{-5} + c_2 x^{-1} + 1/60 \, e^5$

$c_1 = \frac{-117}{120}$ Put in eq A

$+c_2 = 119/60$

$\frac{-117}{120}$

Now let $x = e^t \Rightarrow t = \log x, D \frac{d}{dt}$

$xy = Dy \Rightarrow x^2y^2 = D(D-1)y$

Then $xy = Dy \Rightarrow x^2y = \cancel{\quad} e^{5t}$

$(D^2 - D + 7D + 5)y = e^{5t}$

$(D^2 + 6D + 5)y = e^{5t}$

Char eq is $D^2 + 6D + 5 = 0$

$D^2 + 5D + D + 5 = 0$

$D = -5, -1$

complementary eq is

$C.F = C_1 e^{5t} + C_2 e^{-t}$

P integral

$$P.I \; \frac{1}{D^2 + 6D + 5}$$

$$= \frac{1/2}{5 + 6(5) + 5} e^{5t} \quad \text{replacing } D \text{ by } 5$$

Thus

$$y = Ce^{-5t} + C_2 e^{-t} + \frac{1}{6} e^{5t}$$

$y = C_1 x^{5} + C_2 x^{1} + \frac{1}{60} x^5$

$y' = -5 C_1 x^{-6} \; C_2 x^{-2} + \frac{1}{12} x^4$

$y(0) = 2 \quad x = 0, y = 2$

$2 = C_1 + C_2 + 1/60$

$C_1 + C_2 = 119/60 \rightarrow A$

$y'(1) = 2 \quad x = 1, y = 2$

$2 = -5C_1 - C_2 + 1/12$

$-5C_1 + C_2 = 23/12 \rightarrow B$

A+B $\quad -4C_1 = \frac{234}{60} = 4 = \frac{117}{120}$

Now $y = \frac{117}{120} x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$

$C_1 = \frac{-117}{120}$ Put in eq A

$\frac{-117}{12} + C_2 = \frac{119}{60}$

$C_2 = \frac{119}{60} + \frac{117}{20}$

$= \frac{238 + 117}{120} \quad \frac{-335}{120}$

Scanned with CamScanner

Q5 $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2 \rightarrow (1)$

$x+1 = e^t \Rightarrow x = e^t - 1$

Diff $\log(x+1) = t$

Also $(x+1)\ y' = Dy\ d/dt = D$

$(x+1)^2 y'' = D(D-1) - 3D + 4)y = (e^t - 1)^2$

$(D^2 - 4D + 4)y = e^{2t} - 2e^t + 1)$

Charec eq is $D^2 - 4D + 4 = 0$

$(D-2)^2 = 0$

$D = 2, 2$

Thus complementry function is

$C.f = (c_1 + c_2 t)\ e$

Also particulr integral is

$PI = \frac{1}{(D-2)^2}(e^{2t} - 2e^t + 1)$

$= 1/(D-2)^2\ e^{2t} - 2\ 1/(D-2)^2\ e^t + \frac{1}{(D-2)^2} \quad -(2)$

Now $\frac{1}{(D-2)^2} e^{2t} = \frac{1}{a+2-2} 2e^{2t} = 1/0\ e^{2t}$

case of falure

$1/(D-2)^2 e^{2t} = \frac{t_1}{2(1-2)^2} e^t = \frac{t^2 e^t}{2}$

and $1/(D-2)(1) = 1/(D-2)\ e^{0t} = 1/4$

eq2 $PI = 1/2\ e^2 e^{2t} - 2e^t + 1/4$

Here complete solution is

$y = C.f + P.I$

$y = (c_1 + c_2 t) e^{2t} + \frac{1}{2}(t^2 + \frac{1}{2} t^2 e^{2t} - 2e^t + \frac{1}{4}$

repeat value of t

$y_e = (c_1 + c_2 \log(x+1))(x+1)^2 + \frac{1}{2}[(\log(x+1))$
$(x+1)^2] - 2(x+1) + \frac{1}{4})$

or

$y = c_1 + c_2 \log(x+1)(x+1)^2] - 2x - 7/4$

which is required

Scanned with CamScanner