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SECTION: B

SUBJECT: Applied Calculas

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Q1 :- The Function $g(t)$ is defined dy

$g(t) = 0 \quad t < 0$

$t^2 = \quad 0 \le t \le 3$

$2t + 3 \quad 3 < t \le 4$

$12 \quad t > 4$

(a) STate Any point OF discontinity

(b) Find IF The limit exist $\lim_{t \to 3}$

Solution (a) :-

To cHeck possibility OF discontinouty of the function

2 at $t = 0$ & 4

First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.S

$\lim g(1+h) = \lim (1+h)^2$

$= \lim\limits_{h\to 0} 1 + h^2 + 2H$

Apply Limit :-

$= 1 + 0^2 + 2(0)$

$= 1.$

For L.H.S :-

$\lim\limits_{h\to 0} g(1-h) = 2t+3$

$\lim\limits_{h\to 0} 2(1-h) + 3$

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$\lim_{h \to 0} 2 - 2h + 3$

Apply limit

$2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.S $= g(1) = 5$

$\Rightarrow$ Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

$=$ R.H.S

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$

$\Rightarrow \lim_{h \to 0} 2 + 2h + 3$

$\Rightarrow$ Apply limit

$2 + 2(0) + 3 = 5$

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=> For L.H.S

$$\lim_{h \to 0} g(4-h) = 12$$

=> $g(4)$ R.H.S $\neq$ L.H.S

So :-

Point of discontinouty is at $t = 4$.

(b) :-

Find if They exist

i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.S

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$$\lim_{h\to 3} g(1+h) = \lim_{h\to 3} (1+h)^2$$

$$= \lim_{h\to 3} 1+h^2+2h$$

Apply limits

$$= 1+3^2+2(3) \Rightarrow \boxed{16}$$

L.H.S :-

$$\lim_{h\to 3} g(1-h) = \lim_{h\to 3} 2t+3$$

$$= \lim_{h\to 3} 2(1-h)+3$$

$$= \lim_{h\to 3} 2-2h+3$$

Apply limit

$$= 2-2(3)+3$$

$$= 2-6+3$$

$$= -1.$$

So: L.H.S $\neq$ R.H.S

(Do not exist since L.H.S is -ve)

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Q# 02 :-

$$y(x) = x^2 + \sin x$$

Solution :-

Since we know that the Maclaurin Series is

$$y(x) = y'(x_0)(x - x_0) + y''(x_0)\frac{(x - x_0)^2}{2!} + \ldots$$

Now put $x_0 = 0$

$$y(x) = y(0) + (x - 0)y'(0) + (x - 0)^2 y''(0) + \ldots$$

$$y(x) = y(0) + xy'(0) + \frac{x^2 y''(0)}{2!} + \ldots \quad (1)$$

Now Find :-

$$y(0) = ?$$

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$y(x) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$= 0 + 0$

$= 0$

so $y(0) = 0$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos x$

$y'(0) = 2(0) + \cos 0$

$y'(0) = 0 + 1$

$\boxed{y'(0) = 1}$

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$\Rightarrow$ Since :-

$$y'(x) = 2x + \cos x$$

$$\frac{d}{dx} y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= 2 - \sin x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$y''(0) = 2 - 0 = 2$$

So $\boxed{y''(0) = 2}$

Now :-

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

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$$y'''(0) = -\cos x$$

$$\boxed{y'''(0) = -1}$$

put in equation 1

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \ldots\ldots$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \ldots\ldots$$

$$= x + x^2 - \frac{x^3}{3!} + \ldots\ldots$$

so

$$y(x) = x + x^2 + \frac{x^3}{3!} + \ldots\ldots$$

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part 1

$1+xy+x^2+y^2$

solution :-

Given $1+xy = x^2+y^2$.

taking $\frac{d}{dx}$ on both side

$1+\frac{d}{dx}(xy) = \frac{d}{dx}x^2+\frac{d}{dx}y^2$

$1+\left(x\frac{dy}{dx}+y\frac{dx}{dx}\right) = 2x+2y\,dy/dx$

$1+x\frac{dy}{dx}+y = 2x+2y\,dy/dx$

$1+y+x\frac{dy}{dx} = 2x+2y\frac{dy}{dx}$

$1+y+x\frac{dy}{dx} = 2x+2y\frac{dy}{dx}$

$x\frac{dy}{dx} = 2y \quad \frac{dy}{dx} = 2x - y + 1$

$\Rightarrow (x-2y)\frac{dy}{dx} = 2x - y - 1$

$\Rightarrow \frac{dy}{dx} = \frac{2x-y-1}{x-2y} \rightarrow (1)$

$\Rightarrow y' = \boxed{\frac{2x-y-1}{x-2y}} \rightarrow (1)$

$\Rightarrow$ Diff Again :-

$\frac{d}{dx}y' = \frac{d}{dx}\left(\frac{2x-y-1}{x-2y}\right)$

By Quotient rule.

$y'' = \frac{(x-2y)\frac{d}{dx}(2x-y-1) - (2x-y-1)\frac{d}{dx}(x-2y)}{(x-2y)^2}$

$$\Rightarrow \frac{(x-2y)(2-y)' - (2x-y-1)(1-2y')}{(x-2y)^2}$$

$$\Rightarrow \frac{(x-2y)(2-y') - (2x-y-1)(1-2y')}{(x-2y)^2}$$

Value of
y' from
previous equation

$$\Rightarrow y'' = \frac{(x-2y)\left(2-\left(\frac{2x-2y-1}{x-2y-}\right)\right) - (2x-y-1)\left(1-2\left(\frac{2x-2y-1}{x-2y}\right)\right)}{(x-2y)^2}$$

$$\Rightarrow \frac{(x-2y)(2-2x-y-1)}{(x-2y)^3} - \frac{(2x-y-1)(1-2)(2x-y-1)}{(x-2y)^3}$$

$$\Rightarrow y'' = \frac{-2x-y-3}{(x-2y)^2} - \frac{(2x-y-1)(1-2(2x-y+1)}{(x-2y)^3}$$

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Part b.

Find y by using logarithmic differention

$y = x^3 (1+x)^9 e^{6x}$.

Solution :-

$\ln(y) = \ln (x^3 (1+x)^9 e^{6x}$

$= \ln (x^3 (1+x)^9 + \ln (e^{(6x)})$

$= 3 \ln x + 9 \ln(1+x) + 6x$

Now: $\frac{d}{dx} \ln(y) = (3 \ln x + 9 \ln(1+x) + 6x)$

$= 3 \frac{d}{dx} \ln x + 9 \frac{d}{dx} \ln(1+x) + 6 \frac{dx}{dx}$

$\Rightarrow 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6$

$\Rightarrow \frac{d \ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6.$

Ans

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