

Name # Zohaib Ahmad

ID # 7797

Subject # Differential Equation

Section # A

Exam # Mid Term

$$\textcircled{1} \quad dy/dt = e^{y-t} \sec(y) (1+t^2) y(0) = 0$$

Solution \vdash

$$y(0) \quad \text{So} \quad x=0 \quad y=0$$

$$dy = e^y \cdot e^{-t} \sec(y) (1+t^2) dt$$

$$\frac{1}{e^y \sec(y)} dy = (1+t^2) e^{-t} dt$$

$$\text{As} \quad \cos(y) = \frac{1}{\sec(y)}$$

$$\int e^{-y} \cos y dy = \int (1+t^2) e^{-t} dt$$

using integrating by parts

$$e^{-y} \int \cos y dt - \int (\cos y \frac{d}{dy} e^{-y}) = (1+t^2)$$

$$\int e^{-t} \int (\int e^{-t} \frac{d}{dt} (1+t^2) e^y) \textcircled{1}$$

L.H.S.

$$e^{-y} \int \cos y dx - \int (\cos y \cdot \frac{d}{dy} e^{-y})$$

$$e^{-y} \sin y - \int (\sin e^{-y} (-1))$$

$$e^{-1} \sin y + \int (\sin y \cdot e^{-y})$$

$$e^{-1} \sin y + \int e^{-y} \sin y$$

again using integration by parts.

$$e^{-y} \sin y + e^{-y} (-\cos y) - \int (\sin y \frac{d}{dy} e^{-y})$$

$$e^{-y} \sin y + e^{-y} \cos y - \int (\cos y e^{-y})$$

Since $\int (\cos y e^{-y}) = \text{L.H.S}$

Since is again same to the first one

So L.H.S will become L.H.S = $e^{-y} (\sin y \cos y) - \text{L.H.S.}$

$$2 \text{ L.H.S} = e^{-y} (\sin y - \cos y)$$

$$\text{L.H.S} = \frac{e^{-y} (\sin y - \cos y)}{2}$$

Now taking R.H.S

$$\int (1+t^2) e^{-t} dt$$

$$= -(1+t^2) \int e^{-t} - \int (2t e^{-t} dt)$$

$$= -(1+t^2) e^{-t} - \int (-e^{-t} (2t))$$

$$= -(1+t^2) e^{-t} + \int (2t) e^{-t}$$

again using integration by parts.

$$= -(1+t^2) e^{-t} + (2t \int e^{-t} - \int e^{-t} d/dt (2t))$$

$$= -(1+t^2) e^{-t} + (-2t e^{-t} - \int (-e^{-t} 2))$$

$$= -(1+t^2) e^{-t} + (-2t e^{-t} + \int 2e^{-t})$$

$$= -(1+t^2) e^{-t} + (-2t e^{-t} - 2e^{-t}) + C$$

$$= -(1+t^2) e^{-t} + 2t e^{-t} - 2e^{-t} + C$$

$$= e^{-t} - e^{-t} t^2 - 2t e^{-t} + C$$

$$-(t^2 + 2t + 3)e^{-t} + C = R.H.S$$

Now take L.H.S = R.H.S

$$e^{-y} \left(\frac{\sin y - \cos y}{2} \right) = (t^2 + 2t + 3)e^{-t} + C$$

We know that

$$t = 0 \quad y = 0$$

Put in above

$$= (0 - 1) = -3 + C$$

$$C = 5/2$$

Put value of C

$$\frac{e^{-y}}{2} (\sin y - \cos y) = -(x^2 + 2x + 3)e^{-t} + 5/2 \text{ . Ans}$$

$$\text{Q2) } (\sqrt{x+y} + \sqrt{x-y}) dx - (\sqrt{x+y} - \sqrt{x-y}) dy = 0$$

Solution + $\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}}$

This is homogenous Differential eq in x & y to solve this put.

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + \frac{dv}{dx}$$

They eq ① becomes

$$v + x \frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x \frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x \frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x \frac{dv}{dx} = \frac{1 + \cancel{v} + 1 - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x \frac{dv}{dx} = \frac{\cancel{2} (1 + \sqrt{1-v^2})}{\cancel{2} v}$$

$$v + x \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v}$$

$$\frac{x \, dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v} - v$$

$$\frac{x \, dv}{dx} = \frac{1 + \sqrt{1-v^2} - v^2}{v}$$

$$x \frac{dv}{dx} = \frac{\sqrt{1-v^2} (1 + \sqrt{1-v^2})}{v}$$

$$v \, dv$$

$$\sqrt{1-v^2} (1 + \sqrt{1-v^2}) = \frac{dx}{x}$$

Taking integrals on b/s

$$\int \frac{v \, dv}{\sqrt{1-v^2} (1 + \sqrt{1-v^2})} = \int \frac{dx}{x}$$

$$- \text{put } 1 + \sqrt{1-v^2} = t$$

$$\Rightarrow \frac{1}{2} (1-v^2)^{-1/2} (-2v) \, dv = dt$$

$$\frac{v \, dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$- \ln t = \ln x + \ln c$$

$$- \ln (1 + \sqrt{1-v^2}) = \ln cx$$

$$\ln (1 + \sqrt{1-v^2}) = - \ln cx$$

$$\ln (1 + \sqrt{1-v^2}) = \ln (cx)^{-1}$$

Q2

7797

Page (6)

$$= 1 + \sqrt{1-v^2} = 1/cx$$

$$= 1 + \frac{\sqrt{x^2 - y^2}}{x^2} = 1/cx$$

$$= x + \sqrt{x^2 - y^2} = 1/c$$

$$x + \sqrt{x^2 - y^2} = C_1 \quad \therefore 1/c = C_1$$

which is a Required solution

Question 3)

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

Solution:-

$$\Rightarrow P(D)y = f(x)$$

As it is non homogenous linear equation

So solution will be.

$$y = y_c + y_p \quad \text{--- (1)}$$

Complementary solution y_c

$$D^4 + D^2 = 0 \Rightarrow D^2(D^2 + 1) = 0$$

$$\text{Either } D^2 = 0 \Rightarrow \boxed{D = 0}$$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \Rightarrow D = i \quad \text{or} \quad D = \boxed{0 + i}$$

Roots are real & complex

$$y_c = C_1 e^{0x} + e^{0x} (C_2 \cos x + C_3 \sin x)$$

$$y_c = C_1 + C_2 \cos x + C_3 \sin x$$

$$y_p = \frac{1}{F(D)} F(x)$$

$$y_p = \frac{1}{D^4 + D^2} (3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 + D^2}$$

$$F(D) = D^4 + D^2$$

$$\text{at } D=0 \Rightarrow F(0) = 0$$

$$\text{So } F'(D) = 4D^3 + 2D$$

Now also for $D=0 \Rightarrow F(D) = 0$
again differentiating

$$F''(D) = 12D + 2$$

$$\text{So for } D=0$$

$$F''(0) = 12(0) + 2 = 2$$

So replacing $1/F(D)$ with $\frac{1}{F''(D)}$

$$\Rightarrow y_D \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} 4 \sin x - \frac{x^2}{12D+2} 2 \cos x$$

Putting $D=0$ in all

$$y_p = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 \cdot 4 \sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So: putting in equation (i)

$$y = C_1 + C_2 \cos x + C_3 \sin x + \frac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x.$$

$$y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + \frac{3}{2} x^4$$

Ans