SUMMER EXAM

SUBJECT : STRUCTURAL ANALYSIS II

SECTION : B

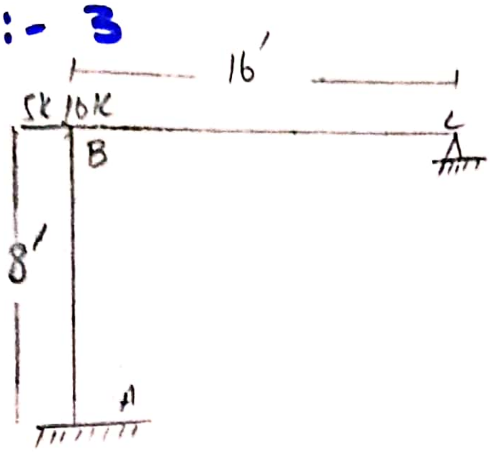
SEMESTER : 6^{th}

SUBMITTED BY : HAMZA EJAZ

SUBMITTED TO : ENGR. ADEED

DATE : 21/08/2020.

QNO:- 3



$E = \text{constant}$

$I_c = I$

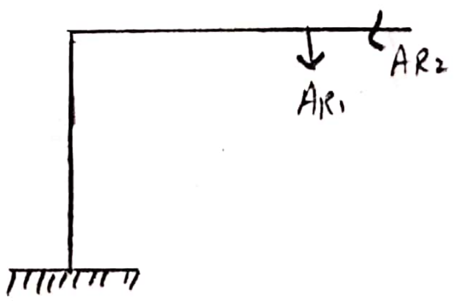
$I_B = 2I$

Sol:-

Total statical indeterminacy

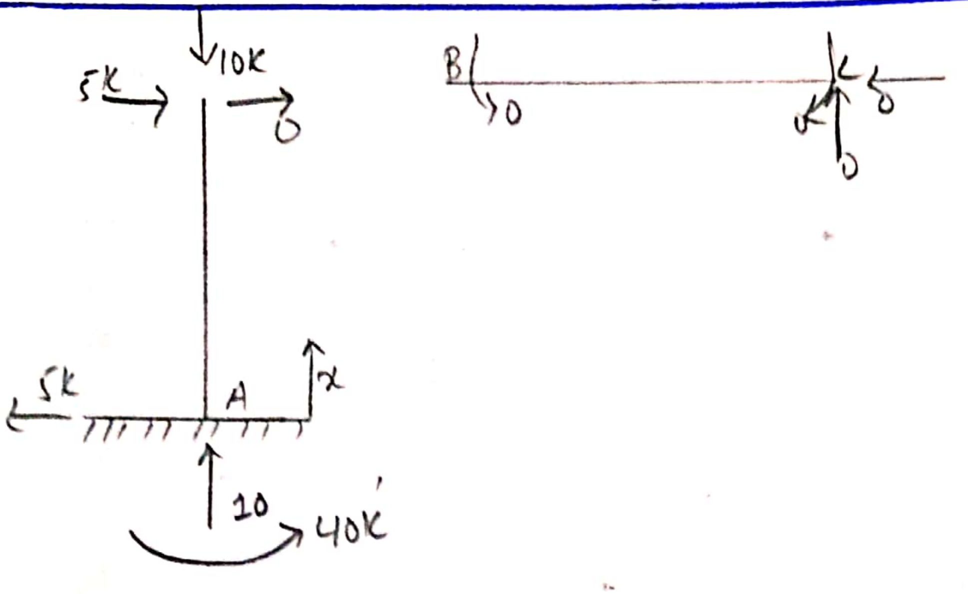
$\Rightarrow R-3 = 5-3 = 2^{\circ}$

Step #01:- Indentify Redundant Actions



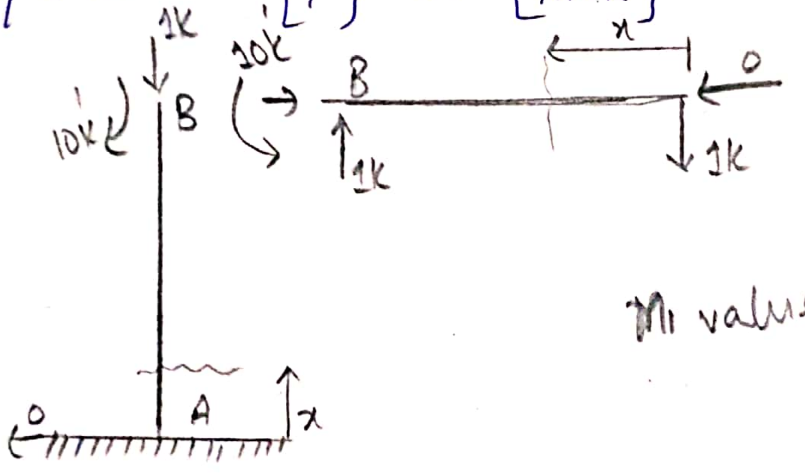
$$\begin{bmatrix} A_{R1} \\ A_{R2} \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} , \quad \begin{bmatrix} D_{RS1} \\ D_{RS2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step #02:- Compute value of $[D_{RL}]$



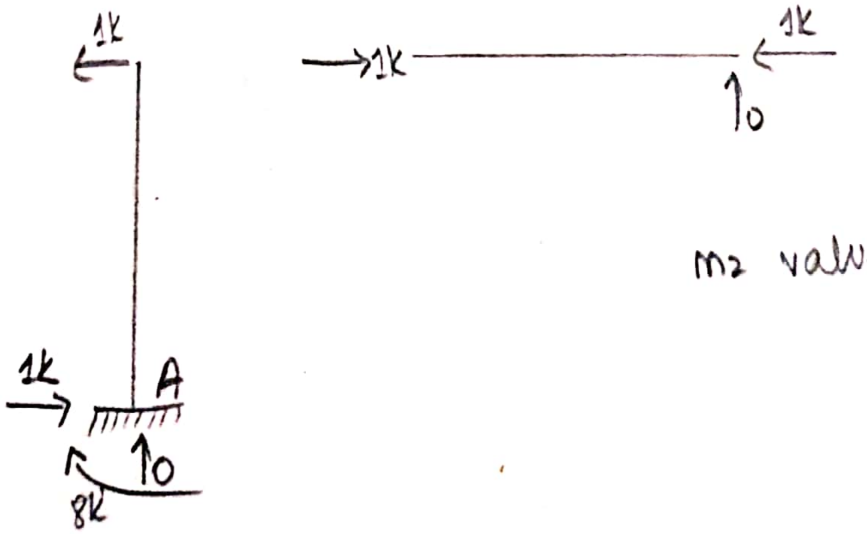
Step # 03 [F] or [AMR]

a)



M1 values

b)



m2 values

Member	AB	BC
origin	A	C
limits	0 - 8	0 - 16
I	I	2I
M	$5x-40$	0
m_1	-16	x
m_2	$8-x$	0

⇒ For finding values of DRL :-

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot m_1(AB)}{EI} + \int_0^{16} \frac{M_{BC} \cdot m_2(BC)}{EI}$$

$$= \int_0^8 \frac{(5x-40)-(16)\,dx}{EI} + \int_0^{16} \frac{0 \cdot x\,dx}{E(2I)}$$

$$DRL_1 = \frac{2560}{EI}$$

$$DRL_2 = \int_0^8 \frac{(5x-40)(8-x)}{EI}\,dx + \int_0^{16} \frac{0 \cdot 0}{E(2I)}\,dx$$

$$DRL_2 = \frac{-853.33}{EI}$$

Compute flexibility Matrix :-

$$F_{2\times2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$\Rightarrow F_{11} = \int_0^8 \frac{m_1^2 (AB)}{EI} + \int_0^{16} \frac{m_1^2 (BC)}{EI}.$$

$$= \int_0^8 \frac{(-16)^2 dx}{EI} + \int_0^{16} \frac{x^2}{E(2I)} dx$$

$$F_{11} = \frac{2730.67}{EI}$$

$$F_{12} = F_{21} = \int_0^8 m_1(AB) \cdot m_2(AB) + \int_0^{16} \frac{m_1(BC) \cdot M_2(BC)}{E(2I)}$$

$$= \int_0^8 \frac{(-16)(8-x)}{EI} dx + \int_0^{16} \frac{(x)(0)}{2EI} dx$$

$$F_{12} = F_{21} = -\frac{512}{EI}$$

$$F_{22} = \int_0^8 (m_2)^2_{AB} dx + \int_0^{16} (m_2)^2_{BC} dx$$

$$= \int_0^8 \frac{(8-x)^2 dx}{EI} + \int_0^{16} \frac{0^2}{2EI} dx$$

$F_{22} = 170.67$

As we know

$$[DRS] = [DRL] + [AR] \times [F]$$

$$\Rightarrow [AR] = \frac{[DRS] - [DRL]}{[F]}$$

$$[AR] = [F]^{-1} \times [DRS - DRL]$$

$$= \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 & -2560 \\ 0 & +853.33 \end{bmatrix}$$

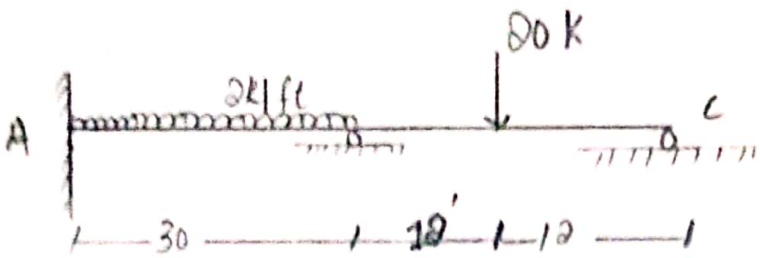
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.00085 \\ 4.997 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Q No 2:-

Force Method	Displacement Method
• Ds < Dk	• Ds > Dk
• Forces are redundent or unknowns	• Displacements are redundent or unknowns
• Starts with equilibrium of forces	• Starts with compatible deformation
• Forces found by compatibility equations of displacement	• Displacement found by equilibrium equations of forces
• No of redundents = Ds	• No of redundents = DK
• Not suitable for computers	• Not suitable for turns.
• Theorem of least work	• Kani's method
• Known as flexibility method eg consistent method of deformations	• Known as stiffness method. eg slope displacement method and moment distribution method.

⇒ Force method is better method because it has variety of use since displacement method can not be applied to trusses. In force method we start it with the equilibrium force to analyze.

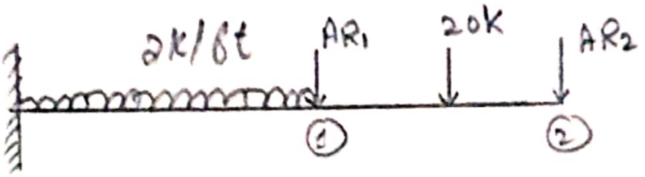
Scanned with CamScanner



Solution:-

Structural Indeterminacy = 2°

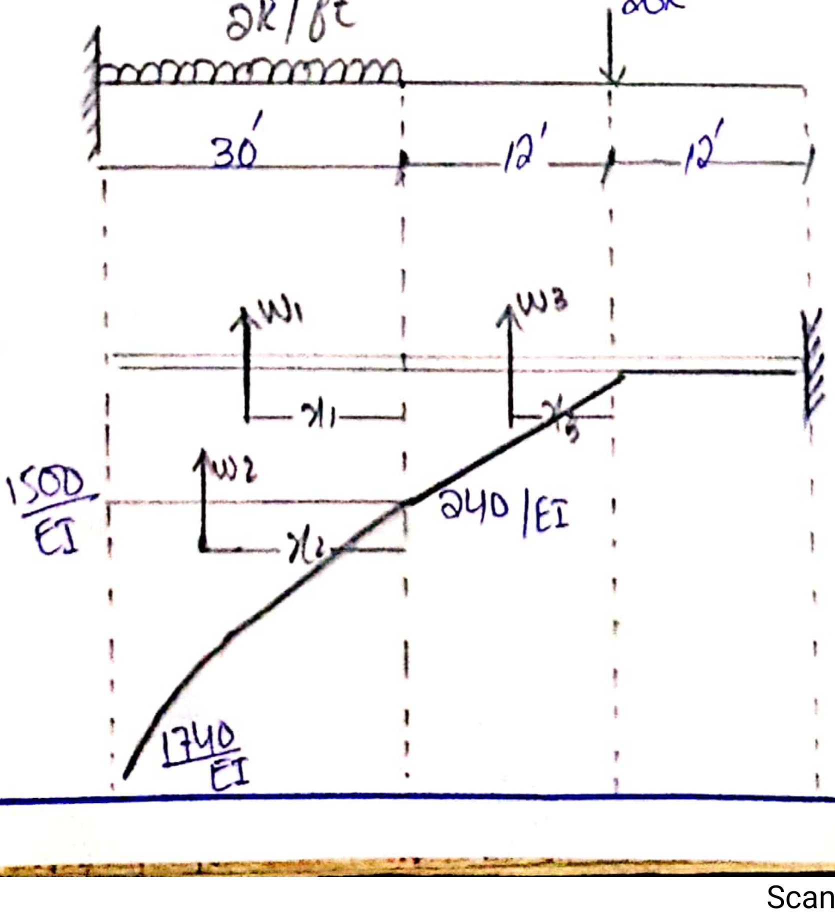
Step # 1:- select Redundent Actions



$$\begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} , \quad \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = [DRL] + [F] \times [AR]$$

Step # 02 Compute the values of [DRL]



$W_1 = 1500 \times 30 = 45000$

$W_2 = 1/3 \times 30 \times 240 = 2400$

$W_3 = 1/2 \times 12 \times 240 = 1440$

$x_1 = b/2 = 30/2 = 15'$

$x_2 = 3/n+2 \times L = 3/2+2 \times 30 = 22.5$

$x_3 = 2/3 \times L = \frac{2}{\cancel{3}} \times \overset{4}{\cancel{12}} = 8'$

Now finding DRL :-

$DRL_2 = W_1 \times (x_1 + 24) + W_2 \times (x_2 + 24) + W_3 \times (x_3 + 12)$

$= 45000 (15 + 24) + 2400 (22.5 + 24) + 1440 (8 + 12)$

$= 1755000 + 111600 + 28800.$

$DRL_2 = 1895400 / EI$

$DRL_1 = W_1 (x_1) + W_2 (x_2)$

$= 45000 (15) + 2400 (22.5)$

$= 675000 + 54000$

$= 729000$

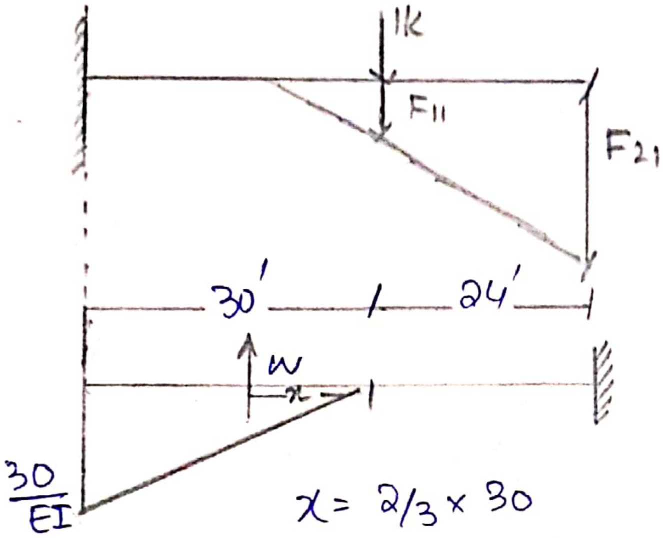
So,

$$DRL = \frac{1}{EI} \begin{bmatrix} 729000 \\ 1895400 \end{bmatrix}$$

Step # 03 Flexibility Matrix

$$[F]_{2\times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

a) Applying unit load on AR_1



$x = 2/3 \times 30$
$= 20'$

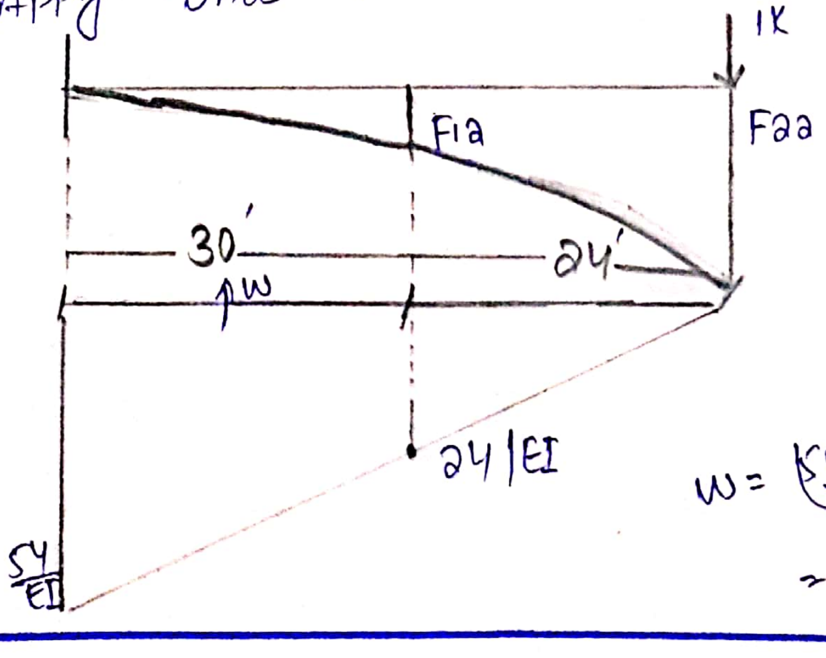
$w = 1/2\left(\frac{30}{EI} \times 30\right)$
$= 450/EI$

So,

$$F_{11} = \frac{450}{EI}(20) = 9000/EI$$

$$F_{21} = \frac{450}{EI}(20+24) = 19800/EI$$

Now Apply unit load on AR_2



$w = \left(\frac{54+24}{2EI}\right) \times 30$
$= 1170/EI$

Now the distance,

$$x = L/3 \left[\frac{b+2(a)}{a+b}\right]$$

$$= 30/3 \left[\frac{24+2(54)}{54+24}\right] = 16.92'$$

$$\Rightarrow F_{12} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$\Rightarrow F_{22} = \frac{1170}{EI} \times (16.92+24) = \frac{47876.4}{EI}$$

Hence

$$F_{2\times2} = \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix} 1/EI$$

Step (4) compute the values of AR

$$[DRS] = [DRL] + [F] \times [AR]$$

$$[AR] = [DRS - DRL] \times [F]^{-1}$$

$$[F]^{-1} = \frac{1}{[F]} \times adj\ F$$

$$= \frac{1}{\begin{vmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{vmatrix}} \times adj \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix}$$

$$|F| = (9000 \times 47876.4 - 19796.4 \times 19800)$$

$$= (430887600 - 391968720)$$

$|F| = 38918880$

$$\Rightarrow Adj\ A = \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

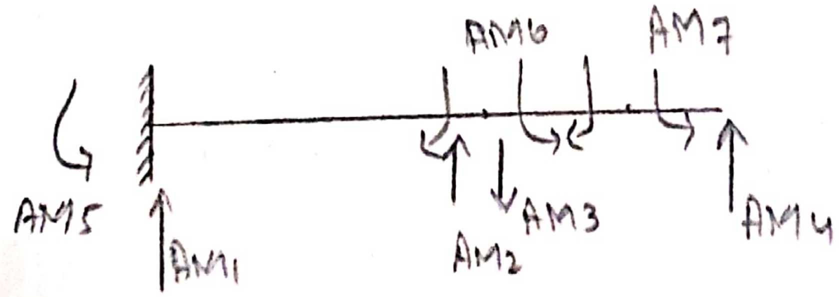
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 & -72900 \\ 0 & -1895400 \end{bmatrix} \frac{1}{EI} \times \frac{1}{38918880} \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$= \begin{bmatrix} -729000 \\ -1895400 \end{bmatrix} \frac{1}{EI} \times \frac{\begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}}{38918880}$$

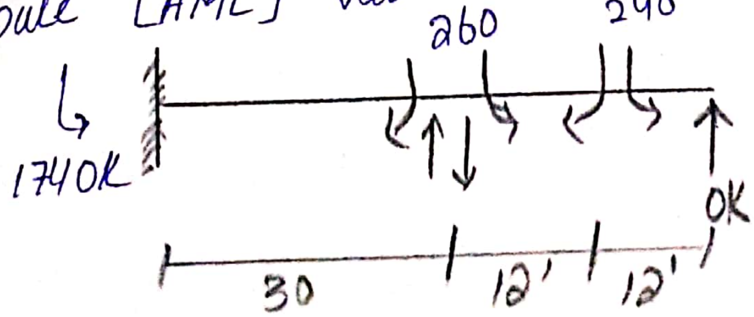
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 66.193 \\ -67.505 \end{bmatrix}$$

step 05 :- compute the members and actions

$$[AM] = [AML] + [AMR] \times [AR]$$

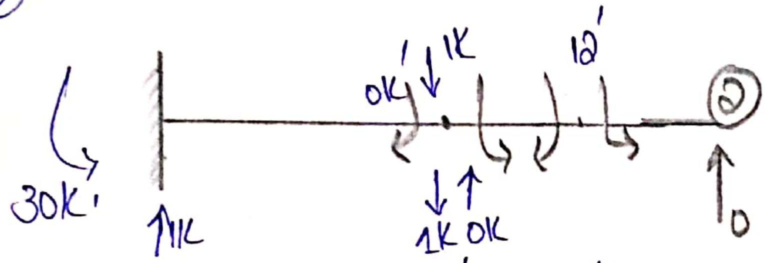


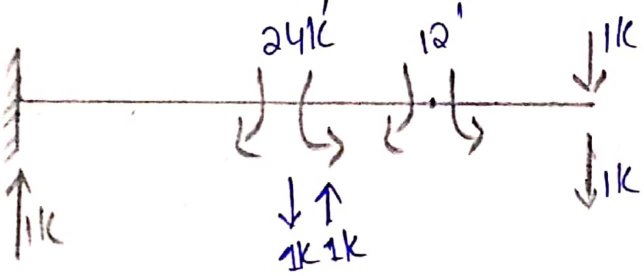
a. compute [AML] values



$$\begin{bmatrix} AML_1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 80k \\ 60k \\ -60k \\ 0 \\ 1740 \\ 1260 \\ 240 \end{bmatrix}$$

b. compute [AMR] values. First apply the unit action at reference point ① and then at reference point ②

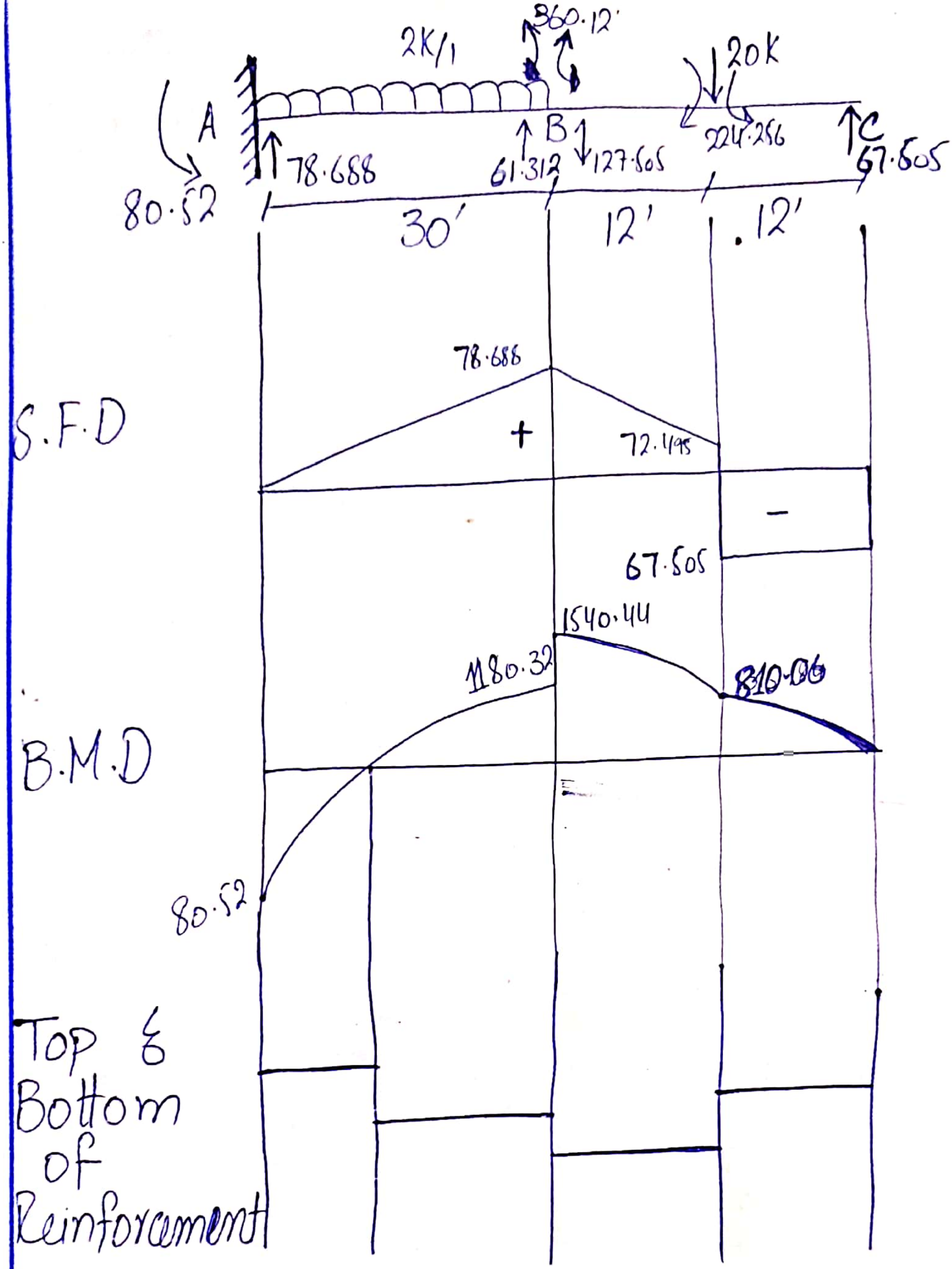




$$AMR = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 30 & 54 \\ 0 & 24 \\ 12 & 12 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 80 \\ 60 \\ -60 \\ 0 \\ 1740 \\ 1260 \\ 240 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & 0 \\ 30 & -1 \\ 0 & 54 \\ 12 & \underset{12}{24} \end{bmatrix} \times \begin{bmatrix} 66.193 \\ -67.505 \end{bmatrix}$$

$$= \begin{bmatrix} 78.688' \\ 61.312' \\ -127.505' \\ 67.505' \\ 80.52' \\ -360.12' \\ 224.256 \end{bmatrix}$$


2K/1
360.12
20K
A
B
C
80.52
78.688
61.312
127.605
224.256
67.605
30'
12'
12'
S.F.D
78.688
+
72.495
-
67.505
B.M.D
1540.44
1180.32
810.06
80.52
Top & Bottom of Reinforcement