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Abdul Aziz 13741 Linear Algebra

Q5

Find Eigen values and Eigen vectors of the given matrix.

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

AS $\cancel{A}x \cancel{=} \lambda \cancel{x}$; $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

=> To find eigen values consider the below equation

$$\det(A - \lambda I) = 0$$

or

$$|A - \lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix}$$

Now taking determinant of The $(A - \lambda I)$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$$|A - \lambda I| = (1-\lambda)(4-\lambda) - (-2)(1)$$

$$|A - \lambda I| = 4 - \lambda - 4\lambda + \lambda^2 + 2 = 0$$

$$4 - 5\lambda + \lambda^2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda - 1)(\lambda - 4) = 0$$

$$\boxed{\lambda = 1, \quad \lambda = 4}$$

eigen values.

Now we have to find The eigen vectors for $\lambda = 1$, $\lambda = 4$.

for eigen vectors for $\lambda = 1, \lambda = 4$

$$(A - \lambda I)x = 0$$

$$\left\{ \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

for $\lambda = 1$

$$\begin{bmatrix} 1-1 & 1 \\ -2 & 4-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$0x_1 + 1x_2 = 0 \rightarrow$ eq (i)

$-2x_1 + 3x_2 = 0 \rightarrow$ eq (ii)

$\Rightarrow$

$$-2x_1 = -3x_2$$

$$2x_1 = 3x_2$$

So $v_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ eigen vector

Simillarly for $\lambda = 4$

$$\begin{bmatrix} 1-4 & 1 \\ -2 & 4-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + x_2 = 0$$

$$-2x_1 \pm 0$$

So $-3x_1 = -x_2$

$3x_1 = x_2$

$N_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is eigen vectr for

$\lambda = 4$

Q 2:-

$L(x, y) = (x+1, y, x+y)$

illustrate if it is Linear Transformation

$L(x, y) = (x+1, y, x+y)$

It is Linear Transformation If It satisfies the Two properties

(1) $L((x_1, y) + (x_2, y_2)) = L(x_1, y_1) + L(x_2, y_2)$

(2) $L(c(x_1, y_1)) = c[L(x_1, y_1)]$

$$L\left((x_1, y_1) + (x_2, y_2)\right) = L\left((x_1 + x_2, y_1 + y_2)\right)$$

$$\cancel{L\left((x_1, y_1) + (x_2, y_2)\right) =}$$

$$L\left((x_1, y_1) + (x_2, y_2)\right) = \left(x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2\right)$$

$\hookrightarrow$ eq (1)

Now finding ~~L(x) and L(y)~~

$L \cancel{(x_1)}$

$$L(x_1, y_1) = (x_1 + 1, y_1, x_1 + y_1)$$

$$L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$$

$$\cancel{L(x_1 + y_1) = L(x_2}$$

$$\cancel{L(x_1, y_1) + L(x_2, y_2)} = x_1 + x_2 + 1$$

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$L(x_1, y_1) + L(x_2, y_2) =$

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$\cancel{(x_1 + x_2 + 1)}$

$L(x_1, y_1) + L(x_2, y_2) = (x_1 + 1, y_1, x_1 + y_1)$

$+ (x_2 + 1, y_2, x_2 + y_2)$

$L(x_1, y_1) + L(x_2, y_2) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

$\hookrightarrow$ eq (2)

eq (1) is not equal to eq (2)

so first property is not satisfied therefore $L(x, y)$ is not a Linear Transformation

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Q 1 :-

express The equation of plane passing through given points.

$A(2,-2,1)$, $B(-1,0,3)$

$C(5,-3,4)$

$\overrightarrow{P_A P_B} = B(-1,0,3) - A(2,-2,1)$

$\overrightarrow{P_A P_B} = (-3,2,2)$

also find $\overrightarrow{P_A P_C}$

$\overrightarrow{P_A P_C} = C(5,-3,4) - (2,-2,1)$

$\overrightarrow{P_A P_C} = (3,-1,3)$

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Now find perpendicular vector

$$n = \vec{P_{AB}} \times \vec{P_{AC}}$$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$n = i\begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix} - j\begin{vmatrix} -3 & 2 \\ 3 & 3 \end{vmatrix} + k\begin{vmatrix} -3 & 2 \\ 3 & -1 \end{vmatrix}$$

$$n = 8i + 15j - 3k$$

as $P_A(x_0, y_0, z_0) = (2, -3, 1)$

$\cancel{n \neq (8, 15, -3)}$

$n(a, b, c) = (8, 15, -3)$

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So equation of plane is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$8(x-2)+15(y+2)-3(z-1)=0$$

$$8x+15y-3z-16+30+3=0$$

$$8x+15y-3z+17=0$$

Q 4:

Find an equation of plane passing through the point $(-1, 3, 2)$ and perpendicular to vector $n = (0, 1, -3)$

The equation of plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$P = (x_0, y_0, z_0) = (-1, 3, 2)$$

$$n = (a, b, c) = (0, 1, -3)$$

putting the values

$$0(x - (-1)) + 1(y - 3) - 3(z - 2) = 0$$

$$\cancel{x+}(y - 3) - 3(z - 2) = 0$$

$$y - 3z - 3 + 6 = 0$$

$$\boxed{y - 3z + 3 = 0}$$

Q3:

using a Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

decode the message 77 54 38 71 49 29 68 51 33 76 48 40 86 53 52

$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}$, $\begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}$, $\begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix}$, $\begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}$,

$\begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$

AS $L(x_2) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax$

for x_1, Since A is non-singular Matrix.

$$x_1 = A^{-1}\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$x_2 = A^{-1}\begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 17 \end{bmatrix}$$

$$X3 = A^{-1}\begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$X \cdot 4 = A^{-1}\begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$X_5 = A^{-1}\begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

Decoded message is

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