MID TERM EXAM

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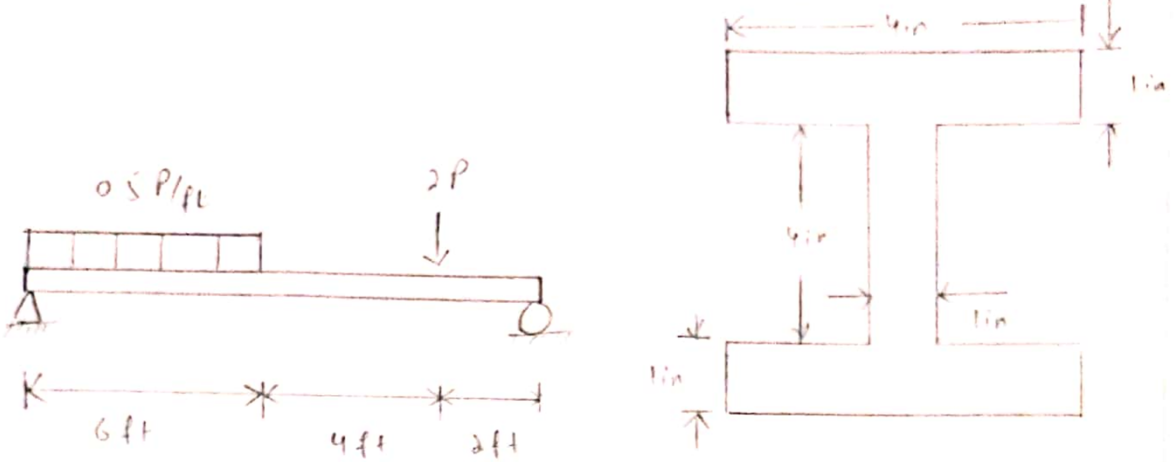
SECTION	B
DEPT	BE (C)
SUBJECT	MOS II
SEMESTER	4th
SUBMITTED TO	Engr. Saqib Khan
DATE	18-April-2020

QUESTION

Construct the Mohr's circle diagram .... the stress transformation equation?



Where as,

The value of $P = 32$

Sol

PART A:

i) REACTIONS:-

$\Sigma F_y = 0$ $\quad$ $+\uparrow$ upward +ive

$\Rightarrow R_A + R_B - (0.5 \times 32 \times 6) - 2(32) = 0$

$\Rightarrow R_A + R_B - 96 - 64 = 0$

$\Rightarrow R_A + R_B = 96 + 64$

$R_A + R_B = 160$ —— (i)

$\Sigma M_A = 0$

$(R_A \times 12) - (64 \times 10) - (96 \times 3) = 0$

$\Rightarrow 12 R_B - 640 - 288 = 0$

$R_B = 928/12$

$$\boxed{R_B = 77.333\ lb}$$

Putting the value of R_B in eq (i)

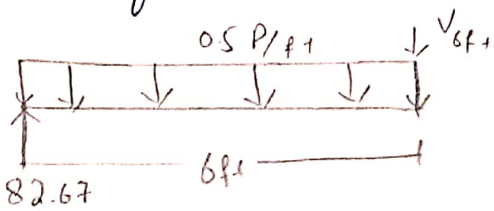
$$R_A + 77.333 = 160$$

$$R_A = 160 - 77.33$$

$$\boxed{R_A = 82.67\ lb}$$

ii) SHEAR FORCE :~

-> First of all we will find the shear force at 6ft.



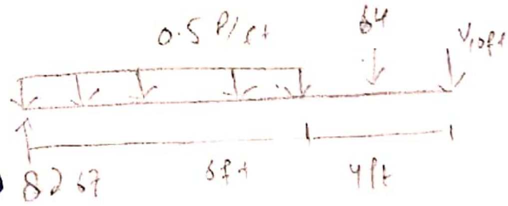
$$\Sigma F_y = 0$$

$$\Rightarrow -V_{6ft} + 82.67 - 96 = 0$$

$$-V_{6ft} - 13.33 = 0$$

$$\boxed{V_{6ft} = -13.33\ lb}$$

-> Secondly we will find at 10ft



$$\Sigma F_y = 0$$

$$\Rightarrow 82.67 - 96 - 64 - V_{10ft} = 0$$

$$\Rightarrow -V_{10ft} - 77.33$$

$$\Rightarrow \boxed{V_{10ft} = -77.33\ lb}$$

iii) BENDING MOMENT :-

$$\Sigma M_{6ft} = -(82.67 \times 6) + (16 \times 6)(6/2)$$

$$\Sigma M_{6ft} = -496.02 + 288$$

$$\boxed{\Sigma M_{6ft} = 208.02 \text{ lbft}}$$

$$\Sigma M_{3ft} = -(82.67 \times 3) + (16 \times 6 \times 3)$$

$$= -248.01 + 288$$

$$\boxed{\Sigma M_{3ft} = 39.99 \text{ lbft}}$$

Now we will find the moment at changing Point

$$\frac{82.62}{x} = \frac{13.33}{6-x}$$

$$82.62 \times (6-x) = 13.33x$$

$$495.72 \cancel{+} 82.62x = 13.33x$$

$$495.72 = 13.33x + 82.62x$$

$$495.72 = 95.95x$$

$$\frac{495.72}{95.95} = x$$

$$\boxed{x = 5.16 \text{ ft}}$$

$$\Sigma M_{5.16ft} = 0$$

$$\Rightarrow M_{5.16ft} - (82.67 \times 5.16) + (.16 \times 6 \times \frac{5.16}{2}) = 0$$

$$\Rightarrow \boxed{M_{5.16ft} = 178.89 \text{ lbft}}$$

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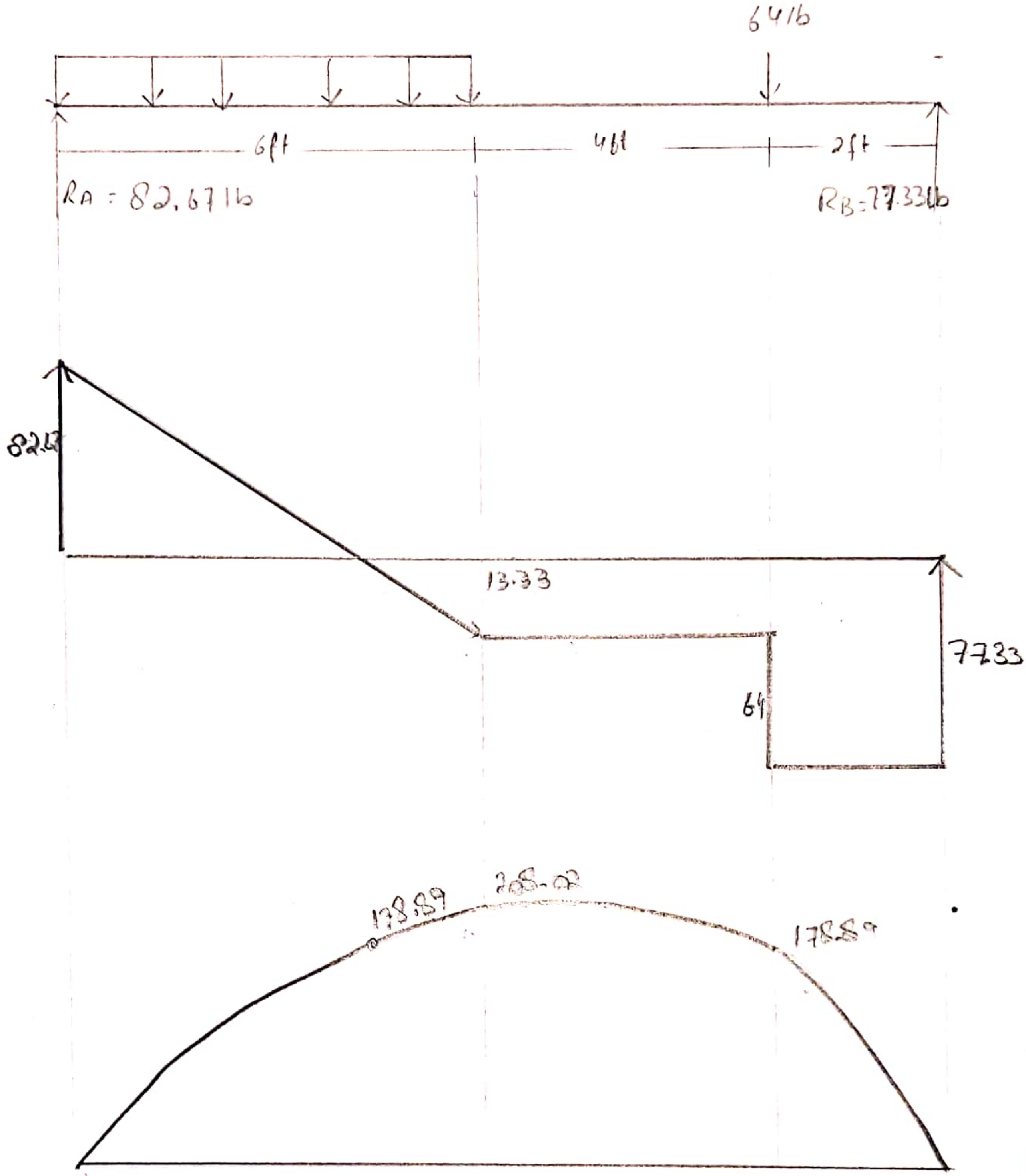
SHEAR FORCE AND BENDING MOMENT DIAGRAM.



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PART B:

Now we have to find the moment of Inertia of the beam cross section

$y_1 = 5.5$

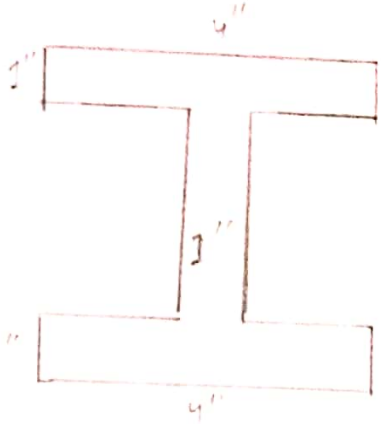
$y_2 = 3$

$y_3 = 0.5$

$A_1 = 4\ in^2$

$A_2 = 4\ in^2$

$A_3 = 4\ in^2$



$$\bar{y} = \frac{(A_1 \times y_1)(A_2 \times y_2) \times (A_3 \times y_3)}{A_1 + A_2 + A_3}$$

$$y = \frac{(4 \times 5.5)(4 \times 3)(4 \times 0.5)}{4 + 4 + 4}$$

$$\boxed{\bar{y} = 3''}$$

Now $I_1 = \dfrac{bh^3}{12}$

$$= \frac{4 \times 1^3}{12}$$

$$\boxed{I_1 = 0.33\ inch^4}$$

$$I_2 = \frac{bh^3}{12}$$

$$I = \frac{1 \times 4^3}{12}$$

$$\boxed{I_2 = 5.33\ inch^4}$$

$\bar{I}_3 = \frac{bh^3}{12}$

$= \frac{4 \times 1^3}{12}$

$\boxed{\bar{I}_3 = 0.33 \text{ inch}^4}$

Now for d

$d_1 = \bar{y} - y_1$

$\Rightarrow d_1 = 3 - 5.5$

$\boxed{d_1 = -2.5}$

Ad^2

$A_1 d_1^2$

$\Rightarrow 4 \times (-2.5)^2$

$\Rightarrow 25 \text{ inch}^4$

$d_2 = \bar{y} - y_2$

$\Rightarrow d_2 = 3 - 3$

$\boxed{d_2 = 0}$

$A_2 d_2$

$= 4 \times (0)^3$

$= 0$

$d_3 = \bar{y} - y_3$

$\Rightarrow d_3 = 3 - 0.5$

$\boxed{d_3 = 2.5}$

$A_3 d_3^3$

$\Rightarrow 4 \times (2.5)^2$

$\Rightarrow 25 \text{ inch}^4$

Now

$\bar{I}_{1x} = \bar{I}_1 + A_1 d_1^2$

$= 0.33 + 25$

$\boxed{\bar{I}_{1x} = 25.33 \text{ inch}^4}$

$\bar{I}_{2x} = \bar{I}_2 \; A_2 d_2^2$

$= 0 + 5.33$

$\boxed{\bar{I}_{2x} = 5.33 \text{ inch}^4}$

$$I_{3x} = I_3 + A_3 d_3^2$$

$$= 0.33 + 25$$

$$\boxed{I_{3x} = 25.33 \text{ inch}^4}$$

Now

$$I_{xx} = I_{1x} + I_{2x} + I_{3x}$$

$$= 25.33 + 5.33 + 25.33$$

$$\boxed{I_{xx} = 55.99 \cong 56 \text{ inch}^4}$$

PART C :

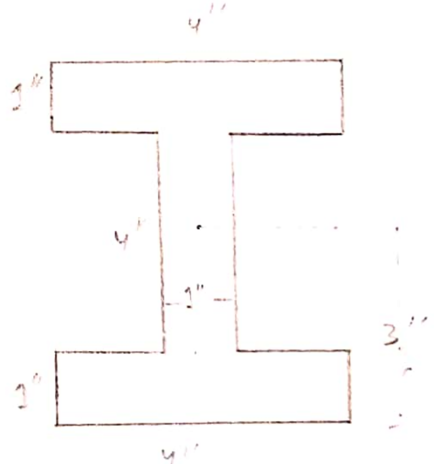
SHEAR STRESS :-

CASE I :

We have to find the shear stress at top fiber.

As we know that

$$\tau = \frac{VQ}{It}$$



As top fiber $A = 0$

$Q = \bar{y}A$ $\quad \because \bar{y} = 3"$

$Q = 3" \times 0 = 0$

$V = 77.33$, $I = 56$, $b = 4 = t$

$$\tau = \frac{77.33 \times 0}{56 \times 4}$$

$\boxed{\tau = 0}$ Psi

CASE II :-

Now find the stress 1 inch below the top fiber but here we have two cases.

$$\bar{y} = 2 + \frac{1}{2} = 2.5$$

$$A = 1.4 \times 4$$

$$Q_A = \bar{y} A$$

$$Q_A = 2.5 \times 4 = 10$$

$$b_A = 4 \text{inch}$$

So

$$\tau_A = \frac{V Q_A}{I b_A}$$

$$= \frac{(77.33)(10)}{56 \times 4}$$

$$\boxed{\tau_A = 3.452 \text{ Psi}}$$

$$\tau_B = \frac{V Q_B}{I b_B} \qquad \because Q = 2.5 \times 4 = 10$$

$$b_B = 1$$

$$= \frac{(7.33)(10)}{56 \times 1}$$

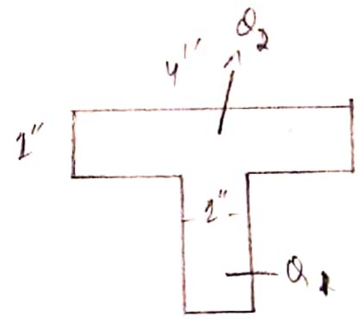
$$\boxed{\tau_B = 13.808 \text{ Psi}}$$

CASE III :-

Find stress at centroidal axis

$$Q = Q_1 + Q_2 \quad -(i)$$

$$Q_1 = \bar{y}, A_1 = \tfrac{2}{2}(1 \times 2) = 2$$



$$Q_2 = 2.5 \times 4 = 10$$

So

$$Q = 2 + 10 \qquad \because b = 1 \text{ inch}$$

$$Q = 12$$

So,

$$\tau_{max} = \frac{VQ}{Ib}$$

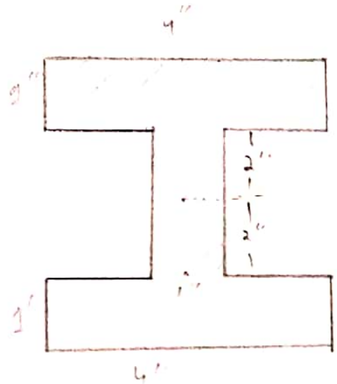
$$= \frac{77.33 \times 12}{56 \times 1}$$

$$\boxed{\tau_{max} = 16.570 \text{ Psi}}$$

CASE IV :-

Find Shear Stress from I in above the fiber

$$\tau_A = \frac{77.33 \times (2.5 \times 4)}{56 \times 1}$$



$$\boxed{\tau_A = 13.808 \text{ Psi}}$$

$$\tau_B = \frac{77.33 \times (2.5 \times 4)}{56 \times 4}$$

$$\boxed{\tau_B = 3.452 \text{ Psi}}$$

CASE V :~

Find shear stress at bottom fiber

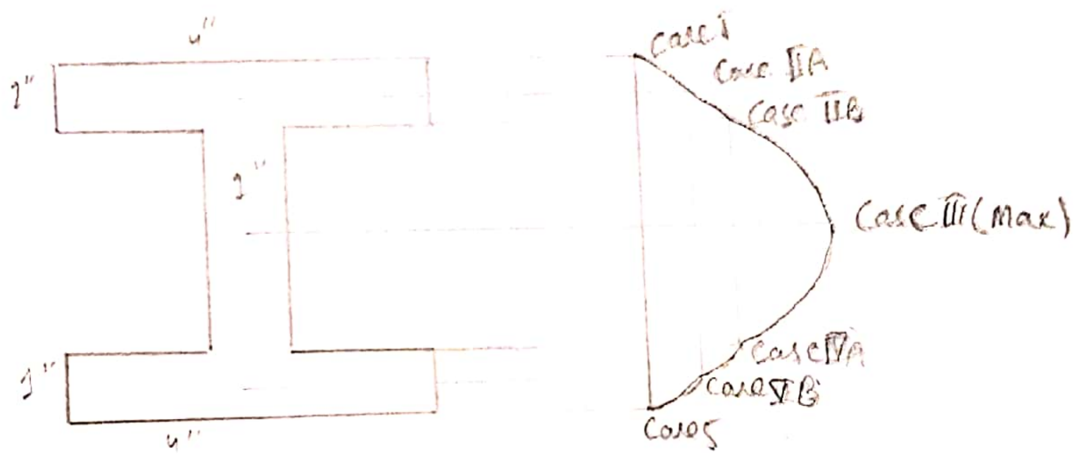
Hence $\bar{y} = 0$ & $Q = 4 \times 0 = 0$

$$\tau = \frac{VQ}{Ib}$$

$$= \frac{77.33 \times 0}{56 \times 1}$$

$$\boxed{\tau = 0 \text{ Psi}}$$

SHEAR STRESS VARIATION DIAGRAM :~



CASE VI :~

We have to find the maximum shear stress at a distance of 6ft from left support of beam along its length

As we know that shear force at 6ft is

$V = 13.33 \text{ lb}$

$Q = 12$ (From case III)

$$\tau_{max} = \frac{VQ}{IB}$$

$$\tau_{max} = \frac{13.33 \times 12}{56 \times 1}$$

$$\boxed{\tau_{max} = 2.856 \text{ Psi}}$$

CASE VII :~

Now we have to find the shear stress at a distance of 6ft from left support and 1 inch below the top fiber.

$$\Rightarrow \tau = \frac{VQ}{IB}$$

$Q = 10$
$b = 4$ inch (from Case IIA)

$$\tau = \frac{83.33 \times 10}{56 \times 4}$$

$$\boxed{\tau = 0.595 \text{ Psi}}$$

FLUXURE STRESS ANALYSIS :-

As we know that

$$\sigma = \frac{My}{I}$$

The maximum bending moment is 256.08 lbft

CASE I :~

Find stress at top fiber

$$\sigma_{Top} = \frac{(208.02)(3)}{56} \qquad \therefore y = 3$$

$$\boxed{\sigma_{Top} = 11.14 \text{ Psi}}$$

CASE II :~

Find stress at 1 inch below Top fiber

$$\sigma_{1in} = \frac{(208.02)(2)}{56}$$

$$\boxed{\sigma_{1inch} = 7.42 \text{ Psi}}$$

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CASE III :~

Find the fluxural stress at Centriodial axis

$$\sigma_{center} = \frac{(208.02)(0)}{56}$$

$$\boxed{\sigma_{center} = 0 \text{ Psi}}$$

CASE IV :~

Find flexure stress at 1 inch above the bottom fiber.

$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{(208.02)(2)}{56} \qquad \because y = 2$$

$$\boxed{\sigma = 7.429 \text{ Psi}}$$

CASE V :~

Find flexural stress at the bottom fiber

$$\sigma_{Bottom} = \frac{My}{I}$$

$$\Rightarrow \sigma_{Bottom} = \frac{208.02 \times 3}{56} \qquad \because y = 3$$

$$\boxed{\sigma_{Bottom} = 11.14 \text{ Psi}}$$

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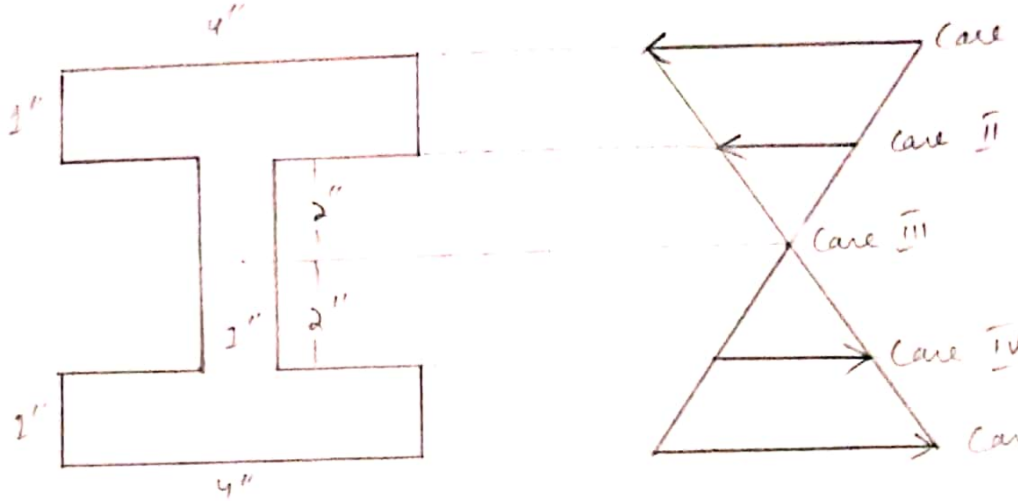
FLEXURE STRESS VARIATION DIAGRAM :-



SHEAR STATE OF A POINT ELEMENT :-

Now we have to find stress state of a point element located at a distance of 3 ft from left support and 1 inch below the top fiber.

As to find the condition of stressed element at point C in this given I section. It require to find all the stresses at this point.

As in the given problem the stresses acting on point C is flexural and shear stresses. There is no torsional stresses forces acting on this beam. due to load symmetry along the beam axis (longitudinal axis)

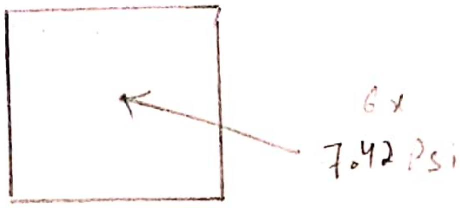
Flexure stress at point C

$\sigma = 7.41$ Psi (from flexural stress Case II)

Shear Stress at Point "C"

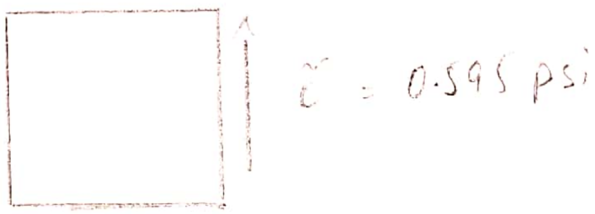
$\tau = 0.595$ Psi from Case VII of Shear stress

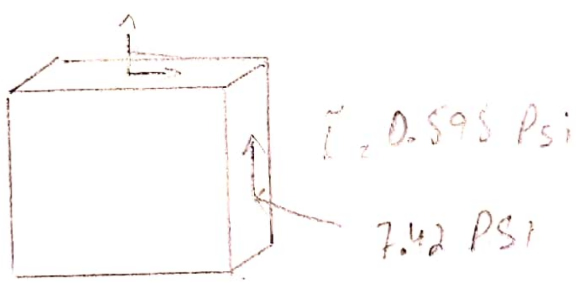
Consider the Point "C" is a planer element



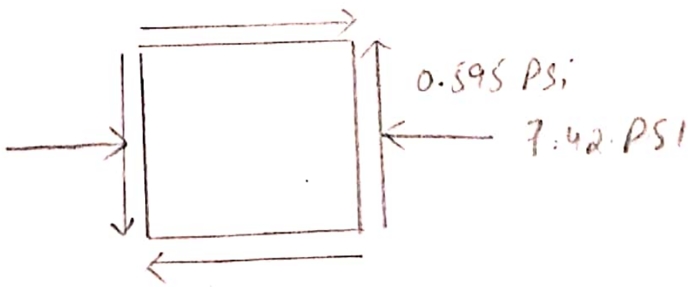
9.14 Psi is compressive because point C lies in compression zone of beam cross section.

If the point C lies below the centroidal than stress would be tensile.





Combining Stresses on 2d element



PART D :-

PRINCIPLE STRESS :-

In this we have to find θp

$$\tan 2\theta p = \frac{\tau}{(\sigma x - \sigma y)/2}$$

$$\tan 2\theta p = \frac{0.595}{-7.42 - 0/2}$$

$$2\theta p = \tan^{-1}(-0.16)$$

$$\theta p = \frac{9.111}{2}$$

$$\boxed{\theta p = -4.55}$$

Now for $\sigma x'$

$$\sigma x' = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\Rightarrow \sigma x' = \frac{-7.42 + 0}{2} + \left(\frac{-7.42 + 0}{2}\right) \cos 2(-4.55) + 0.595 \times \sin 2(-4.55)$$

$$\boxed{\sigma x' = -7.467 \text{ psi}}$$ (Compression)

$\Rightarrow$ Now for $\sigma y'$

$$\sigma y' = \frac{\sigma x + \sigma y}{2} - \frac{\sigma x - \sigma y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\Rightarrow \sigma y' = \frac{7.42 + 0}{2} - \left(\frac{7.42 - 0}{2}\right) \cos 2(-4.55) + 0.595 \times \sin 2(455)$$

$$\boxed{\sigma y' = -0.140 \text{ Psi}}$$ Compression

Now

Shear Plane

$$\tan 2\theta_p = \frac{(-\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$= \frac{(-7.42 - 0)/2}{0.596}$$

$$2\theta_p = \tan^{-1}(-2.80716)$$

$$\theta_p = \frac{-70.39}{2}$$

$$\boxed{\theta_p = 35.196}$$

$$\tau_{x'y'} = -\left(\frac{-\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-7.42 - 0}{2}\right)\sin 2(35.196) + 0.595\cos 2(35.196)$$

$$\Rightarrow \boxed{\tau_{x'y'} = 3.694}$$

STRESS TRANSFORMATION :-

Now we have to find the stress state condition of point C at a clockwise orientation where $\theta = -25^\circ$ (assumed)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\Rightarrow \sigma_{x'} = \left(\frac{-7.42 + 0}{2}\right) + \left(\frac{-7.42 - 0}{2}\right)\cos 2(-25) + 0.598\sin 2(-25)$$

$$\Rightarrow \boxed{\sigma_{x'} = -6.550}$$ Compression

$$\sigma_{y'} = \frac{\sigma_x - \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\Rightarrow \sigma_{y'} = \frac{-7.42+0}{2} - \frac{-7.42-0}{2}\cos 2(25) - 0.595\sin 2(-25)$$

$$\boxed{\sigma_{y'} = -0.869\ Psi}$$ Compression

Now

$$\tau_{x'y'} = \frac{-\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y'} = \frac{7.42-0}{2}\sin 2(-25) + 0.595\cos 2(-25)$$

$$\boxed{\tau_{x'y'} = -2.45\ Psi}$$

MOHR'S CIRCLE :-

Coordinates :-

$$(h, K) = \left(\frac{-7.42}{2}, 0\right)$$

$$(h, K) = (-3.71, 0)$$

RADIUS :-

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{-7.42-0}{2}\right)^2 + (0.595)^2}$$

$$\boxed{r = 3.64}$$

$V = 7.42$

Scale

$1 psi = 1 cm$

$\tau_{x'y'} = 0.595$

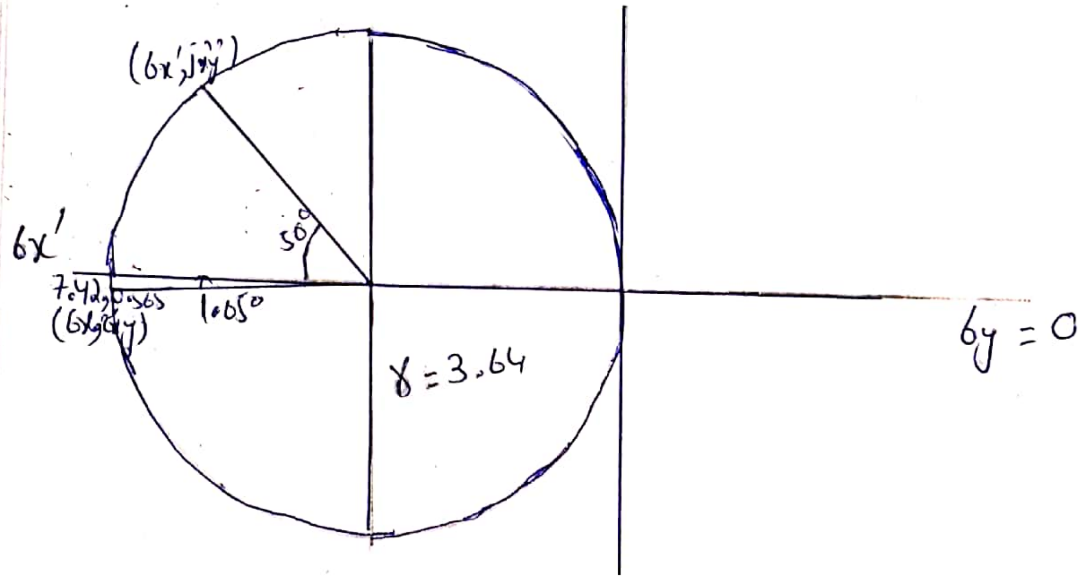
Now

$\theta = \tan^{-1}\left(\frac{0.596}{7.42+25}\right)$

$\Rightarrow \theta = 1.05^\circ$

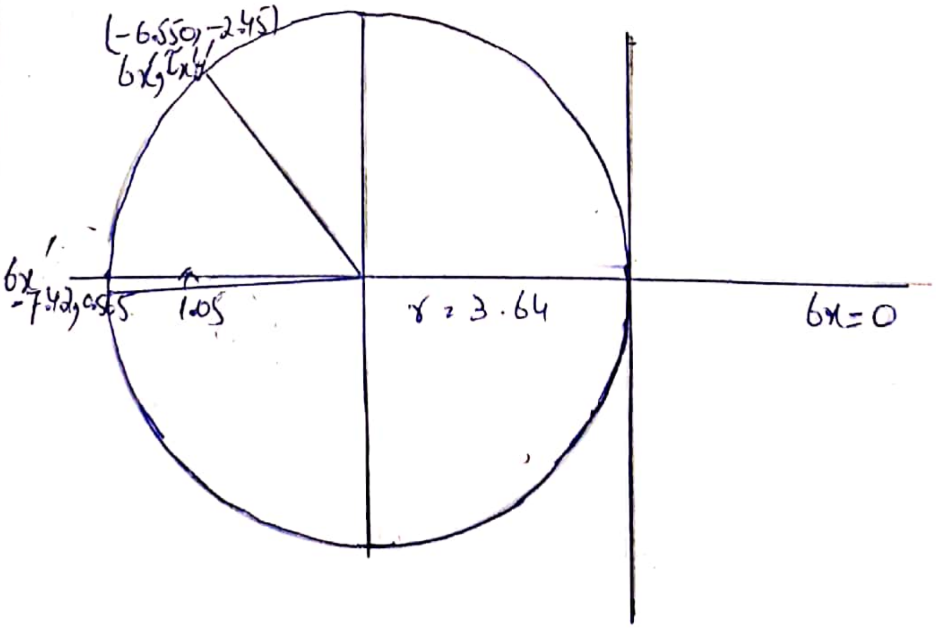
MOHR'S CIRCLE :-

$\sigma_x = -7.42$, $\sigma_y = 0$ $\tau_{xy} = 0.595$



FOR NEW ORIENTATION (-25°) Clockwise)

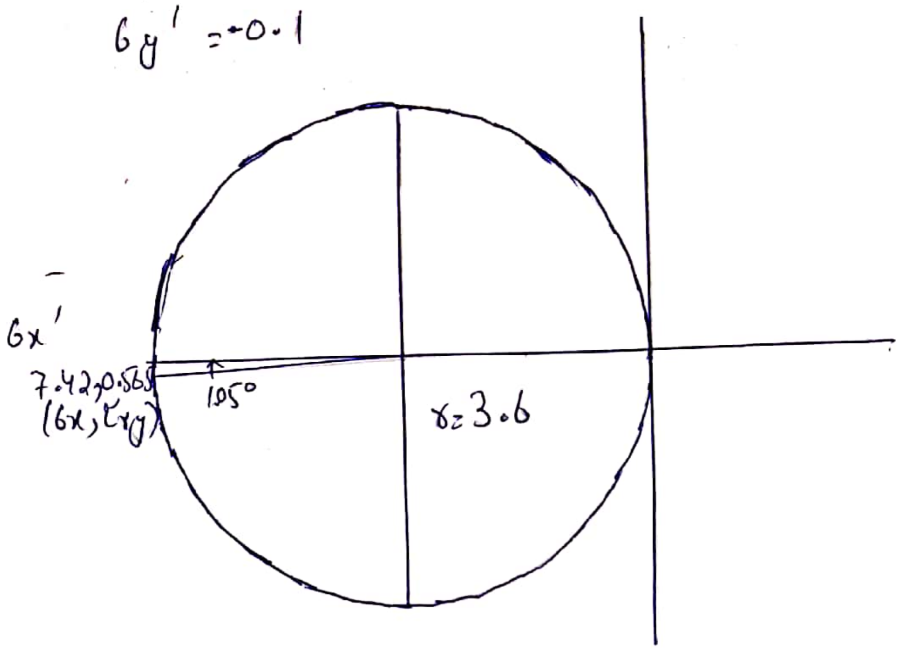
$\sigma_{x'} = -6.550$ Psi $\tau_{x'y'} = -2.45$ Psi



For Principle Stresses :-

$\sigma_{x'} = -7.467$

$\sigma_{y'} = -0.1$



Scale 1psi = 1cm

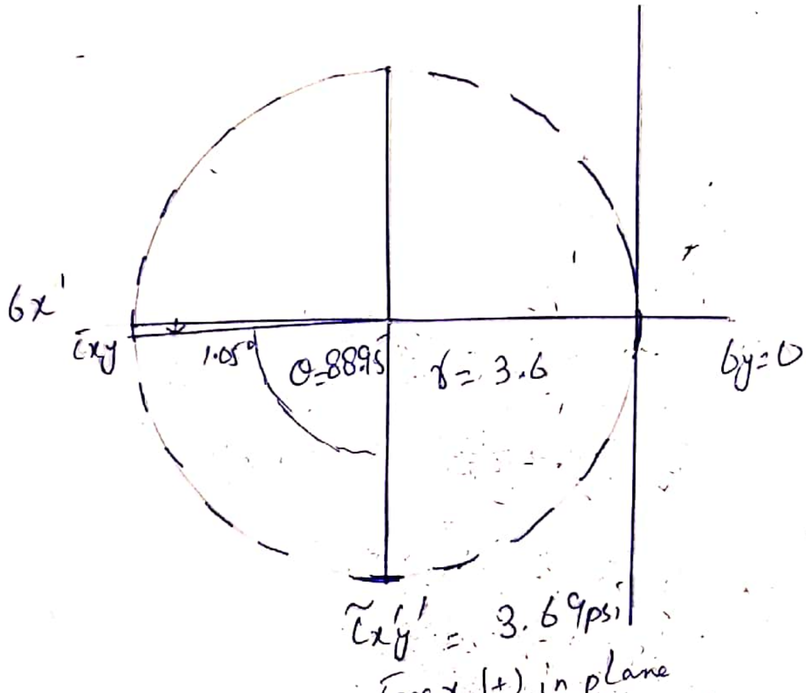
$r = 3.6$

$\theta_p = \frac{1.05}{2}$

$\theta_p = 0.525$

For Shear Stresses :~

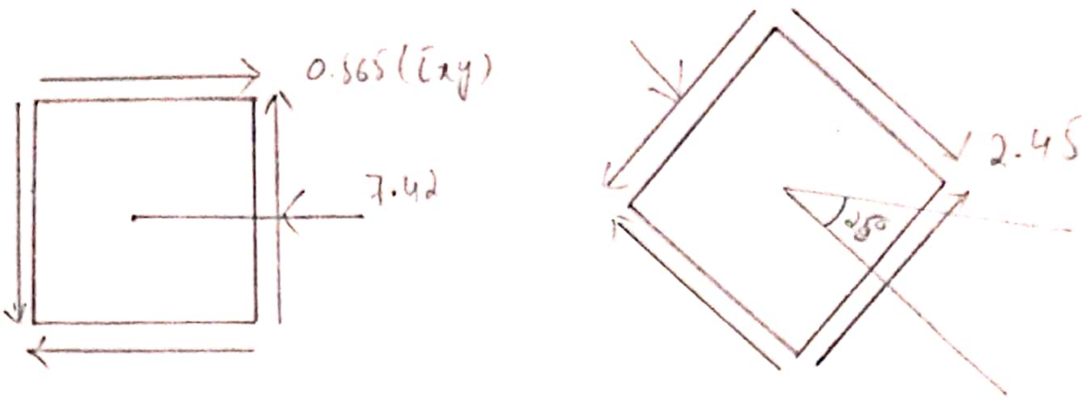
$\tau_{x'y'} = 3.69$



$\theta = \frac{88.95}{2}$

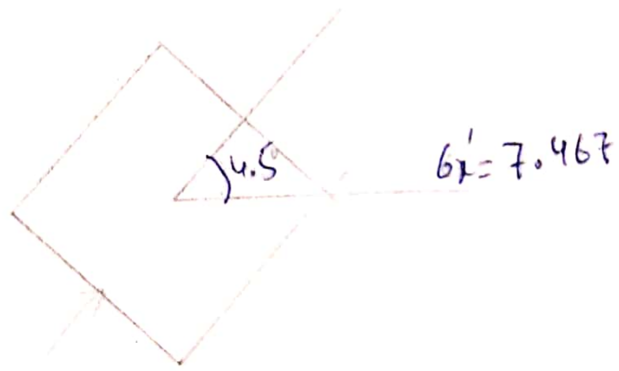
$\theta_3 = 44.475°$

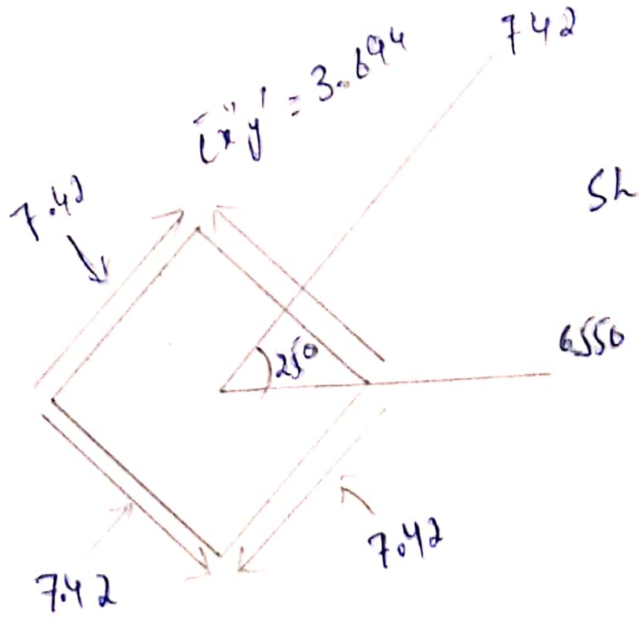
Comparison Of Mohr's Circle With Shear Transformation :-



Principle And Plan Shear Stress :-

Principle





Shear

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