Question NO (01)

(i) The order of Matrix AB is $\underline{m \times n}$.

(ii) Answer:

Non-Zero rows in echelon form is the rank of matrix. Normally we have one non-zero row.

(iii) $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$; $a = ?$

Solution:

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$0 = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$0 = 1 \times a - 2 \times 4$$

$$0 = a - 8$$

$$8 = a$$

$$\Rightarrow \boxed{a = 8.}$$ Answer

(IV)

$$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix} \; ; \; |A| = ?$$

Solution:.

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = (2i \times -i) - (i \times i)$$

$$= (-2i^2) - (i^2) \qquad \therefore i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= +2 + 1$$

$$= 3$$

(V) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

is a Scalar Matrix..

Because in Scalar matrix diagonal element are Same (non-zero) and non-diagonal elements are zero.

(vi) $\frac{dy}{dx} + 2xy = y$.

Solution:

$$\frac{dy}{dx} + 2xy = y$$

$$\Rightarrow \frac{dy}{dx} = y - 2xy$$

$$\Rightarrow \frac{dy}{dx} = y(1-2x)$$

$$\Rightarrow \frac{dy}{y} = (1-2x)\,dx$$

Now integrating both sides

$$\Rightarrow \int \frac{dy}{y} = \int (1-2x)\,dx$$

$\therefore \int \frac{1}{y} dy = \ln y$

$$\Rightarrow \int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\Rightarrow \ln y = \int 1\,dx - \int 2x\,dx$$

$$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c.$$

$$\Rightarrow \ln y = x - x^2 + c.$$

$$\Rightarrow e^{\ln y} = e^{x - x^2 + c.}$$

$$\Rightarrow \boxed{y = e^{x(1-x)+c.}}$$

(vii)

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Order = 1

degree = 3

★ order is highest derivative operating

★ degree is exponent power of highest derivative.

(viii)

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$

Order of this differential equation is 2.

degree of differential equation is 1.

(ix) $2\frac{dy}{dx} + x^2 y = 2x+3$, $y(0) = 5$.

Solution.

$$2\frac{dy}{dx} + x^2 y = 2x+3 \; ; \; y(0) = 5.$$

dividing both sides by 2.

$$\frac{\cancel{2} y'}{\cancel{2}} + \frac{x^2 y}{2} = \frac{2x+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right) y = \tfrac{1}{2}(2x+3).$$

$$y' + \left(\frac{x^2}{2}\right) y = \tfrac{1}{2}(2x+3).$$

let $t = \frac{x^2}{2}$.

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}.$$

Multiplying $e^{x^3/6}$ on both sides

$$e^{x^3/6} y' + e^{x^3/6}\left(\frac{x^2}{2}\right) y = e^{x^3/6}\left(\tfrac{1}{2}\right)(2x+3).$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + c}{2e^{x^3/6}}.$$

$y(0) = \frac{0+3}{2} = 3/2.$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + 3/2 \text{ Answer}$$

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = ?$

Solution:

Let $|A| = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Expanding By R_1.

$|A| = 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$

$|A| = 1(bc^2 - cb^2) - a(c^2 - b^2) + a^2(c - b)$.

$|A| = bc^2 - cb^2 - ac^2 + ab^2 + a^2c - a^2b$.

$|A| = bc(c-b) - a(c^2+b^2) + a^2(c-b)$.

$|A| = (bc - a + a^2)(c-b)(c^2+b^2)$. Answer.

$\cancel{|A|}$

Q No (2) (i) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Solution :-

Let $|A| = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Expanding by R_1.

$$|A| = a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$|A| = a\left(b^2c^3 - b^3c^2\right) - b\left(a^2c^3 - a^3c^2\right) + c\left(a^2b^3 - a^3b^2\right)$$

$$|A| = ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

$$|A| = abc\left(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b\right) \quad \text{or}$$

$$|A| = abc\left(bc[c-b] - ac[c+a] + ab[b-a]\right) \quad \text{Answer.}$$

Q No (2)(ii)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$; Find Eigen Value?

Solution:-

Let $A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

We know that

$$A-\lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Q No (2) Part (ii) Remaining Part

Now Expanding by R_1.

$$A-\lambda I = 2-\lambda\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$+(-1)\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 0 \text{ ——— } (A)$$

Now again expanding the first term by R_1.

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \Rightarrow 3-\lambda\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$+(-1)\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow 3-\lambda\left[(\lambda^2-5\lambda+6)-1\right]+1\left((-2+\lambda)-1\right)-1\left(1-(-3+\lambda)\right)$$

$$\Rightarrow 3-\lambda\left(\lambda^2-5\lambda+6-1\right)+1\left(-3+\lambda\right)-1\left(1+3-\lambda\right)$$

$$\Rightarrow -\lambda^3+8\lambda^2-20\lambda+15-3+\lambda-4+\lambda$$

$$\Rightarrow \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \text{ ——— } (a).$$

Q No(2) Part(ii) Remaining Part:-

$$\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Now Expanding by C_1.

$$\Rightarrow +(-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0.$$

$$\Rightarrow -1\left((\lambda^2-5\lambda+6)-(1)\right) + 1\left((-2+\lambda)-1\right)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda.$$

$$\Rightarrow -\lambda^2+6\lambda-8 \text{ ———— } (b).$$

$$\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Now Expanding by C_1.

$$\Rightarrow +(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0.$$

$$\Rightarrow -1(-2+\lambda-1) + 1(\lambda^2-5\lambda+6-1) + 0$$

$$\Rightarrow 3-\lambda+\lambda^2-5\lambda+5 \Rightarrow \lambda^2-6\lambda+8 \text{ ———— } (c)$$

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Solving the Eq (a) (b) and (c)

$\Rightarrow (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\left(-\lambda^2+6\lambda-8\right)-\left(\lambda^2-6\lambda+8\right)$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda+\lambda^2+6\lambda+8-\lambda^2-16\lambda+8$

$\Rightarrow \cancel{\lambda^4-10\lambda^3+34\lambda^2-44\lambda+32=0}$

$= \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

Now By synthetic division.

$$\begin{array}{c|cccc} & 1 & -10 & 32 & -32 \\ 2 & & 2 & -16 & +32 \\ \hline & 1 & -8 & 16 & \boxed{R=0} \end{array}$$

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$\lambda_1=0 \quad \lambda_2=2, \quad \lambda^2-8\lambda+16=0$

Now factorizing.

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)=0$

$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=5}$ Answer.

Q No (3) $(x^2+3y^2)dx - 2xydy = 0$ at $x=2, y=6$

Solution:

$$(x^2+3y^2)dx - 2xydy = 0$$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

Dividing both sides by $2xydy$.

$$\frac{(x^2+3y^2)\,dx}{2xy\,dy} = \frac{\cancel{2xydy}}{\cancel{2xydy}}$$

Now moving dx/dy on the other side.

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = 1/2\left(\frac{x}{y} + \frac{3y}{x}\right)$$

$$\frac{dy}{dx} = 1/2\left(\frac{x}{y} + \frac{3y}{x}\right) \text{ --- } (a)$$

QNo(3) Remaining Part

let us Suppose.

$$y = Vx$$

Now differentiating (Product rule).

$$dy = V\,dx + x\,dv.$$

$$\frac{dy}{dx} = \frac{V\,dx}{dx} + \frac{x\,dv}{dx}$$ dividing b/sides by dx

$$\frac{dy}{dx} = V + x\frac{dv}{dx} \quad \text{——— (b)}$$

Putting eq (b) in eq (a).

$$V + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{V\cancel{x}} + \frac{3V\cancel{x}}{\cancel{x}}\right]$$

$$V + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{V} + 3V\right]$$

Multiplying b/sides by 2.

$$2V + 2x\frac{dv}{dx} = \cancel{2}\cdot\frac{1}{\cancel{2}}\left[\frac{1}{V} + 3V\right]$$

$$2v + 2x\frac{dv}{dx} = \frac{1}{V} + 3V.$$

Q No(3) Remaining Part

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v.$$

$$\frac{2x\,dv}{dx} = \frac{1}{v} + v.$$

$$\frac{2x\,dv}{dx} = \frac{1+v^2}{v}.$$

$$\frac{2x\,\cancel{dv}}{\cancel{dx}} \times \frac{\cancel{dx}}{\cancel{dv}} = \frac{1+v^2}{v}\left(\frac{dx}{dv}\right)$$ Multiplying b/sides by dx/dv.

$$2x = \frac{1+v^2}{v}\,\frac{dx}{dv}$$

$$2x \times dv = \frac{1+v^2}{v}\,dx$$

Multiply b/sides by $\frac{v}{x(1+v^2)}$.

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx.$$

Now integrating both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx.$$

$$\ln|1+v^2| = \ln x + \ln C.$$

Q No (3) Remaining Part

$\ln|1+v^2| = \ln x + \ln C$

taking e on b/sides.

$e^{\ln|1+v^2|} = e^{\ln|x C|}$

$1+v^2 = xC$ —— (c)

Put $v = y/x$ in eq (c)

$1+\left(\frac{y}{x}\right)^2 = xC$.

$1+\frac{y^2}{x^2} = xC$.

$\frac{x^2+y^2}{x^2} = xC$

Multiplying b/sides by x^2

$x^2+y^2 = x^3C$ —— (d)

Put $x=2$, $y=6$ in eq (d)

$(2)^2+(6)^2 = (2)^3 C$

$\frac{4+36}{8} = C$

$\Rightarrow \boxed{C = 5}$

Q No (3) Remaining Part

We know that from eq (d)

$$x^2+y^2=5x^3.$$

$$y^2=5x^3-x^2$$

$$y^2=x^2(5x-1).$$

Now taking Sq root on b/sides.

$$\sqrt{y^2}=\sqrt{x^2(5x-1)}$$

$$y=+x\sqrt{5x-1}\ ,\ y=-x\sqrt{5x-1}$$

$$\boxed{y=\pm x\sqrt{5x-1}}$$ Answer.

End.