Department of Electrical Engineering

Assignment

Date:13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th

Instructor: Pir Meher Ali Sha **Total Marks:** 30

Student Details

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Q1

(a) Consider the following analog signal *Marks 5, CLO 1*

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

i. Determine the minimum sampling rate required to avoid aliasing.

ii. Suppose that the signal is sampled at the rate F_s = 100Hz. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.

iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

(b) Consider a discrete time signal which is given by *Marks 5, CLO 1*

$$x(n) = \begin{cases} 0.5n\ , & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate F_s= 200Hz.

i. Draw the sampled signal.

ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .

iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.

Q2

(a) Determine the response of the system to the following input signal with given impulse response *Marks 5, CLO 2*

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \qquad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

	(b)	Compute the convolution y(n) of the following signal $$x(n)=\begin{cases} a^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n)=\begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i) $$X(n)\begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{8})^{-n}, & n < 0 \end{cases}$$ ii) $$X(n)=\begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$$	Marks 10 CLO 2

Name : Muhammad Kamran

ID # : 13752

Subject : Digital signal processing

Q1: (a)

Considerd the following analog Signal:

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine the minimum Sampling rate required to avoid aliasing.

Solution:

As we have two frequency components So we have to find maximum one.

$3\cos 100\pi t \Rightarrow$

$$w = 100\pi$$

$$2\pi F_1 = 100\pi$$

$$F_1 = \frac{100\cancel{\pi}}{\cancel{2}\cancel{\pi}} = 50Hz$$

$4\sin 200\pi t \Rightarrow$

$$W = 200\pi$$

$$2\pi F_2 = 200\pi$$

$$F_2 = \frac{200\pi}{2\pi}$$

$$F_2 = 100Hz$$

As $F_2 = 100Hz$ So $F_{max} = 100Hz$

minimum nyquist rate:

$$F_s = 2F_m$$

$$F_s = 2 \times 100$$

$$F_s = 200Hz$$

(ii) Suppose that the Signal is Sampled at the rate $F_s = 100Hz$. What is the Discrete-time Signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time Signal.

$$F_s = 100Hz$$

We Know that:

$$F_s = 2F_{max}$$

$$100Hz = 2F_{max}$$

$$F_{max} = 50Hz$$

As $t = nT = n/F_s$ (by sampling)

Muhammad Kamran (3) ID# 13752

As $x(t) = 3\cos 100\pi t + 4\sin 200\pi t$

$$x(n) = 3\cos \frac{100\pi n}{100} + 4\sin \frac{200 n\pi}{100}$$

$$x(n) = 3\cos n\pi + 4\sin 2\pi n$$

As $F_{max} = 50Hz$

Also we Know

$$F_0 = F_K - KF_{Max}$$

Put $K=1$

$$F_0 = F_1 - F_s$$

$$F_0 = 50 - 50 = 0Hz$$

put $K=2$

$$F_0 = F_2 - 2F_{Max}$$

$$F_0 = 100 - 2\times 50$$

$$F_0 = 100 - 100 = 0Hz$$

As both frequency are not more than folding maximum frequency thus Their will be no effect of sampling over newly genereted Discrete time Signal

(iii) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

As we obtained:

$$3\cos n\pi + 4\sin 2n\pi$$

converting to time domain

$$3\cos n\pi \frac{100}{100} + 4\sin \frac{2n\pi}{100} \times 100 \quad \because F_s = 100$$

$$3\cos 100\pi t + 4\sin 200\pi t \quad \because n/F_s = t$$

Thus we have two frequeny components in the above which is 50 Hz and 100 Hz upon which sampling was done. Thus recoverd signal is

$$x(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

which is axactly same as original one.

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Q1 (b):

consider a discrete time signal which is given by:

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

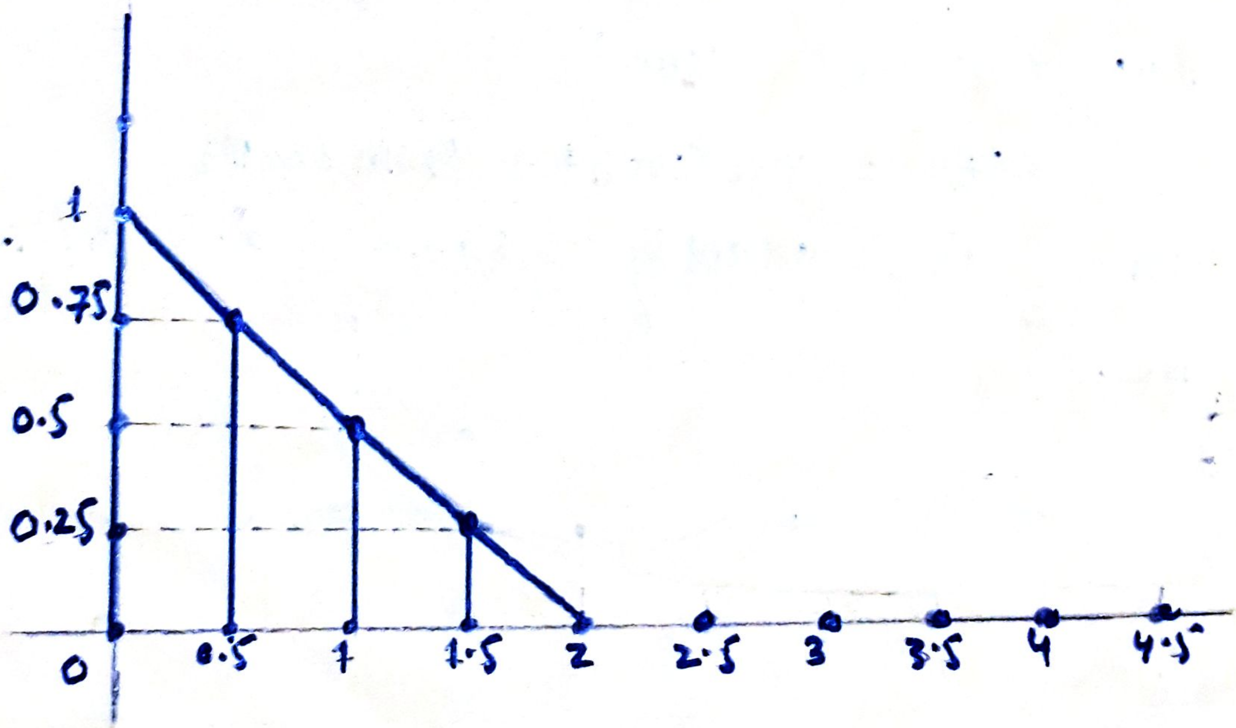
This is signal is sampled at the $F_s = 2 Hs$

(i) Draw the sampled signal.

$F_s = 2Hz$

$T_s = 1/F_s$

$T_s = 1/2 = 0.5 sec$



Muhammad Kamran (6) 13752

(ii) The samples of the signals are intended to carray 3 bits per sample Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part(i)

Here $n = 3$ bits per sample

$L = 2^n$ (L is quantization level)

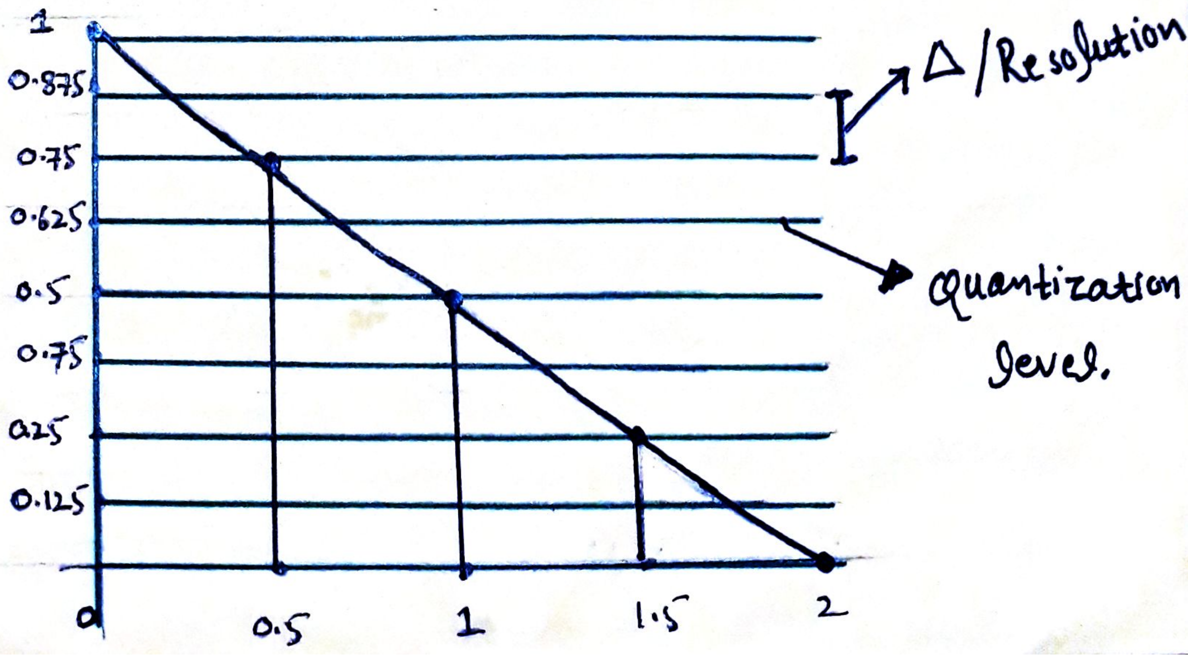
$L = 2^3 = 8$

$L = 8$ levels

Quantization resulation / steps size Δ is

$$\Delta = \frac{x_{max} - x_{min}}{L-1}$$

$$\Delta = \frac{1-0}{8-1} = \frac{1}{7} = 0.142$$



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iii) Perform the process of truncation and rounding off number of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.

n	$x(n)$ Discrete Time signal	$x_q(n)$ Truncation	$x_q(n)$ Rounding	$e_q(n) = x_q(n) - x(n)$ Rounding
0	1	1.0	1.0	0.0
1	0.875	0.8	0.9	-0.1
2	0.75	0.7	0.8	-0.1
3	0.625	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.375	0.3	0.4	-0.1
6	0.25	0.2	0.3	-0.1
7	0.125	0.1	0.1	0.0

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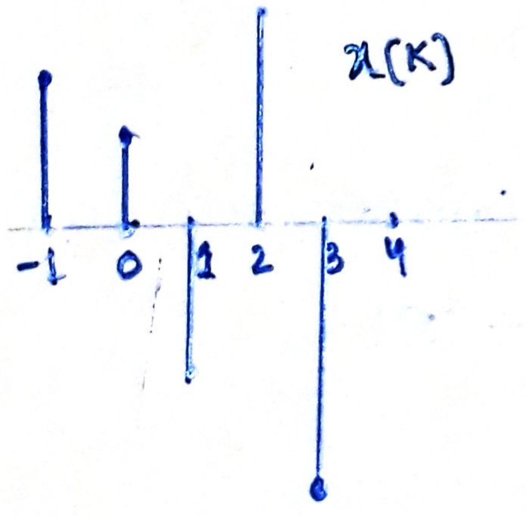
Q 2 (a):

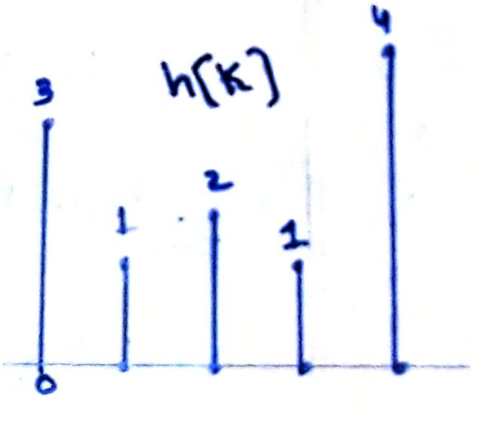
Determine the responce of the system to the following input signal with given impulse responce.

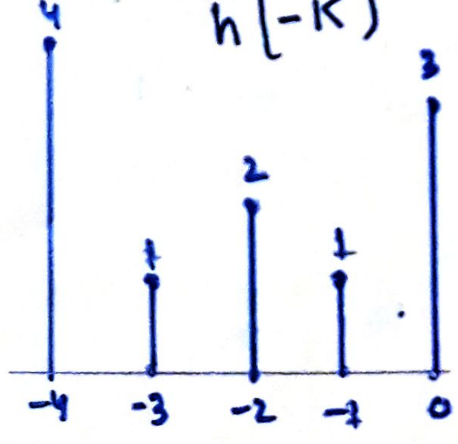
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

$$y[n] = \sum_{K=-\infty}^{\infty} x(K)h[n-K]$$







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$$y[0] = \sum_{K=-1}^{0} x[-1]\,h(-1) + x(0)\,h(0)$$

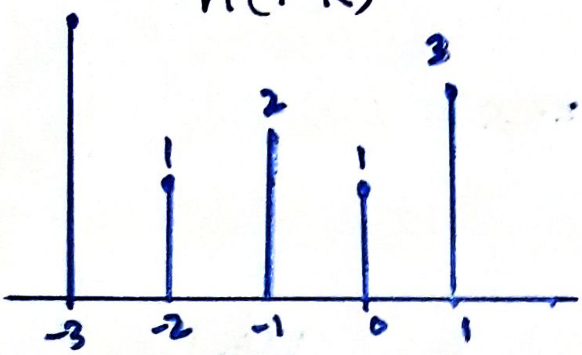
$$= 2\times 1 + (1)3$$

$$= 2+3$$

$$= 5$$

For $n=1$

$h(1-K)$



$$Y[1] = \sum_{K=-1}^{1} x(n)\,h[1-k]$$

$$= x(-1)\,h[-1] + x(0)\,h(0) + x[1]\,h[1]$$

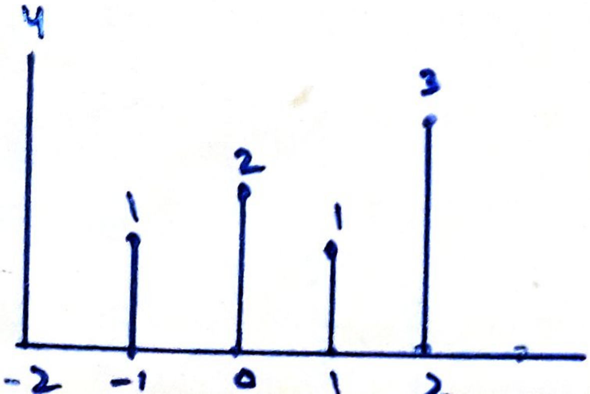
$$= (2)(2) + (1)(1) + (3)(-2)$$

$$= 4+1-6$$

$$= -1$$

$n=2$

$h[2-K]$



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$$Y[2] = \sum_{K=-1} x(n) h(2-K)$$

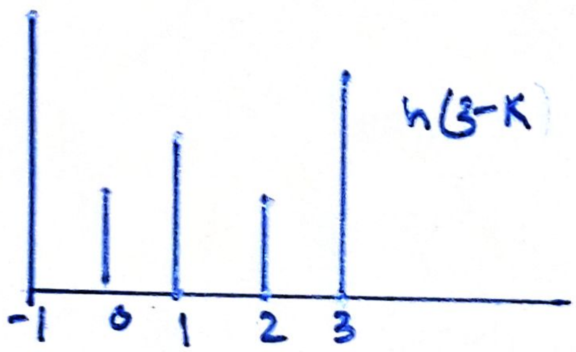
$$Y(2) = x(-1)h(-1) + x(0) + h(0) + x(1)h(1) + x(2)h(2)$$

$$Y(2) = (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$

$$Y[2] = 2+2-2+9$$

$$Y[2) = 11$$

$n = 3$



$$Y[3] = \sum_{K=-1}^{3} x(n) h(3-K)$$

$$x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 2 \times 4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$

$$= 4 + 1 - 4 + 3 - 12$$

$$= -8$$

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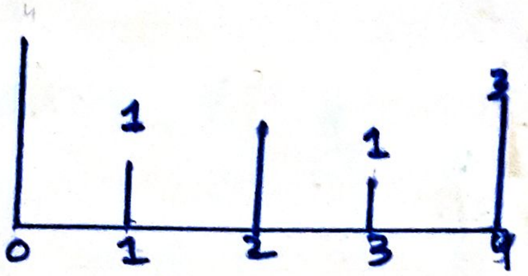
$\underline{n = 4}$

$h(4-K)$



$$y(4) = \sum_{K=0}^{3} x(n)\, h(4-K)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

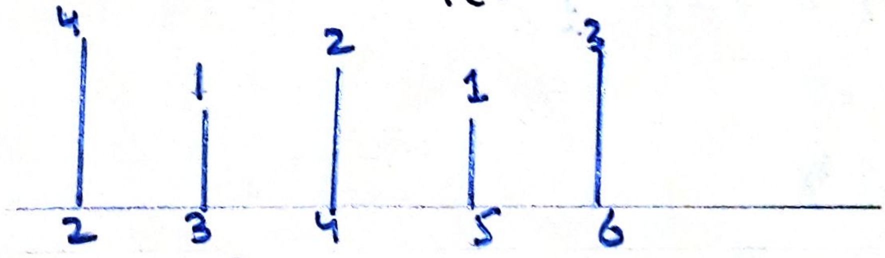
$$1 \times 4 + (-2)(1) + 3(2) + (-4)(1)$$

$$4 - 2 + 6 - 4$$

$$y(4) = 4$$

$\underline{n = 5}$

$h(5-K$



$$y(5) = \sum_{K=1}^{3} x(n)\, h(5-K)$$

$$x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= (-2)(4) + (3)(1) + (-4)(2)$$

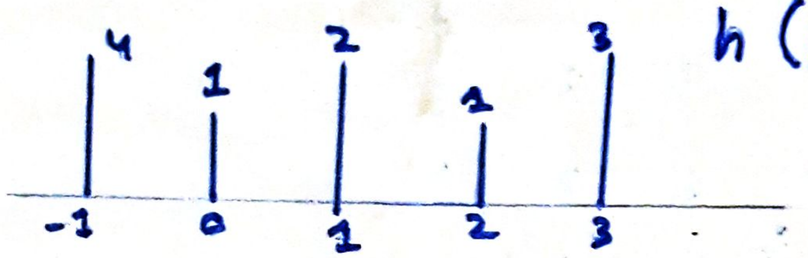
$$= -8 + 3 - 8$$

$$= -13$$

$= 2+2-2+9$

$= 11$

$\underline{n = 3}$



$h(3-k)$.

$$y(3) = \sum_{k=-1}^{3} x(n)\, n(3-k).$$

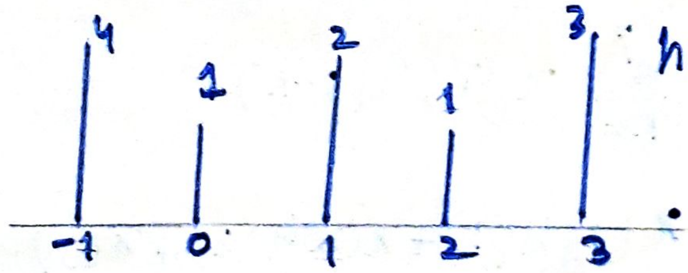
$= x(-1)h(-1) + x(0)h(3) + x(1)h(1) +$
$x(2)(\cancel{x}h(2) + x(3)h(3)$

$= 2\times 4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3).$

$= 4+1-4+3-12$

$= -8.$

$\underline{n = 4}$



$h(4-k)$

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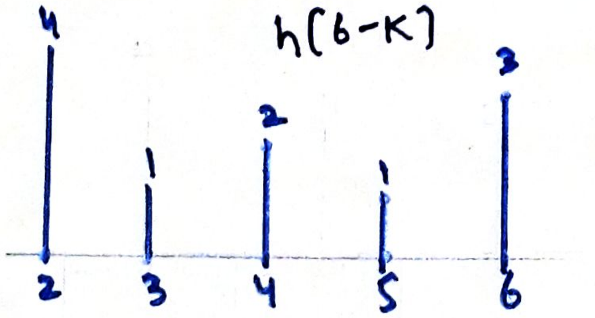
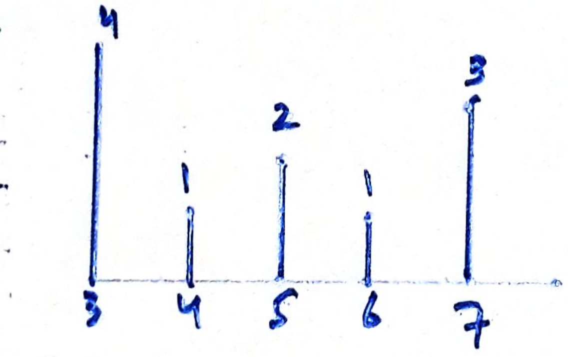
$n = 6$

$$Y(6) = \sum_{K=2}^{K=3} x[2]\, h(2) + x(3)\, h(3)$$

$$Y(6) = (3)(4) + (1)(-4)$$
$$= 12 - 4$$
$$= 8$$

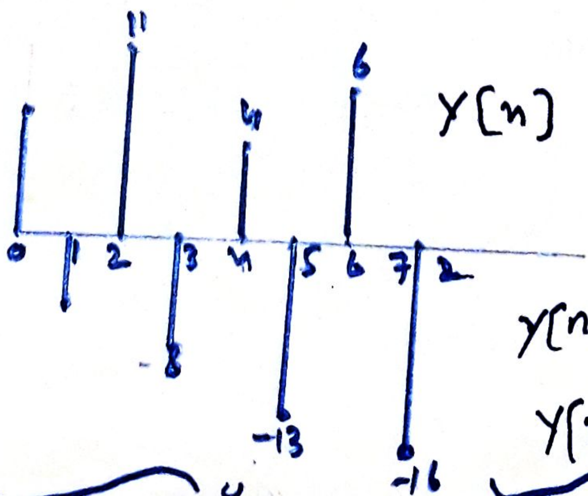


$n = 7$



$$Y(7) = x(3)\, h(3)$$
$$= 4 \times (-4)$$
$$= -16$$

Y(n]



$Y[n] = -1 + 11 + (-8) + 4 + (-13) + 8 + 16$

$Y[n] = 17$

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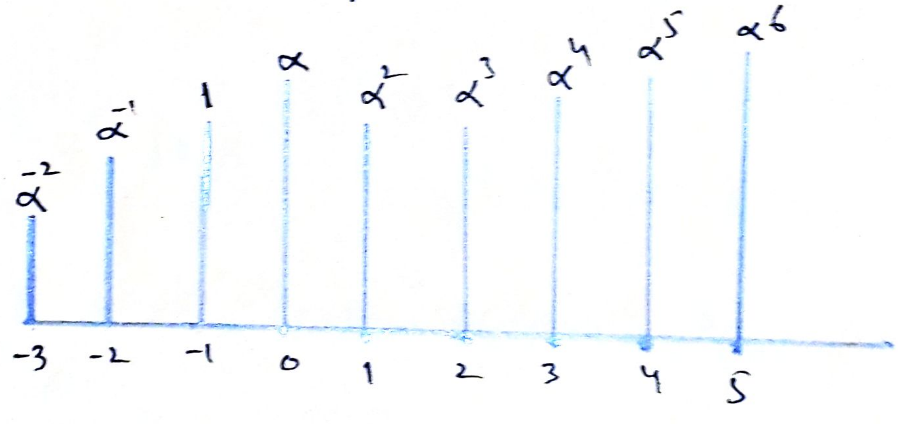
Q2 (b)

compute the convolution y(n) of the following signal.

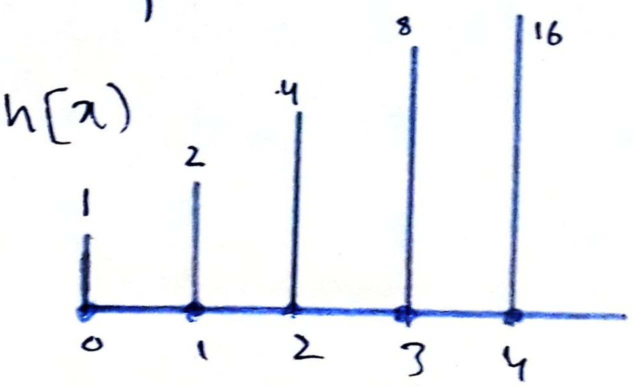
$$x(n) = \begin{cases} \alpha^{n+1} & , -3 \le n \le 5 \\ 0 & , \text{else} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & , 0 \le n \le 4 \\ 0 & , \text{else.} \end{cases}$$

Solution: $x(n) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$



$x(n) = \{1, 2, 4, 8, 16\}$



Muhammad Kamran (15) (13752)

$$Y[-3] = (\alpha^{-2})(1)$$

$$Y[-3] = \alpha^{-2}$$

$$Y[-2] = (\alpha^{-1})(1) + (\alpha^{-2})(2)$$

$$Y[-2] = \alpha^{-1} + \alpha^{-2}$$

$$Y[-2] = \alpha^{-3}$$

$$Y[-1] = (\alpha^{1})(1) + (2)(4) + (\alpha^{-1})(4) + (\alpha^{-2})(8)$$

$$Y[-1] = 1 \times 1 + 2\alpha^{-1} + 4\alpha^{-2}$$

$$Y[-1] = 6\alpha^{-3} + 1$$

$$Y[0] = (8)(\alpha^{-2}) + (\alpha^{-1})(4) + (1)(2) + (\alpha)(1)$$

$$Y[0] = 8\alpha^{-2} + 4\alpha^{-1} + 2 + \alpha$$

$$Y[0] = 12\alpha^{-3} + 2 + \alpha$$

$$Y[1] = \alpha^{-1}(8) + (1)(4) + (\alpha)(2) + \alpha^{2}(1)$$

$$= 8\alpha^{-1} + 4 + 2\alpha + \alpha^{2}$$

$$Y[2] = (\alpha^{-1})(16) + (1)(8) + (\alpha)(4) + (\alpha^{2})(2) + \alpha^{3}(1)$$

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$$Y[2] = (\alpha^{-1})(16) + (1)(8) + (\alpha)(4) + (\alpha^2)(2) + (\alpha^3)(1)$$

$$Y[2] = 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$$

$$Y[3] = (1)(16) + (\alpha)(8) + (\alpha^2)(4) + (\alpha^3)(2) + (\alpha^4)(1)$$

$$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$$

$$Y[4] = (\alpha)(16) + (\alpha^2)(8) + (\alpha^3)(4) + (\alpha^4)(2) + (\alpha^5)(1)$$

$$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$$

$$Y[5] = (\alpha^2)(16) + (\alpha^3)(8) + \alpha^4(4) + \alpha^5(2) + \alpha^6(1)$$

$$Y[5] = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$$

$$Y[6] = \alpha^3(16) + \alpha^4(8) + \alpha^5(4) + \alpha^6(2)$$

$$Y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$

$$Y[7] = (16)(\alpha^4) + \alpha^5(8) + \alpha^6(4)$$

$$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$

$$Y[8] = \alpha^5(16) + \alpha^6(8)$$

$$Y[9] = 16\alpha^6$$

$Y[10] = 0$ so No overlap in $Y[10]$

Q3:

Determine the z-transform of the following signals and Also sketch its region of convergence (ROC)

(i) $x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3)^{-n}, & n < 0 \end{cases}$

Solution;

As $$X(z) = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$$

$$= \sum_{-\alpha}^{\alpha} (1/4)^n u(n)$$

$$= \sum_{-\alpha}^{0} (1/3)^{-n} z^{-n} + \sum_{0}^{\alpha} (1/4)^n z^{-n}$$

$$= \sum_{-\alpha}^{0} (1/3\, z)^n + \sum_{0}^{\alpha} (1/4\, z^{-1})^n$$

$$= \frac{1}{1 - 1/3\, z} - 1 + \frac{1}{1 - 1/4\, z^{-1}}$$

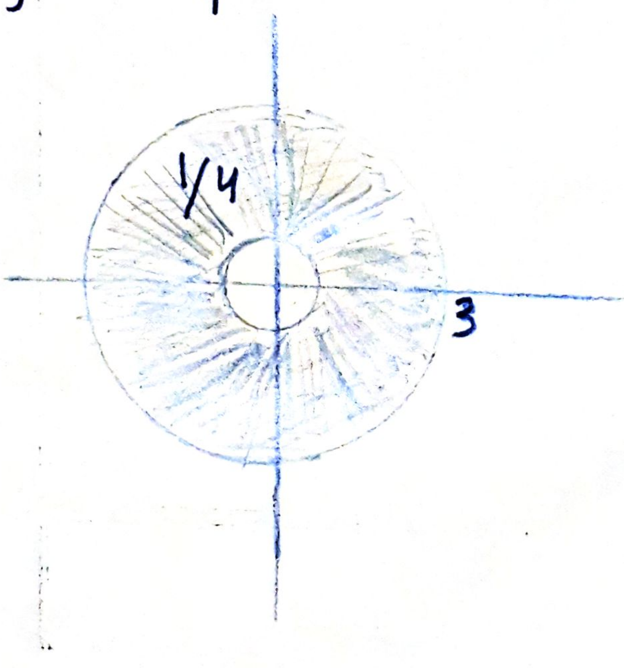
$$= \frac{(1 - 1/4\, z^{-1}) - (1 - 1/3\, z)(1 - 1/4\, z^{-1}) + (1 - 1/3\, z)}{(1 - 1/3\, z)(1 - 1/4\, z^{-1})}$$

$$= \frac{1 - \frac{1}{4}z^{-1} - \left(1 - \frac{1}{4}z^{-1} - \frac{1}{3}z + \frac{1}{12}\right) + 1 - \frac{1}{3}z}{\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{\cancel{1 - \frac{1}{4}z^{-1}} \cancel{- 1} + \cancel{\frac{1}{4}z^{-1}} + \cancel{\frac{1}{3}z} - \frac{1}{12} + 1 \cancel{- \frac{1}{3}z}}{\left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{1 - \frac{1}{12}}{\left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{11/12}{\left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right.}$$



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(ii) $x(n) = \begin{cases} (1/2)^n - 3, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

solution: $X(z) = \sum_{0}^{\alpha} (1/2)^n z^{-n} - \sum_{0}^{\alpha} 3^n z^{-n}$

$$= \sum_{n=0}^{\alpha} (1/2 z^{-1})^n - \sum_{n=0}^{\alpha} (3z^{-1})^n$$

$$X(z) = \frac{1}{1 - 1/2 z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{(1 - 3z^{-1}) - (1 - 1/2 z^{-1})}{(1 - 1/2 z^{-1})(1 - 3z^{-1})}$$

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} + 1/2 z^{-1}}{(1 - 1/2 z^{-1})(1 - 3z^{-1})}$$

$$= \frac{-3z^{-1} + 1/2 z^{-1}}{(1 - 1/2 z^{-1})(1 - 3z^{-1})}$$

$$\frac{z^{-1}(1/2 - 3)}{(1 - 1/2 z^{-1})(1 - 3z^{-1})}$$

$$X(z) = \frac{-\frac{5}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

So ROC is $|z| > 3$

$|z| > \frac{1}{2}$

Thus over all ROC is $|z| > \frac{1}{2}$