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Assigment:- linear
Algebra.

Pg No1 15772.

Q1:-

Sol:- $V_1 = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$, $V_2 = \begin{bmatrix} 5 \\ 7 \\ 7 \end{bmatrix}$, $V_3 = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

independent values.

$a_1V_1 + a_2V_2 + a_3V_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 0 \\ 5 & 7 & 9 & 0 \\ 7 & 7 & 3 & 0 \end{array}\right]$$

$R_2 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 0 \\ 0 & 18 & 21 & 0 \\ 7 & 7 & 3 & 0 \end{array}\right]$$

$R_3 - 7R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 0 \\ 0 & 18 & 21 & 0 \\ 0 & -30 & -36 & 0 \end{array}\right]$$

$\frac{1}{-30} R_3$ $\left[\begin{array}{ccc|c} 1 & 5 & 6 & \\ 0 & 18 & 21 & 0 \\ 0 & 1 & -36/30 & 0 \\ & & & 0 \end{array}\right]$. PgNO2 15772

$R_1 - 5R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -36 & 0 \\ 0 & -18 & 21 & 0 \\ 0 & 1 & -36/30 & 0 \end{array}\right].$$

$R_3 - R_1$

$$\left[\begin{array}{ccc} 1 & 0 & 144 \\ 0 & -18 & 21 \\ 0 & 0 & -36/30 \end{array}\right].$$

$a_1 + 144$

$a_1 = -144$.

Pg 4. 15772

Pg 3

15772

Q2 (Part b) Explain the linear transformation properties with the help of above problem as an example.

- $T(u+v) = T(u) + T(v)$
- $T(cu) = cT(u)$

(a) $Tc\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \longrightarrow \begin{bmatrix} x-y \\ x+y \\ 2+x \end{bmatrix}$

(b) $w\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \longrightarrow \begin{bmatrix} x+y \\ y+z \end{bmatrix}$

$= T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \longrightarrow \begin{bmatrix} x+y \\ x+y \\ 2y \end{bmatrix}$

$= T\left(\begin{bmatrix} 4 \\ 15 \end{bmatrix}\right) \longrightarrow \begin{bmatrix} 4-15 \\ 4+15 \\ 2(4) \end{bmatrix} = \begin{bmatrix} 11 \\ 19 \\ 8 \end{bmatrix}$

$= T(u+v) \longrightarrow T(u) + T(v)$

$= u \begin{bmatrix} a1 \\ a2 \end{bmatrix} \quad v = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$= T\begin{bmatrix} a1 \\ a2 \end{bmatrix} + \begin{bmatrix} b1 \\ b_2 \end{bmatrix} = T\left[\begin{pmatrix} a_1 + b_1 \\ a_2 \quad b_2 \end{pmatrix}\right]$

$= \begin{bmatrix} a_1 + b_1 - (a_2 + b_2) \\ a1 + b1 + (a_2 + b_2) \\ 2(a_1 + b_1) \end{bmatrix}$

Pg 4. 15772

$$\begin{bmatrix} a_1+b_1 - a_2 - b_2 \\ a_1+b_1 + b_1+b_2 \\ 2a_1 + 2b_1 \end{bmatrix} \quad T(u+v)$$

$$T(u) + T(v)$$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) + T\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} a_1 - a_2 \\ a_1 + a_2 \\ 2a_1 \end{bmatrix} + \begin{bmatrix} b_1 - b_2 \\ b_1 + b_2 \\ 2b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1 - a_2 + b_1 - b_2 \\ a_1 + a_2 + b_1 + b_2 \\ 2a_1 + 2b_1 \end{bmatrix} = T(u+v)$$

$$T(U+V) = T(U)$$

- $T(cU) = cT(U)$

$$U = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$T\, c\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad T\begin{bmatrix} c \cdot a_1 \\ c \cdot a_2 \end{bmatrix}$$

$$= \begin{bmatrix} ca_1 + ca_2 \\ ca_1 + ca_2 \\ 2 \cdot c \cdot a \end{bmatrix}$$

$$c \cdot T\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$= c\begin{bmatrix} a_1 - a_2 \\ a_1 + a_2 \\ 2a \end{bmatrix}$$

$$\begin{bmatrix} c\,(a_1 - a_2) \\ (a_1 + a_2) \end{bmatrix}^{2a} \quad c(2a)$$

$$= c \cdot T(U)$$

pg No 5 15772

Q4(a) Determent $M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2x2

Q4(a) For which values of det M does have inverse.

Sol:-

$$M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

$$M^{-1}=\text{Adj}M\cdot\frac{1}{|M|}$$

$$|M|=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$|M|=ad-bc$$

$$M=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Adj}M=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$M=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$M^{-1}=\frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{[ad-bc]}$$

$$\boxed{M^{-1}=[ad-bc]\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

15772 3 Pg No 6

Ⓑ Write down 2x2 bit matrices with determinant 1 (Remember bits are either 0 or 1 and 1+1=0 in bits).

Sol:-

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$|D| = 1 - 0$$

$$\boxed{|D| = 1}$$

Ⓒ Write down 2x2 matrices with determinant 0.

Sol:-

$$B = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}.$$

$$|B| = \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}$$

$$|B| = |4 \times 1| - |2 \times 2|$$

$$|B| = 4 - 4$$

$$|B| = 0$$

Pg No 7   1577-2

Q4(d):- Compute det A for below 3x3 Matrices?

SOL:- $A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 7 & 5 \\ 7 & 1 & 2 \end{bmatrix}$

$|A| = 3\begin{vmatrix} 7 & 5 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 5 & 5 \\ 7 & 2 \end{vmatrix} + 1\begin{vmatrix} 5 & 7 \\ 7 & 1 \end{vmatrix}$

$|A| = 3\,|14-5| - 1\,|10-35| + 1\,|5-49|$

$3\,(9) - 1\,(-25) + 1\,(-45)$

$|A| = 27 + 25 - 45$

$\boxed{|A| = 7}$

Pg 8 15772

Q3:- What are the main four things we need to define for a vector space? Which of the following is vector space over R? For those that are not a vector, modify one part of the definition to make it into a vector space?

**Vector Space:-**

A set of numbers with properties if $a, b, \in F$, then a+b, a-b, a·b and a/b are in F

Here F is a field

**Main things that define vector space:-**

A vector space R consists of a set "V" along with two operation i.e "+" and "•" (multiplication), subjected to the following conditions

a) for any $s, t \in V$

$\therefore s+t \in V$

(2) for any $s, t \in V$

$s \cdot t \in V$

Pg No 9 15772

(3) Also follow scalar multiplication i.e

for any $r \in R$ and $u \in V$ the scalar multiplication $r \cdot u = v$

It also in according with distributive and associative

$$(s+t) \cdot v = s \cdot v + t \cdot v$$

$$st \cdot v = t(s \cdot v)$$

**Q 3(a):- V = 2×2 matrices with entries in R, usual matrix addition?**

The set of V of all matrices of the form $\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}$ where $a, b \in R$ over R with standard addition and scalar multiplication

Note that V is not closed under addition for $a, b, c, d \in R$ we have $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix}$ but

$$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 2 & a+c \\ b+d & 2 \end{pmatrix} \notin V$$

Pg No 10 15772

We have

$$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+c \end{pmatrix} \in R$$

$$k \circ \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix} - \begin{pmatrix} 1 & ka \\ kb & 1 \end{pmatrix} \in V.$$

Q3(b):- V= { Polynomials with complete conficients of degree $\leq 3$ }, with usual additon and scale?

Sol:-

let $V_1 \cdot V_2 = [x_1, y_1, z_1] \cdot [x_2, y_2, z_2] = x_1x_2 + y_1y_2$

$+ z_1 \cdot z_2$

$V_1 \cdot V_2 = x_2x_1 + y_2y_1 + z_2z_1$

$V_1 \cdot V_2 = V_2 \cdot V_1 \leq 3.$