MULTIVARIATE CALCULUS (Mid-Term Summer)

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Q1: If $(x+yi)/i = (7+9i)$ where x & y are real what is the value of $(x+yi)(x-yi)$?

Solution:

It is given that;

$$(x+yi)/i = 7+9i$$

$$\Rightarrow x+yi = (7+9i)i \quad \because \text{Multiplying by } i$$

$$\Rightarrow x+yi = 7i+9i^2 = 7i+9(-1). \quad \because i^2 = -1$$

$$\Rightarrow x+yi = 7i-9 = -9+7i$$

By equating real & imaginary parts; we have: $\boxed{x = -9 \ , \ y = 7}$ — (i)

Now $(x+yi)(x-yi) = x^2-(yi)^2$ $\boxed{\because (a+b)(a-b) = a^2+b^2}$

$$\Rightarrow (x+yi)(x-yi) = x^2-y^2i^2 = x^2-y^2(-1) \because i^2=-1$$

$$\Rightarrow (x+yi)(x-yi) = x^2+y^2$$

P.T.O

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Putting values from eq (i)

$\Rightarrow (-9)^2 + 7^2$

$= 81 + 49$ So,

$(x+yi)(x-yi) =$ **130** Answer

Q2: Find the values of x & y in the following equation, given further that $x \in R$, $y \in R$

$(x+iy)(2+i) = 3-i$

Solution:

$\Rightarrow 2(x+iy) + i(x+iy) = 3-i$

$\Rightarrow 2x + i2y + ix + i^2y = 3-i$

$\Rightarrow 2x + i(2y+x) + (-1)y = 3-i \quad \because i^2 = -1$

$\Rightarrow (2x-y) + i(x+2y) = 3 + (-1)i$

By equating real & imaginary parts we have;

$2x - y = 3$ ——— (i) , $x + 2y = -1$ ——— (ii)

Multiplying eq (i) by (ii)

$(2x - y = 3) \times 2 \Rightarrow 4x - 4y = 6$

P.T.O to pg 3

Now adding the product with eq (ii)

$$4x - 2y = 6$$
$$x + 2y = -1$$
$$5x = 5$$ So,

$$\Rightarrow \boxed{x = 1}$$

Now we put $x = 1$ into eq(ii)

$$1 + 2y = -1 \Rightarrow 2y = -1 - 1 \Rightarrow 2y = -2$$

Subtracting 1 from both sides we get;

$$\boxed{y = -1}$$ Answer

Q3: Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in C$

Solution:- Comparing with $az^2 + bz + c = 0$ we have;

$a = 2$, $b = -2i$, $c = -5$

Since $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow z = \frac{-(-2i) \pm \sqrt{(-2i)^2 - 4(2)(-5)}}{2(2)}$$

P.T.O to Pg (4)

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$$= \frac{2i \pm \sqrt{4i^2 + 8 \times 5}}{4} = \frac{2i \pm \sqrt{-4+40}}{4}$$

$$\Rightarrow z = \frac{2i \pm \sqrt{36}}{4} = \frac{2i \pm 6}{4} = \frac{2i}{4} \pm \frac{6}{4}$$

$$\Rightarrow z = \frac{1}{2}i \pm \frac{3}{2} = \pm\frac{3}{2} + \frac{1}{2}i$$

$$\therefore \text{Solution Set} = \left\{-\frac{3}{2} + \frac{1}{2}i \;,\; \frac{3}{2} + \frac{1}{2}i\right\}$$

$$= \boxed{\left\{\frac{-3+i}{2} \;,\; \frac{3+i}{2}\right\}} \quad \text{Answer}$$

Q4: Express $4 - \sqrt{5}i$ in polar form

Solution:

Let $r\text{cis}\theta$ be the polar form of $4 - \sqrt{5}i$

$$\Rightarrow r\cos\theta + i\, r\sin\theta = 4 - \sqrt{5}i$$

$$\Rightarrow r\cos\theta = 4 \;,\quad r\sin\theta = -\sqrt{5}$$

Squaring & Adding: $r^2\cos^2\theta + r^2\sin^2\theta = 4^2 + (-\sqrt{5})^2$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 16 + 5$$

$$\Rightarrow r^2 = 21 \;, \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

$$\Rightarrow r = \sqrt{21}$$

P.T.O to pg 5

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Now $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{r\sin\theta}{r\cos\theta} = \frac{-\sqrt{5}}{4}$

$\Rightarrow \theta = \tan^{-1}(-\sqrt{5}/4) = -\tan^{-1}(\sqrt{5}/4)$

i.e $\boxed{\theta = -\tan^{-1}(\sqrt{5}/4)}$

$\therefore r\,\text{cis}\theta = \sqrt{21}\,\text{Cis}\left(-\tan^{-1}\frac{\sqrt{5}}{4}\right)$ Answer.

Q5: Find the limit $\lim_{z\to 8} \frac{2z^2-17z+8}{8-z}$

Solution:-

$\left(= \frac{2(8)^2-17(8)+8}{8-8} = \frac{0}{0} \text{ form}\right)$

Consider $2z^2-17z+8 = 2z^2-16z-z+8$

$= 2z(z-8)-1(z-8)$

$= (2z-1)(z-8)$

$\therefore \lim_{z\to 8} \frac{2z^2-17z+8}{8-z} = \lim_{z\to 8} \frac{(2z-1)(z-8)}{8-z}$

$= \lim_{z\to 8} \frac{(2z-1)(-1)(8-z)}{(8-z)}$

$= \lim_{z\to 8} (1-2z)$

PTO to Pg 6

$= 1-2(8)$

$= 1-16$

$= \boxed{-15}$ Answer

Q6: Differentiate

(i) $f(x) = (\ln x)^4$

Sol:

Differentiate w.r.t x

$$\frac{d}{dx} f(x) = \frac{d}{dx}(\ln x)^4$$

$$= 4(\ln x)^{4-1} \frac{d}{dx} \ln x$$

$$= 4(\ln x)^3 \frac{1}{x}$$

$$\boxed{f'(x) = \frac{4}{x}(\ln x)^3}$$ Answer

(ii) $g(x) = x^2 . \ln x$

Sol:

Differentiate w.r.t x :-

$$\frac{d}{dx} g(x) = \frac{d}{dx}(x^2 \ln x)$$

$$g'(x) = x^2 \frac{d}{dx} \ln x + \ln x \frac{dx^2}{dx}$$

PTO to Pg 7

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$$= x^x \frac{1}{x} + \ln x \cdot 2x^{2-1} \frac{dx}{dx}$$

$$= x + \ln x \cdot 2x$$

$$= x + 2x \ln x$$

$$= \boxed{x(1 + 2\ln x)}$$ Answer