Name:- Qazi Azmat Ullah
ID:- 14448
Subject:- Calculus and Analytical geometry
Program:- (BSSE)

Question # 1

A:

$$\frac{3x^2-5x^2+5}{x^2+1}$$

Change into product form

$(3x^2-5x^2+5)(x^2+1)^{-1}$

Let $y = (3x^2-5x^2+5)(x^2+1)^{-1}$

differenit with respect to "x"

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{(3x^2-5x^2+5)}_{\mathrm{I}}\underbrace{(x^2+1)^{-1}}_{\mathrm{II}}$$

$$= (3x^2-5x^2+5)\frac{d}{dx}(x^2+1)^{-1}+(x^2+1)^{-1}\frac{d}{dx}(3x^2-5x^2+5)$$

$$= (3x^2-5x^2+5)(-1(x^2+1)\frac{d}{dx}(x^2+1)+1(x^2+1)^{-1}(9x-10x)$$

$$(3x^2-5x^2+5)(-x^2-1)^{-2}(2x)+(x^2+1)^{-1}(9x-10x)$$

$$= 3x^2-5x^2+5)(-x^2-1)^{-2}(2x)+(x^2+1)^{-1}(9x-10x)$$

$$= (10x^3-10x^3+10x)(-x^2-1)^{-2}+(x^2+1)^{-1}(9x-10x)$$

Question 1:-

B:- $\frac{(x^2+1)^2}{x^2-1}$

Solution

let $y = \frac{(x^2+1)^2}{x^2-1}$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(x^2+1)^2}{x^2-1}\right]$$

$$= \frac{(x^2-1)\,d/dx\,(x^2+1)^2 - (x^2+1)^2\,d/dx\,(x^2-1)}{(x^2-1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2-1)\,2(x^2+1)^{2-1}\,d/dx\,(x^2+1) - (x^2+1)^2 (2x)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\,2(x^2+1)(2x) - (x^2+1)^2 (2x)}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)\left[2(x^2-1) - (x^2+1)\right]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)\left[2x^2-2-2x^2-1\right]}{(x^2-1)^2}$$

$$\boxed{= \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}}$$ Ans.

Page # 3

Question 2:-

A:-

$Y = (1+2\sqrt{x})^3 \cdot x^{2/3}$:-

$Y = (1+2\sqrt{x})^3 \cdot x^{2/3}$

$dy/dx = d/dx\,(1+2\sqrt{x})^3 \cdot x^{2/3}$

$= (1+2\sqrt{x})^3 \; d/dx\; x^{2/3} + x^{2/3}\; d/dx\,(1+2\sqrt{x})^3$

$= (1+2\sqrt{x})^3 \; 2/3\; x^{2/3-1} \; d/dx + x^{2/3}\; (3(1+2\sqrt{x})^2 \; d/dx\,(1+2\sqrt{x})$

$= (1+2\sqrt{x})^3 \cdot 2/3\; x^{2/3} + x^{2/3} \cdot 3(1+2\sqrt{x})^2 \; 0 + x\, 1/2\; x^{1/2-1}\; d/dx\, x$

$(1+2\sqrt{x})^3 \cdot 2/3\, x^{-1/3} + x^{2/3} \cdot 3(1+2\sqrt{x})^2 + x^{-1/2}$

$= 2/3\, x^{-1/3}\,(1+2\sqrt{x})^3 + 3x^{2/3}\,(1+2\sqrt{x})^2 \left(\frac{+1}{\sqrt{x}}\right)$

Ay

B:-

$Y = \sqrt{\frac{1-x}{1+x}}$

Or Put $u = \frac{1-x}{1+x}$

So $Y = \sqrt{u}$

Differentiate "u" w.r.t. x

$du/dx = d/dx \left(\frac{1-x}{1+x}\right)$

$\frac{du}{dx} = \frac{(1+x)\, d/dx\,(1-x) - (1-x)\, d/dx\,(1+x)}{(1+x)^2}$

$$\frac{du}{dx} = \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$\frac{du}{dx} = \frac{-1-x-1+x}{(1+x)^2}$$

$$dy/dx = \frac{-2}{(1+x)^2}$$

Now different y w.r.t u.

$$\frac{dy}{du} = d/du\, u^{1/2}$$

$$dy/du = 1/2\, u^{-1/2}$$

$$\frac{dy}{du} = 1/2 \left(\frac{1-x}{1+x}\right)^{-1/2}$$

$$\frac{dy}{du} = 1/2 \left(\frac{1+x}{1-x}\right)^{1/2}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = 1/2 \left(\frac{1+x}{1-x}\right)^{1/2} \cdot \frac{-2}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(1-x)^{1/2}(1+x)^{2-1/2}}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x}\,(1+x)^{2/3}}$$

Question #3.

A.

Solution:

$$\int \frac{1}{\sqrt{x^3}} \, dx$$

$$\int \frac{1}{\sqrt{x}} \, dx = \int \frac{1}{\sqrt{x^3}} \, dx$$

$$= \int x^{3/2} \, dx$$

$$= \frac{x^{3/2+1}}{3/2+1}$$

$$\frac{3+2}{2}$$

$$= \frac{x}{\frac{3+2}{2}}$$

$$= \frac{x^{5/2}}{5/2} + C$$

$$\int \frac{1}{\sqrt{3}} \, dx = \frac{x^{5/2}}{5/2} + C \quad \text{Ans}$$

Page #6

Question No = 3

. B .-

$$\int \frac{1}{(6x+7)^6} dx$$

Solution =

$$\int \frac{1}{(6x+7)^6} dx$$

$$\int \frac{1}{(6x+7)^6} dx = \int 1 (6x+7)^{-6} dx$$

$$= \int (6x+7)^{-6} dx$$

$$= \frac{(6x+7)^{-6+1}}{-6+1}$$

$$= \frac{(6x+7)^{-5}}{-5} = \frac{1}{-5}(6x+7)^{-5}$$

$$\int \frac{1}{(6x+7)^6} dx = \frac{-1}{5}(6x+7)^{-5} + c$$