PAGE NO: 01

Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

Name:	Fawad Rehman	**Student ID:**	5053

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$	Marks 5 CLO 2

Page No: 02 ID: 5053

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^n, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Question NO: 1

(a) Consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

i) Determine the minimum sampling rate required to avoid aliasing.

As $f_s \geq 2f_{max}$ $f = \frac{\omega}{2\pi}$

So

$$f_1 = \frac{100\pi}{2\pi} \qquad f_2 = \frac{200\pi}{2\pi}$$

$$f_1 = 50\,Hz \qquad f_2 = 100\,Hz$$

So f_2 is max greater than f_1.

$f_s \geq 2 \times 100\,Hz$ is sample frequency to avoid aliasing.

ii) Suppose that the signal is sampled at rate $F_s = 100\,Hz$ .... As given in Q-Paper.

As $F_s = 100\,Hz$

f_1 becomes $f_1' = \frac{f_1}{f_s} = \frac{50}{100} = 0.5\,Hz$

And

f_2 becomes $f_2' = \frac{f_2}{f_s} = \frac{100}{100} = 1\,Hz$

So $\omega_1' = 2\pi f_1'$ $\omega_2' = 2\pi f_2'$

$\omega_1' = 2\pi \times 0.5$ $\omega_2' = 2\pi \times 1$

$\omega_1' = \pi$ $\omega_2' = 2\pi$.

$x[n] = 3\cos 100\pi t + 4\sin 200\pi t$

thus the signal are

$x[n] = 3\cos \pi n + 4\sin 2\pi n$

→ The effect of sampling rate on the newly generated discrete time signal is that there will be no aliasing means there will be no unwanted component in the Reconstruction of the signals.

the reconstruct original signal.

$\omega_1 = 100\pi$ $f_2 = 100 Hz$

$f_1 = \frac{100\pi}{2\pi} \Rightarrow f_1 = 50$

iii) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

As we know that

$\text{folding frequency} = \frac{F_s}{2}$

$= \frac{100}{2} = 50 Hz$

we have frequency of original signal

$f_1 = 50 Hz$, $f_2 = 100 Hz$

So both the frequency are either equal or greater than the folding frequency

Hence for ideal interpolation we can construct the original signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

The original signal is constructed because we use sampling frequency at low sampling rate. We can also reconstruct the signal for sampling frequency above the low sampling rate.

Question No: 1

(b) Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

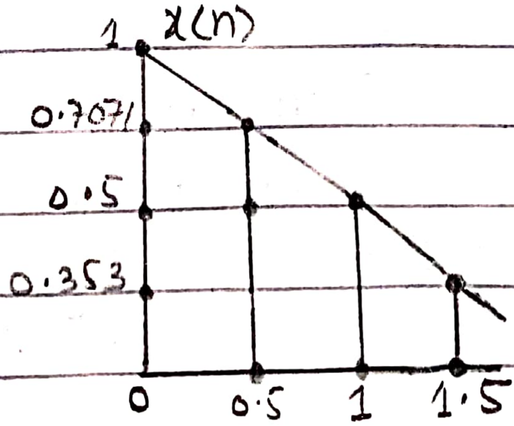
This signal is sampled at the rate $F_s = 2Hz$.

As $F_s = 2Hz$

$$F_s = \frac{1}{T} \rightarrow T = \frac{1}{F_s} = 0.5 \text{ sec}$$

i) Draw the sampled signal.

x_n	$= 0.5^n$
0	1
0.5	0.7071
1	0.5
1.5	0.353



As $T = 0.5$ sec

ii) The sample of the signals are intended to carry 3 bits pe------- ---- Given in detail in Question Paper.

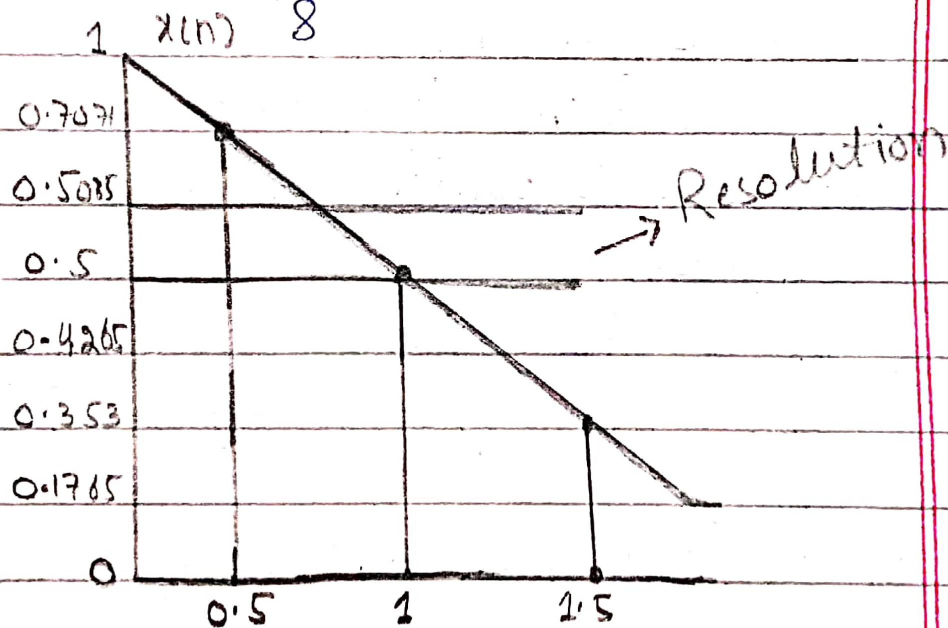
$L = 2^n$

$n = \text{bits} = 3$

$L = 2^3 = 8$ Levels

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$



iii) Perform the process of truncation and rounding off on all the values of the sampled signal ---- -- as given in Question Paper.

P-T-O

	Discrete time Signal	Truncation	Reading	error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

Question No: 2.

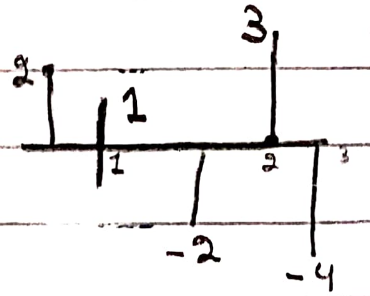
(a) Determine the response of the system to the following input signal with given impulse response.

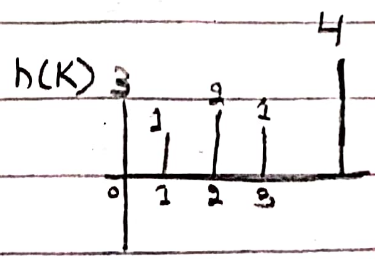
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution:

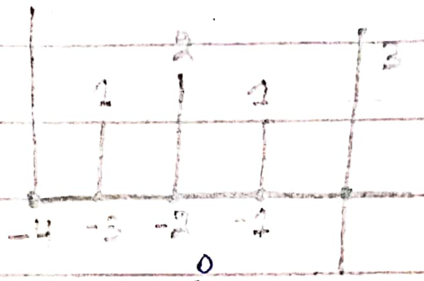
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, h(n-k)$$





P-T-O

h (-K) folded signal



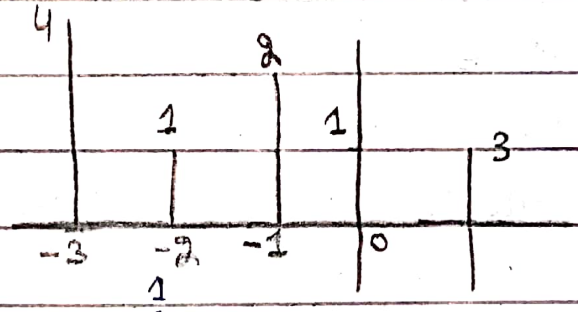
$$X[0] = \sum_{K=1}^{0} x(-1)h(-1) + x(0)h(0)$$

$$X[0] = (2)(1) + (1)(3)$$

$$= 2+3 = 5$$

For $n = 1$

$h(1-K)$



$$y(1) = \sum_{K=1}^{1} x(n)\, h(1-K)$$

$$= x(-1)h(-1) + x(0)h(0) + x(1) + h(1)$$

$$+ (K)\; h(1) + x(2)h(2) + x(3)h(3)$$

$$y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1)$$

$$+ (-4)(3)$$

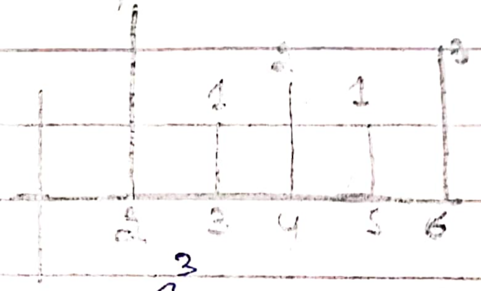
$$= 8+1-4+3-12$$

$$= -4$$

$n = 3$

$h(3-K)$

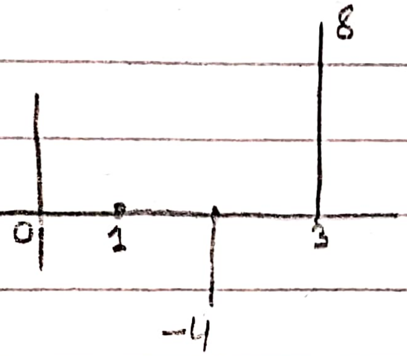
P - T - O



$$Y(3) = \sum_{K=2}^{3} x(n)\, h(n-K)$$

$$= x(2)\, h(2) + x(3)\, h(3)$$

$$(3)(4) + (-4)(1) = 12 - 4 = 8$$



Question No: 3

Determine the Z-transform of the following signals & also sketch its Regions of Convergence (ROC).

i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$

<u>Solution:</u>

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

Writting in the form of Z-transform

$$X(Z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n Z^{-n} + \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n Z^n - 1$$

using geomatric series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}} - 1$$

$$= \frac{1-\frac{1}{\frac{1}{4}}z^{-1} + 1 - \frac{1}{\frac{1}{1}} - 1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-}\right)} - 1$$

$$= \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-\left(1+\frac{1}{3}z-\frac{1}{4}z^{-1}+\frac{1}{12}\cancel{z^{-1}}\cancel{z}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\cancel{\frac{1}{3}z}+1-\cancel{\frac{1}{4}z^{-1}}-1+\cancel{\frac{1}{3}z}+\cancel{\frac{1}{4}z^{-1}}+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{\frac{13}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

Hence the ROC is $\frac{1}{4} < |z| < 3$.

Question No: 3

ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

Solution:

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0; & \text{elsewhere} \end{cases}$$

Writting in the form z-transform

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

Using geometric series

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} + \frac{1}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

$$= \frac{-\frac{5}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

Hence the ROC is $|z| > 3$.