Iqra National University

Subject : Basic Mathematics

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Program: B.B.A

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Question No 1:

Transpose to make 'x' the subject.

(a) $y = 3x$

Solution:-

$$y = 3x$$

$$y/3 = x$$

$$\boxed{x = y/3}$$

(b) $y = 1/x$

Solution:-

$$y = 1/x$$

$$xy = 1$$

$$\boxed{x = 1/y}$$

(c) $y = 7x - 5$

Solution:-

$$y = 7x - 5$$

$$y + 5 = 7x$$

$$\frac{y+5}{7} = x$$

$$y/7 + 5/7 = x$$

$$\Rightarrow \boxed{x = y/7 + 5/7}$$

(d) $y = 1/2\,x - 7$

Solution:-

$$y = 1/2\,x - 7$$

$$y + 7 = 1/2\,x$$

$$2(y+7) = x$$

$$2y + 2(7) = x$$

$$2y + 14 = x$$

$$\boxed{x = 2y + 14}$$

Question No 2:-

Solve the fallowings:

(a) $\frac{1}{2}x + \frac{3}{2}(x-4) = 6$

$-21x + 12 = -6 - 3x$

Solution:-

$$\frac{1}{2}x + \frac{3}{2}(x-4) = 6 \longrightarrow (1)$$

$$-21x + 12 = -6 - 3x \longrightarrow (2)$$

From equation (1)

$$\frac{1}{2}x + \frac{3}{2}(x-4) = 6$$

$$\frac{1}{2}x + \frac{3}{2}x - \frac{12}{2} = 6$$

$$\frac{x + 3x}{2} - 6 = 6$$

$$\frac{4x}{2} - 6 = 6$$

$$\frac{4x}{2} = 6 + 6$$

$$2x = 12$$

$$x = 12/2$$

$$\boxed{x = 6}$$

From equation ②

$$-21x + 12 = -6 - 3x$$

$$-21x + 3x = -6 - 12$$

$$-18x = -18$$

$$\boxed{x = 1}$$

(b) $4x + \frac{1}{2}(2x - 4) = 18$

$-1 - 7m = -8m + 7$

Solution:-

$$4x + \frac{1}{2}(2x - 4) = 18 \longrightarrow ①$$

$$-1 - 7m = -8m + 7 \longrightarrow ②$$

From equation ①

$$4x + x - 4/2 = 18$$

$$4x + x - 2 = 18$$

$$5x = 18 + 2$$

$$5x = 20$$

$$x = 20/5$$

$$\boxed{x = 4}$$

From equation (2)

$$-1 \quad -7m = -8m + 7$$

$$-7m + 8m = 7 + 1$$

$$\boxed{m = 8}$$

Question No 3:

Solve the Simaltaneous equations.

(a) $2x + y = 2$

$3x + 7y = 14$

Solution:-

$$2x + y = 2 \longrightarrow (1)$$

$$3x + 7y = 14 \longrightarrow (2)$$

Multiply equation (1) with 3 and equation (2) with 2 and Subtract them.

$$\begin{array}{r} 6x + 3y = 6 \\ -6x + 14y = -28 \\ \hline -11y = -22 \end{array}$$

$$11y = 22$$

$$y = \frac{22}{11}$$

$$\boxed{y = 2}$$

Put $y = 2$ in equation (1)

$$2x + 2 = 2$$

$$2x = 2 - 2$$

$$2x = 0$$

$$\boxed{x = 0}$$

$x = 0$, $y = 2$.

(b) $2x + y = -7$

$5x + 3y = -21$

Solution:-

$2x + y = -7 \rightarrow (1)$

$5x + 3y = -21 \rightarrow (2)$

Multiply equation (1) with 5 and equation (2) with 2 and subtract them.

$$10x + 5y = -35$$

$$-19x + 6y = -42$$

$$-y = 7$$

$$\boxed{y = -7}$$

Put $y = -7$ in equation (1)

$$2x + (y) = -7$$

$$2x - 7 = -7$$

$$2x = -7 + 7$$

$$2x = 0$$

$$\boxed{x = 0}$$

$$x = 0 \text{ , } y = -7$$

(C) $x + 5y = 15$

$-3x + 2y = 6$

Solution:-

$$x + 5y = 15 \rightarrow (1)$$

$$-3x + 2y = 6 \rightarrow (2)$$

Multiply equation (1) with 2 and equation (2) with 5 and subtract both equations.

$$2x + 10y = 30$$

$$+ -15x - 10y = -30$$

$$17x = 0$$

$$\boxed{x = 0}$$

Put $x = 0$ in equation (1).

$$0 + 5y = 15$$

$$5y = 15$$

$$y = 15/3$$

$$\boxed{y = 5}$$

$x = 0$, $y = 5$.

(d) $2x + 3y = -10$

$7x + y = 3$

Solution:-

$2x + 3y = -10 \rightarrow (1)$

$7x + y = 3 \rightarrow (2)$

Multiply equation (2) with 3 and subtract equation (2) from (1).

$$2x + 3y = -10$$

$$\underline{-21x + 3y = -9}$$

$$-19x = -19$$

$$\boxed{x = 1}$$

Put $x = 1$ in equation (1)

$$2(1) + 3y = -10$$

$$3y = -10 - 2$$

$$3y = -12$$

$$y = -12/3$$

$$\boxed{y = -4}$$

(e) $-2x + 4y = -16$

$y = -2$

Solution:-

$$-2x + 4y = -16 \rightarrow (1)$$

Put $y = -2$ in equation (1)

$$-2x + 4(-2) = -16$$

$$-2x - 8 = -16$$

$$-2x = -16 + 8$$

$$-2x = -8$$

$$x = 8/2$$

$$\boxed{x = 4}$$

$x = 4$, $y = -2$.

(f) $-2x + 2y = -22$

$-5x - 7y = -19$

Solution:-

$-2x + 2y = -22 \rightarrow (1)$

$-5x - 7y = -19 \rightarrow (2)$

Multiply equation (1) with 7 and equation (2) with 2 and Add both equations.

$-14x + 14y = -154$

$-10x - 14y = -38$

$-24x = -192$

$x = 192/24$

$\boxed{x = 8}$

Put $x = 8$ in equation (1)

$-2(8) + 2y = -22$

$-16 + 2y = -22$

$2y = -22 + 16$

$2y = -6$

$y = -6/2$, $\Rightarrow \boxed{y = -3}$

Question No 4:

If $A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$. Find $|A|$?

Solution:-

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$$

$$|A| = ?$$

$$|A| = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix}$$

$$= 9 - (-2)$$

$$= 9 + 2 = 11$$

$$\boxed{|A| = 11}$$

Question No 5:

Write Each Product as a single Matrix.

(i) $\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$

Solution:-

$$A = \begin{bmatrix} 3 & +1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0-1 & -3+2+0 \\ 0+0+2 & 0-2+0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix} \text{ Ans}$$

(ii) $\begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

Solution:-

$$A = \begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -4 & -4 \end{bmatrix} \text{ Ans}$$

(iii) $\begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$

Solution:-

$$A = \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2+2+1 & -4+2+2 & 10+(-6)+(-4) \\ -1+(-1)+2 & -2+(-1)+4 & 5+3+(-8) \\ -1+0+1 & -2+(-0)+2 & 5+0+(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 10-6-4 \\ 0 & 1 & 8-8 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Ans}$$

(iv) $\begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

Solution:-

$$A = \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+(-2)+5 & 2+(-2)+0 & 1+4+(-5) \\ -2+(-1)+3 & 2+(-1)+0 & 1+2+(-3) \\ -2+(-2)+4 & 2+(-2)+0 & 1+4+(-4) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Ans}$$

Question No 6:

If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. Find $A^2 + BC$?

Solution:-

$$A^2 = A \cdot A$$

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9+(-3) & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 6 & 4 \\ 8 & 9 \end{bmatrix} \text{ Ans}$$

Question No 7:

Show that if $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$. Then

(a) $(A+B)(A+B) \neq A^2 + 2AB + B^2$

(b) $(A+B)(A-B) \neq A^2 - B^2$

Solution:-

(a)

$$A + B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 2+0 \\ 0+(-1) & 1+2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & -2+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B \cdot B = B^2 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1+0 & 0+0 \\ -1+(-2) & 0+4 \end{bmatrix}.$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

Now

$$A^2 + 2AB + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} -1+(-2) & 0+4 \\ 0+(-1) & 0+2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -6 & 8 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -5 & 8 \\ -5 & 8 \end{bmatrix}$$

$$A^2 + 2AB + B^2 = \begin{bmatrix} -4 & 8 \\ -5 & 9 \end{bmatrix} \longrightarrow (1)$$

$$(A+B)(A+B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+(-2) & 0+6 \\ 0+(-3) & -2+9 \end{bmatrix}$$

$$(A+B)(A+B) = \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix} \longrightarrow (2)$$

From equation (1) and (2)

$(A+B)(A+B) \neq A^2 + 2AB + B^2$ Proved

(b)

$$(A-B) = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+(-1) & 2-0 \\ 0+(-1) & 1+(-2) \end{bmatrix}$$

$$(A-B) = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$(A+B)(A-B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2 & 0+(-2) \\ 2+3 & -2+(-3) \end{bmatrix}$$

$$(A+B)(A-B) = \begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix} \longrightarrow (1)$$

$$A^2 - B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix} \longrightarrow (2)$$

From equation (1) and (2)

$(A+B)(A-B) \neq A^2 - B^2$ Proved.

Question: NO 8:

Show that

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -a + 2b + 3c \\ 2a + b \\ 3a + 5b - c \end{bmatrix}$$

Solution:-

$\because$ $m \times n$

L.H.S

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} (-1)(a) + (2)(b) + (3)(c) \\ (2)(a) + (1)(b) + (0)(c) \\ (3)(a) + (5)(b) + (-1)(c) \end{bmatrix}$$

$$= \begin{bmatrix} -a + 2b + 3c \\ 2a + b \\ 3a + 5b - c \end{bmatrix}$$