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SECTION A

Subject Earthquake & Dynamic

Subb-to : ENGR * YASEEN
MAHMOOD

Q NO 01

GIVEN DATA

$$E = 29000 \text{ KSI}$$

$$I = 150 \text{ in}^2$$

$$\delta_1 = 7767$$



Solution:-

The general E.O.M for SDOF System is

$$kU + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is unclamped ($c=0$) undergoing free vibration ($P(t)=0$)

Hence general EOM became

$$kU + m\ddot{u} = 0 \quad \dots \textcircled{1}$$

$$k = 3EI/L^3$$

put value.

$$k = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$k = 7.55 \text{ k/in}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft, sec or kg, m, sec

$$k = 7.55 \text{ k/in} = 90625 \text{ lb/ft}$$

$$m = \frac{7767 \text{ lb/ft}^2}{32.2 \text{ ft}}$$

$$m = 241.2$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90625}{241.2}}$$

$$\omega_n = 19.95 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.95} = 0.3149 \text{ sec}$$

Substituting the corresponding values

in eq (1)

$$kU + m\ddot{U} = 0$$

$$90625U + 241.2\ddot{U} = 0$$

Where "k" is in lb/ft and "m" is in
lb sec²/ft²

General Solution to the EOM for
Undamped free vibration is

$$U(t) = U(0) \cos(\omega_n t) + \frac{\dot{U}(0)}{\omega_n} \sin(\omega_n t)$$

$$U(0) = \frac{1}{2}, \quad \frac{1}{2 \times 12} = \frac{1}{247t} \quad \text{and} \quad \dot{U}(0) = 0$$

equivalent static force at any
time "t" is

$$F_s(t) = k \cdot U(t) = \frac{90625 \times \cos(19.95t)}{24}$$

$$F_s(t) = \cancel{241.2} = 3776 \cos(19.95t)$$

P.T.O

Amplitude of dynamic displacement
 U_0 for Undamped free vibration
is

$$U_0 = \sqrt{U(0)^2 + \left(\frac{\dot{U}(0)}{\omega_n}\right)^2}$$

$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0} \quad U(0) = 0$$

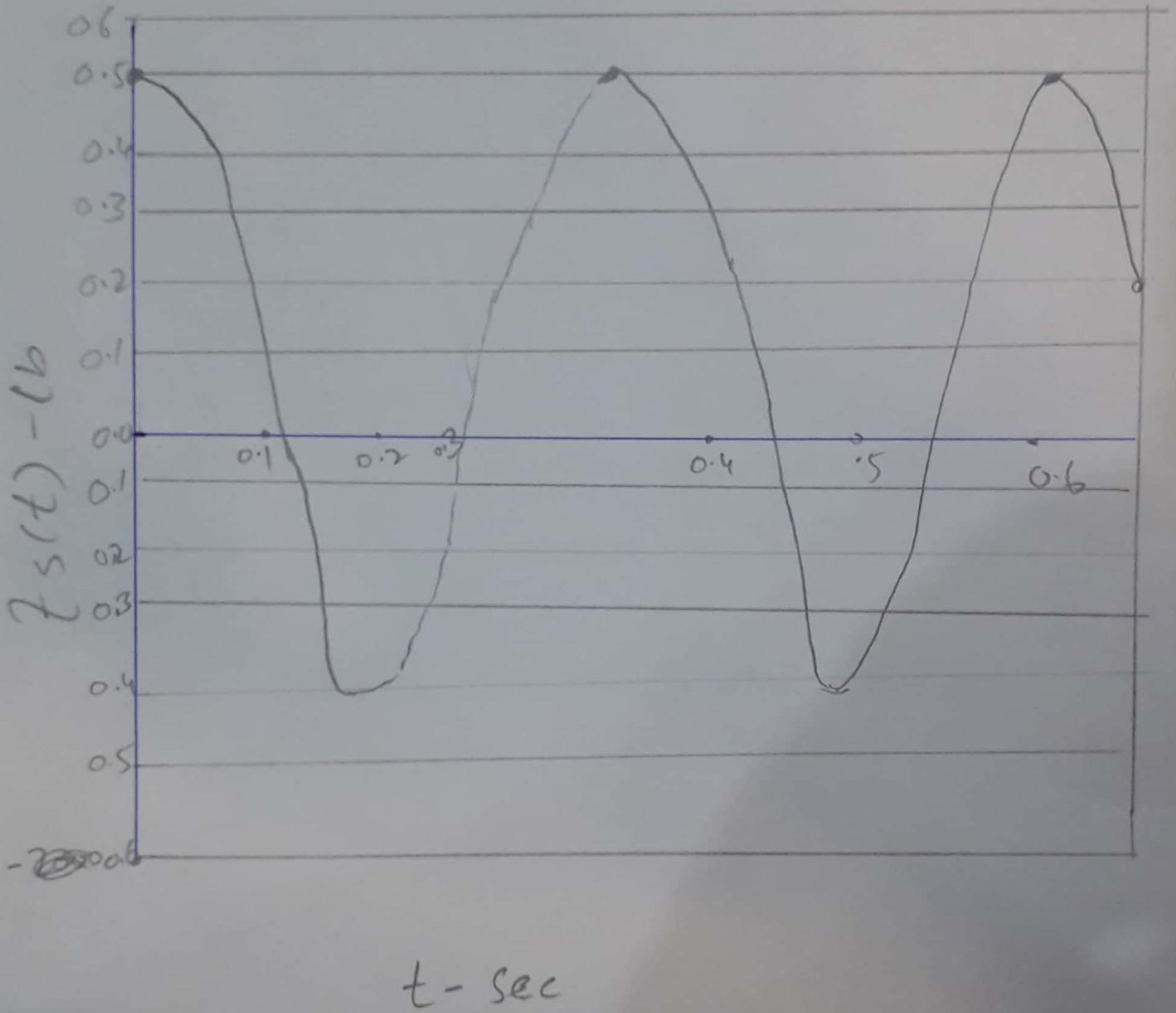
$$U_0 = \frac{1}{24} \text{ ft}$$

Amplitude of equivalent static force

F_{so}

$$kU_0 = 90625 \times \frac{1}{24} = 3776 \text{ lb}$$

Undamped free vibration



Variation of displacement with time.

Q NO # 03 2

ξ (Damping ratio) of Reinforced
Concrete with Considerable Cracking

= 3-5%

= 3%

Using Data of beam given in
Question 1

Required

• Develop & Solve the equation Showing
Variation in Equivalent Static Force
with Time

• Draw graph to Show the Variation
of displacement with Time &
the Variation of equivalent Static
force with time.

Sol:

(2)

E.O.M for damped free vibration is

$$kx + c\dot{x} + m\ddot{x} = 0 \quad - (1)$$

for Question 1

$$k = 90625 \text{ lb/ft}$$

$$m = 241.2$$

$$\omega_n = 19.95 \text{ rad/sec}$$

$$c = \zeta \times 2m\omega_n \quad \text{put value.}$$

$$c = (0.03) \times 2(241.2) \times (19.95)$$

$$c = 288.71 \text{ lb sec/ft}$$

Put value in eq \rightarrow (1)

$$kx + c\dot{x} + m\ddot{x} = 0 \quad - (1) \quad \text{put value.}$$

$$= 90625x + 288.71\dot{x} + 241.2\ddot{x} = 0$$

Solution to the EOM for damped free vibration is

$$U(t) = e^{-\zeta \omega_n t} \left(U(0) \cos(\omega_D t) + \frac{1}{\omega_D} \left[\dot{U}(0) + \zeta \omega_n U(0) \right] \sin(\omega_D t) \right)$$

$$\omega_D = 19.95 \text{ rad/sec}$$

$$U(t) = e^{-0.03 \times 19.95} + \left[\frac{1}{24} \times \cos(19.95t) + \frac{1}{19.95} \right] \times \left[0 + \frac{1}{24} \times 0.03 \times 19.95 \right] \times \sin(19.95t)$$

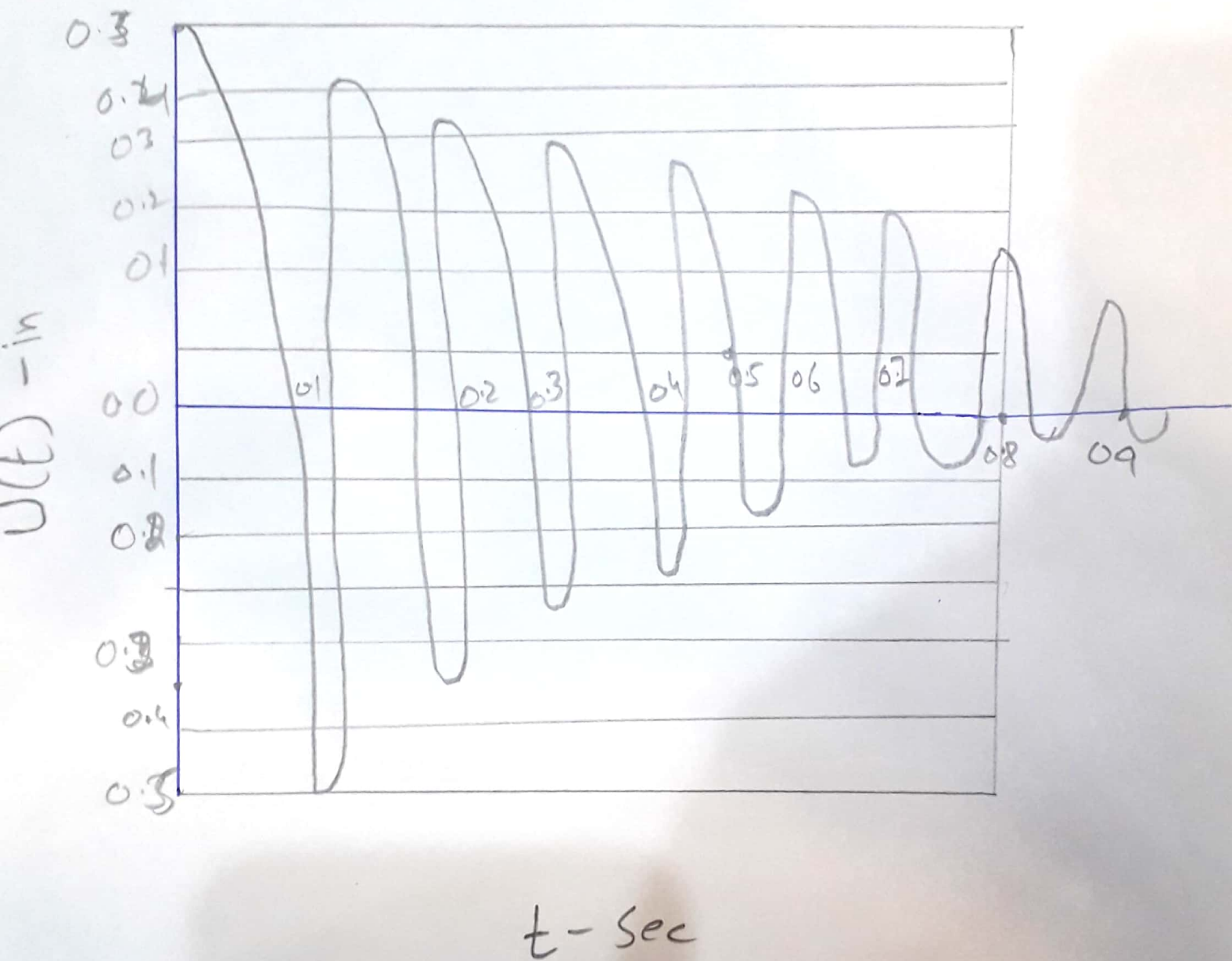
$$U(t) = e^{-0.598} + \left[0.041 \times \cos(19.95t) + 0.00125 \right] \times \sin(19.95t)$$

$$f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u(t)$$

$$f_s(t) = e^{-0.598t} \left[(90625 \times 0.041) \cos(19.95t) \right. \\ \left. + (90625 \times 0.00125) \sin(19.95t) \right]$$

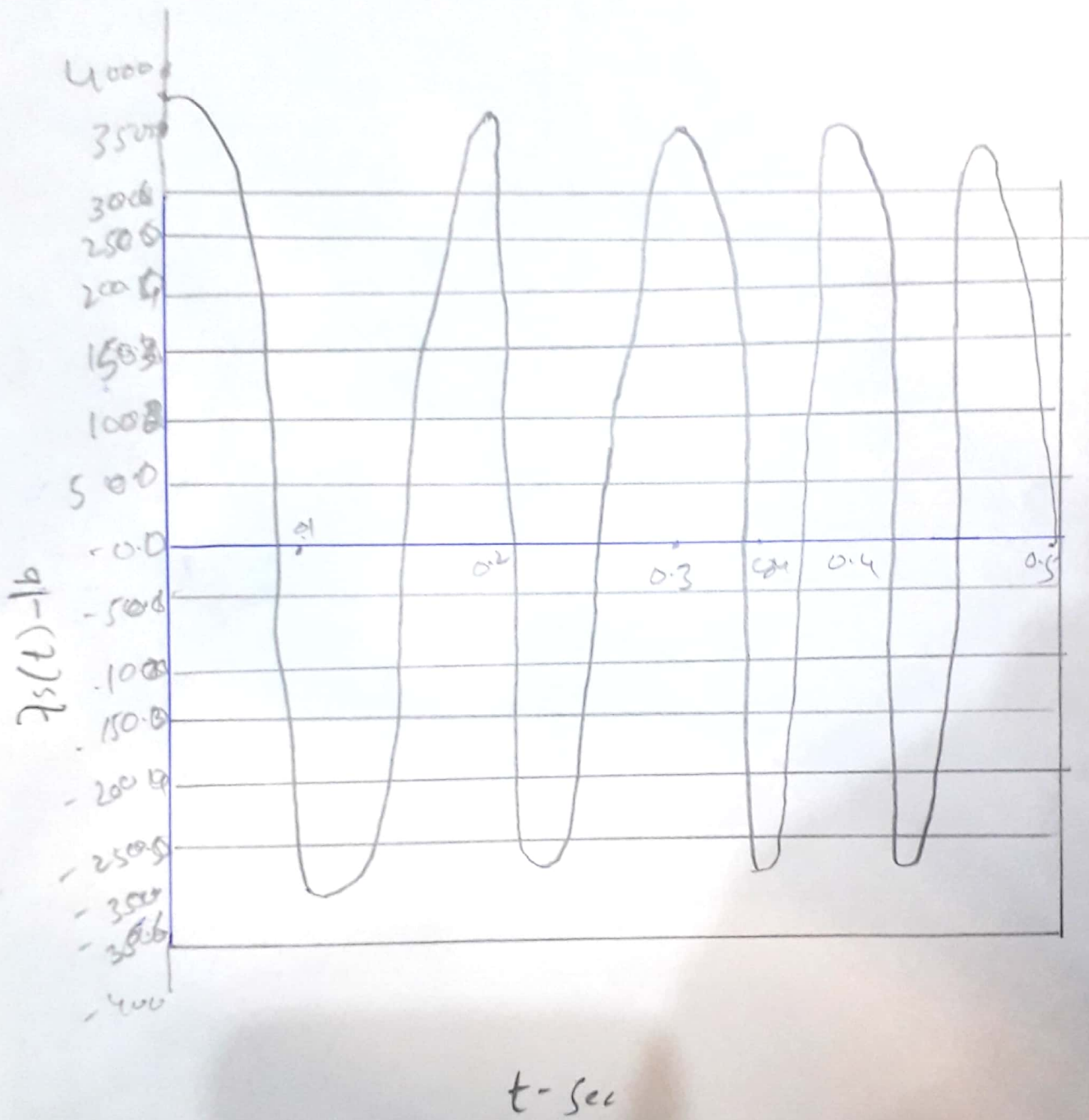
$$\Rightarrow e^{-0.598t} \left(3715.62 \cos(19.95t) + 113.28 \sin(19.95t) \right)$$

$$= 1941.89$$



Variation of displacement with time.

Damped Free Variation



Variation of Equivalent Static Forces
with time -

Q NO NO 3

GIVEN DATA :-

Amplitude cable force = 60 kips

• Horizontal displacement of tank = $\frac{7767}{1000}$
= 7.767 in

• Cycles = 7

• Cycle Completion Time = 3.57 sec

• Amplitude of displacement = 2.286 cm
= 0.9 in

Required DATA :-

- Damping ratio
- Natural period of undamped vibration
- Stiffness of Structures
- Weight of tank
- Damping coefficient
- Number of cycle to reduce the displacement amplitude to = 0.5

Solution:-

As given in equation

$$u_1 = 7.767 \text{ in}$$

After $J = 7$

$$u_{T+1} = u_8 = 2.286 \text{ cm} = 0.9 \text{ in}$$

a = ζ = Damping ratio = ?

$$J = \frac{1}{2\pi\zeta} \ln \left[\frac{u_1}{u_{s+1}} \right]$$

By putting values we get

$$7 = \frac{1}{2\pi\zeta} \times \ln \left(\frac{7.767}{0.9} \right)$$

$$\zeta = 0.0490 = 4.90\% \text{ Ans}$$

$$\zeta = 4.90\%$$

$$\boxed{\zeta = 4.90\%} \text{ Ans}$$

(b) Natural period of undamped (1)
Vibration = $T_n = ?$

As the 7 cycles of vibration are
Completed in 3.57 Sec

⇒ Time required to complete one
cycle, $T_D = \frac{3.57}{7} = 0.51 \text{ sec}$

$$T_D = 0.51 \text{ Sec}$$

Now,

$$\omega_D = \omega_n \sqrt{1 - \zeta^2}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n(1 - \zeta^2)}$$

$$\Rightarrow T_D = \frac{T_n}{(1 - \zeta^2)}$$

$$T_n = T_D \times \sqrt{1 - \zeta^2}$$

$$T_n = 0.51 \times \sqrt{1 - (0.0490)^2}$$

$$T_n = 0.5094 = 0.51 \text{ sec}$$

C Stiffness Structure $K = ?$

$$K = \frac{60 \times \cos 60}{7.767} = 3.862 \text{ K/in}$$

$$K = 46344 \text{ lb/ft}$$

D weight of tank $W = ?$

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\frac{W}{g}}} = \sqrt{\frac{K \cdot g}{W}}$$

$$\Rightarrow W_n^2 = \frac{K \cdot g}{W}$$

$$W = \frac{K \cdot g}{W_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$|x| = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46344 \text{ lb}}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2} \right] \times \left(\frac{0.51 \text{ sec}}{4\pi^2} \right)$$

$$W = 9831.73 \text{ lb} = 9.8319$$

e Damping coefficient, $c = ?$

It is known that

$$c = \frac{c}{2m\omega_n}$$

$$C = c \times 2m\omega_n$$

P.T.O

$$C = (x \cdot 2m) \times \left(\frac{2\pi}{T_n} \right)$$

$$C = (0.0490) \times 4 \times \pi \left(\frac{9831.73}{32.2} \right)$$

$$0.51$$

$$C = 368.46 \text{ lb}\cdot\text{sec}/\text{ft}$$

7 Number of cycles to reduce
the displacement amplitude to 0.5"

$$J = \frac{1}{2\pi f} \ln \left(\frac{U_i}{U_{i+1}} \right)$$

$$J = \frac{1}{2\pi \times 0.0490} \ln \left(\frac{7.767}{0.5} \right)$$

$$J = 8.90 \text{ or } 9 \text{ cycles}$$