(1)

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SEMESTER 4th
COURSE DIFFERENTIAL EQUATION
DATE 21/09/2020

Solve & graph the solution.

1- $x^2y'' - 4xy' + 6y = 0$ $y'(1) = 0.4,\ y'(1) = 0$

Solution:-

(1) lets substitute

$$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$$

into the given ODE. This give

$$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0$$

$$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{-2}} - 4\cancel{x}mx^m\cancel{x^{-1}} + 6x^m = 0$$

we can see that x^m is a common factor. dropping it gives.

$$m(m-1) - 4m + 6 = 0$$

$$m^2 - 5m + 6 = 0$$

2/ let's find the root of equation

$$m^2 - 5m + 6 = 0 \implies m_{1/2} = \frac{5 \pm \sqrt{(-5)^2 + 4 \cdot 6}}{2}$$

$$m_{1/2} = \frac{5 \pm 1}{2}$$

$$m_1 = 3 \ \wedge\ m_2 = 2$$

$$y_1 = x^{m_1} = x^3 \ \wedge\ y_2 = x^{m_2} = x^2$$

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So, the general solution is

$$y = C_1 y_1 + C_2 y_2$$

$$= \boxed{C_1 x^3 + C_2 x^2}$$

$$\Rightarrow y' = 3C_1 x^2 + 2C_2 x$$

3/ Now, all we need to do is to do is to determine C_1 and C_2 from IVP:

$$\Rightarrow \begin{cases} 0.4 = y(1) = C_1 \cdot 1^3 + C_2 \cdot 1^2 \\ 0 = y'(1) = 3C_1 \cdot 1^2 + 2C_2 \cdot 1 \end{cases}$$

$$= \begin{cases} 0.4 = C_1 + C_2 \\ 0 = 3C_1 + 2C_2 \end{cases}$$

$$= \begin{cases} 0.4 - C_2 = C_1 \\ 0 = 3(0.4 - C_2) + 2C_2 \end{cases}$$

$$= \begin{cases} 0.4 - C_2 = C_1 \\ 0 = 1.2 - C_2 \end{cases}$$

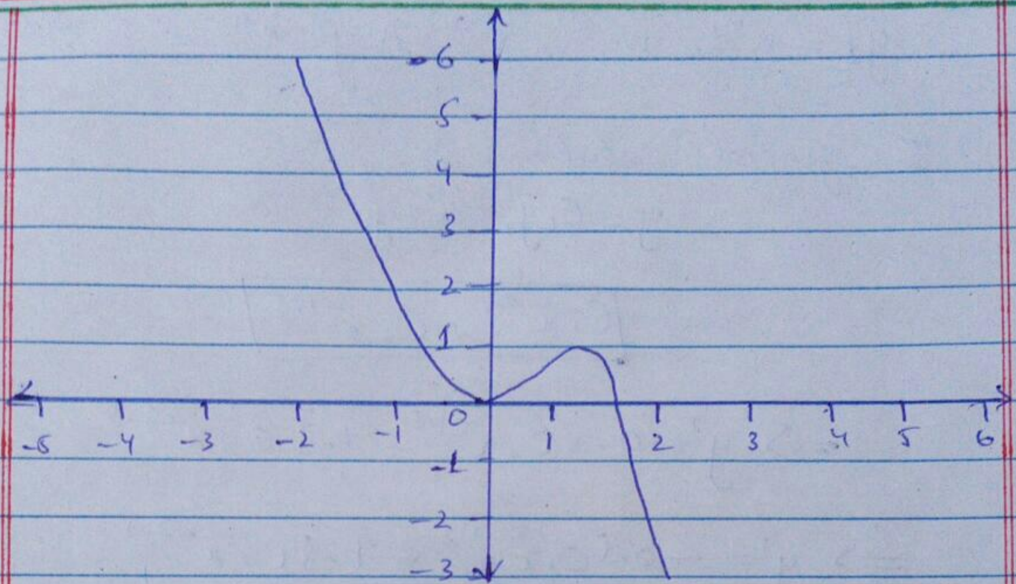
$$= \begin{cases} 0.4 - C_2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$= \begin{cases} 0.4 - 1.2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$= \begin{cases} -0.8 = C_1 \\ 1.2 = C_2 \end{cases}$$

The particular solution of the IVP is.

$$\boxed{y = -0.8x^3 + 1.2x^2}$$



Q2) $x^2y'' + 3xy' + 0.75y = 0$, $y(1) = 1$, $y'(1) = -1.5$

Solution:-

(1) Let's substitute:

$$y = x^m,\quad y' = mx^{m-1},\quad y'' = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} + 3x m x^{m-1} + 0.75x^m = 0$$

$$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{-2}} + 3\cancel{x} m x^m \cdot \cancel{x^{-1}} + 0.75\cancel{x^m} = 0$$

(2) We can see that x^m is common factor.

$$m(m-1) + 3m + 0.75 = 0$$

$$m^2 + 2m + 0.75 = 0$$

Let's find the root of the equation.

$$m^2 + 2m + 0.75 = 0 \Rightarrow m_{1/2} = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot 0.75}}{2}$$

$$\Rightarrow m_{1/2} = \frac{-2 \pm 1}{2}$$

So, it has the distinct real roots.

$$m_1 = -\frac{1}{2} \wedge m_2 = -\frac{3}{2}$$

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$y_1 = x^{m_1} = x^{-\frac{1}{2}} = x^{-0.5} \wedge y_2 = x^{m_2} = x^{-\frac{3}{2}} = x^{-1.5}$

The general solution is

$$y = C_1 y_1 + C_2 y_2$$

$$= \boxed{C_1 x^{-0.5} + C_2 x^{-1.5}}$$

$$\Rightarrow y' = -0.5 C_1 x^{-1.5} - 1.5 C_2 x^{-2.5}$$

$$\Rightarrow y' = -0.5 C_1 x^{-1.5} - 1.5 C_2 x^{-2.5}$$

3/ Now, all we need to determine C_1 and C_2 from IVP:

$$\begin{cases} 1 = y(1) = C_1 \cdot 1^{-0.5} + C_2 \cdot 1^{-1.5} \\ -1.5 = y'(1) = -0.5 C_1 \cdot 1^{-1.5} - 1.5 C_2 \cdot 1^{-2.5} \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 + C_2 \\ -1.5 = -0.5 C_1 - 1.5 C_2 \;/\; (: -0.5) \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 + C_2 \\ 3 = C_1 + 3C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 1 - C_2 = C_1 \\ 3 = 1 - C_2 + 3C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 1 - C_2 = C_1 \\ 2 = 2C_2 \;/\; (: 2) \end{cases}$$

$$\Rightarrow \begin{cases} 1 - C_2 = C_1 \\ 1 = C_2 \end{cases} \Rightarrow \begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$

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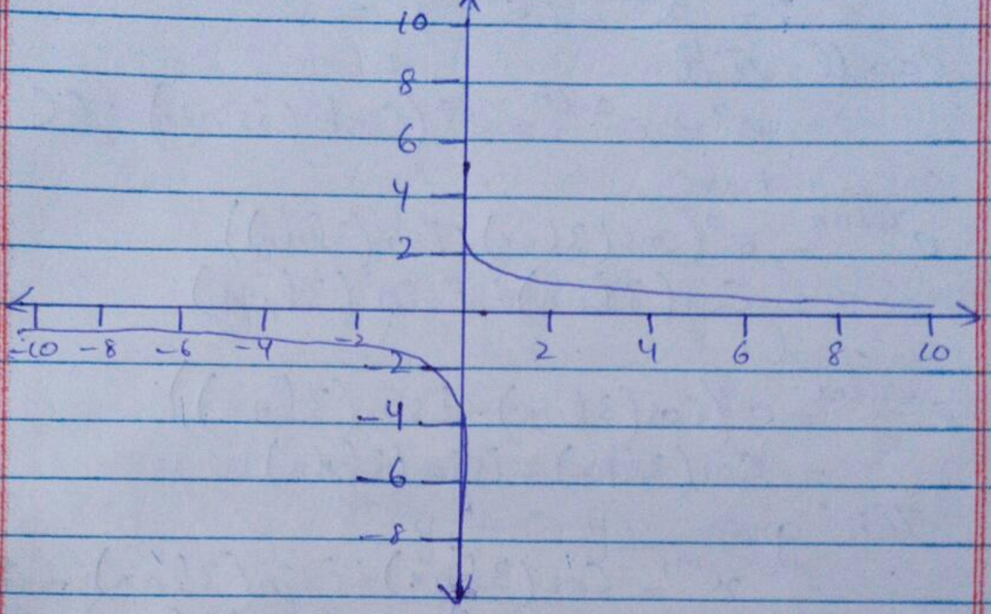
The particular solution of the IVP is
$y = x^{-1.5}$



Q(3) $x^2y'' + xy' + 9y = 0$, $y(1) = 0$, $y'(1) = 2.5$
Solution:-
Let's substitute:
$$y = x^m, \; y'' = m(m-1)x^{m-2}$$
This gives
$$x^2 m(m-1)x^{m-2} + mx^m + 9x^m = 0$$
$$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{-2}} + mx^m + 9x^m = 0$$
$$m(m-1) + m + 9 = 0 \Rightarrow m^2 - \cancel{m} + \cancel{m} + 9 = 0$$
$$\Rightarrow m^2 + 9 = 0$$

Let's find the roots of the equation
$$m^2 + 9 = 0 \Rightarrow m^2 - (3i)^2 = 0$$
$$\Rightarrow (m - 3i)(m + 3i) = 0$$
So, it has the complex conjugate root
$$m_1 = 3i \;\wedge\; m_2 = -3i$$

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Now, we use the fact that $x = e^{\ln x}$

$$x^{m_1} = x^{3i} = (e^{\ln x})^{3i} = e^{3i\ln x}$$

$$x^{m_2} = x^{-3i} = (e^{\ln x})^{-3i} = e^{-3i\ln x}$$

Recall that

$$e^{x} = e^{a+ib} = e^{a}(\cos b + i\sin b), \; x \in C$$

So we have.

$$e^{3i\ln x} = e^{0}(\cos(3\ln x) + i\sin(3\ln x))$$

$$= \cos(3\ln x) + i\sin(3\ln x)$$

and

$$e^{-3i\ln x} = e^{0}(\cos(3\ln x) - i\sin(3\ln x))$$

$$= \cos(3\ln x) - i\sin(3\ln x)$$

This gives

$$x^{m_1} = \cos(3\ln x) + i\sin(3\ln x) \quad -(1)$$

$$x^{m_2} = \cos(3\ln x) - i\sin(3\ln x) \quad -(2)$$

Adding (1) and (2) formulas give

$$x^{m_1} + x^{m_2} = \cos(3\ln x) + \cancel{i\sin(3\ln x)} + \cos(3\ln x) - \cancel{i\sin(3\ln x)}$$

$$= 2\cos(3\ln x)$$

Now divide it by 2

$$\frac{x^{m_1} + x^{m_2}}{2} = \frac{\cancel{2}\cos(3\ln x)}{\cancel{2}}$$

$$= \cos(3\ln x)$$

Now, substract (2) from (1) and divide it by $2i$ after that.

$$x^{m_1} - x^{m_2} = \cos(3\ln x) + i\sin(3\ln x) - \cos(3\ln x) + i\sin(3\ln x)$$

$$= 2i\sin(3\ln x)$$

divide it by $2i$:

$$\frac{x^{m_1} - x^{m_2}}{2i} = \frac{\cancel{2i}\sin(3\ln x)}{\cancel{2i}} = \sin(3\ln x)$$

$\cos(3\ln x)$ and $\sin(3\ln x)$ are the solution of the Euler-Cauchy equation. Their quotient is not constant, so the solution.

$y_1 = \cos(3\ln x) \wedge y_2 = \sin(3\ln x)$ are linearly independent and form a basis of solutions.

So it general solution is:

$$y = C_1 y_1 + C_2 y_2$$

$$= \boxed{C_1 \cos(3\ln x) + C_2 \sin(3\ln x)}$$

$$\Rightarrow y' = -C_1 \sin(3\ln x) \cdot (3\ln x)' + C_2 \cos(3\ln x) \cdot (3\ln x)$$

$$= -\frac{3C_1}{x}\sin(3\ln x) + \frac{3C_2}{x}\cos(3\ln x)$$

determine C_1 & C_2 from IVP.

$$\begin{cases} 0 = y(1) = C_1 \cos(3\ln 1) + C_2 \sin(3\ln 1) \\ 2.5 = y'(1) = -3C_1 \sin(3\ln 1) + 3C_2 \cos(3\ln 1) \end{cases}$$

$$\Rightarrow \begin{cases} 0 = C_1 \cos(0) + C_2 \sin(0) \\ 2.5 = -3C_1 \sin(0) + 3C_2 \cos(0) \end{cases}$$

$$\Rightarrow \begin{cases} 0 = C_1 \\ 5/2 = 3C_2 \quad / :3 \end{cases}$$

$$= \begin{cases} 0 = C_1 \\ 5/6 = C_2 \end{cases}$$

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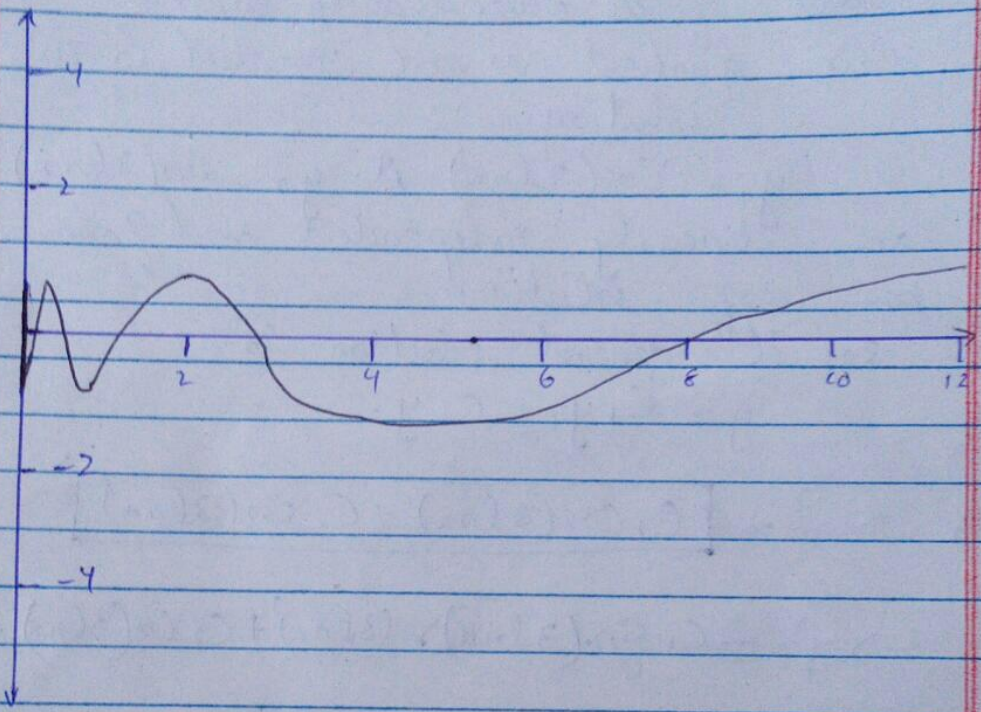
The particular solution of the IVP is

$$\boxed{y = \frac{5}{6}\sin(3\ln x)}$$



Q4) $x^2y'' + 3xy' + y = 0$, $y(1) = 3.6$, $y'(1) = 0.4$

Solution:-

let's substitute

$$y = x^m, \quad y'' = m(m-1)x^{m-2}$$

This gives

$$x^2 m(m-1)x^{m-2} + 3mx^m + x^m = 0$$

$$\cancel{x^2} m(m-1)\, x^m \cdot \cancel{x^{-2}} + 3mx^m + x^m = 0$$

$$m(m-1) + 3m + 1 = 0 \Rightarrow m^2 - m + 3m + 1 = 0$$

$$\Rightarrow m^2 + 2m + 1 = 0 \quad \text{— } (*)$$

find the root of equation $(*)$

$$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0$$

$$m = 1$$

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$$y_1 = x^m = x^{-1} = \frac{1}{x}$$

$$y'' + \frac{3}{x} \cdot y' + \frac{1}{x^2} \cdot y = 0$$

Now, we can see that:

$$p(x) = 3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = 3\ln|x|$$

put

$$y_2 = u y_1$$

where

$$u = \int U\,dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int p\,dx}$$

Let's find U:

$$e^{-\int p\,dx} = e^{-3\ln|x|} = \left(e^{\ln|x|}\right)^{-3} = x^{-3}$$

$$\Rightarrow U = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$$

By integration, we have:

$$u = \int \frac{dx}{x} = \ln|x|$$

So

$$y_2 = u y_1 = y_1 \ln x = \frac{1}{x} \cdot \ln x$$

So, the general solution is:

$$y = C_1 y_1 + C_2 y_2$$

$$= C_1 \cdot \frac{1}{x} + C_2 \cdot \frac{1}{x} \cdot \ln x$$

$$= \boxed{\frac{1}{x} \cdot \left(C_1 + C_2 \ln x\right)}$$

$$\Rightarrow y' = (x^{-1})'(C_1 + C_2 \ln x) + x^{-1}(C_1 + C_2 \ln x)'$$

$$= -x^{-2}(C_1 + C_2 \ln x) + \frac{1}{x} C_2 \cdot \frac{1}{x}$$

$$= \frac{1}{x^2}(-C_1 - C_2 \ln x + C_2)$$

determine C_1 & C_2 from IVP:

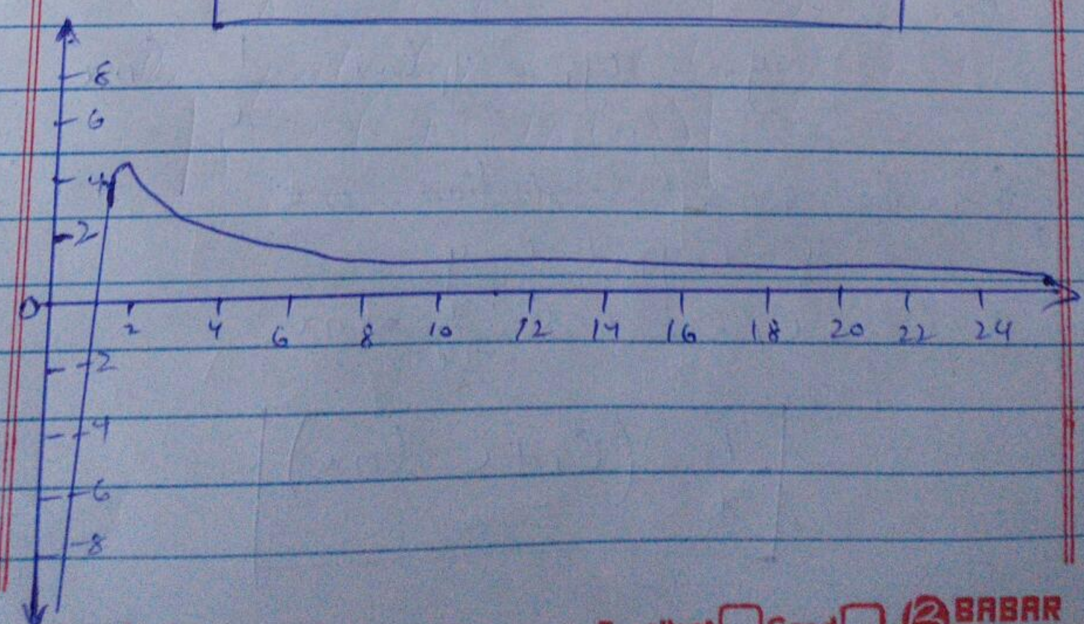
$$\begin{cases} 3.6 = y(1) = 1(C_1 + C_2 \ln 1) \\ 0.4 = y'(1) = \frac{1}{1^2}(-C_1 - C_2 \ln 1 + C_2) \end{cases}$$

$$\Rightarrow \begin{cases} 3.6 = C_1 \\ 0.4 = -C_1 + C_2 \end{cases}$$

$$= \begin{cases} 3.6 = C_1 \\ 0.4 = -3.6 + C_2 \end{cases}$$

$$= \begin{cases} 3.6 = C_1 \\ 4.0 = C_2 \end{cases}$$

$$\boxed{y = (3.6 + 4.0 \ln x) \cdot \frac{1}{x}}$$



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Q(5) $(x^2D^2 - 3xD + 4I)y = 0$, $y(1) = -\pi$, $y'(1) = 2\pi$

Solution:-

$$x^2D^2y - 3xDy + 4Iy = x^2D(Dy) - 3xDy + 4y$$
$$= x^2y'' - 3xy' + 4y$$

$$x^2y'' - 3xy' + 4y = 0$$

Substitute.

$$y = x^m ,\ y' = mx^{m-1} ,\ y'' = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} - 3xmx^{m-1} + 4x^m = 0$$
$$\cancel{x^2} m(m-1)x^m . \cancel{x^{-2}} - 3\cancel{x}mx^m . \cancel{x^{-1}} + 4x^m = 0$$

x^m is a common factor.

$$m(m-1) - 3m + 4 = 0 \iff m^2 - 4m + 4 = 0$$
$$m^2 - 4m + 4 = 0 \iff (m-2)^2 = 0$$
$$m = 2$$
$$y_1 = x^m = x^2$$

$$y'' - \frac{3}{x} . y' + \frac{4}{x^2} . y = 0$$

$$p(x) = -3 . \frac{1}{x} \Rightarrow \int p\,dx = -3\ln|x|$$

put:

$$y_2 = uy_1$$

where $u = \int U\,dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int p dx}$

let's find U

$$e^{-\int p dx} = e^{3\ln|x|} = \left(e^{\ln|x|}\right)^3 = x^3$$
$$\Rightarrow U = x^3 . \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}$$

By integration, we have:

$$u = \int \frac{dx}{x} = \ln|x|$$

So. $y_2 = u y_1 = y_1 \ln x = x^2 \ln x$

The general solution is

$$y = C_1 y_1 + C_2 y_2$$
$$= C_1 x^2 + x^2 \ln x$$
$$= \boxed{x^2 (C_1 + C_2 \ln x)}$$

Applying product rule.

$$\Rightarrow y' = (x^2)'(C_1 + C_2 \ln x) + x^2 (C_1 + C_2 \ln x)'$$
$$= 2x(C_1 + C_2 \ln x) + C_2 x^2 \cdot \frac{1}{x}$$
$$= 2C_1 x + 2C_2 x \ln x + C_2 x$$
$$= 2C_1 x + C_2 x(2\ln x + 1)$$

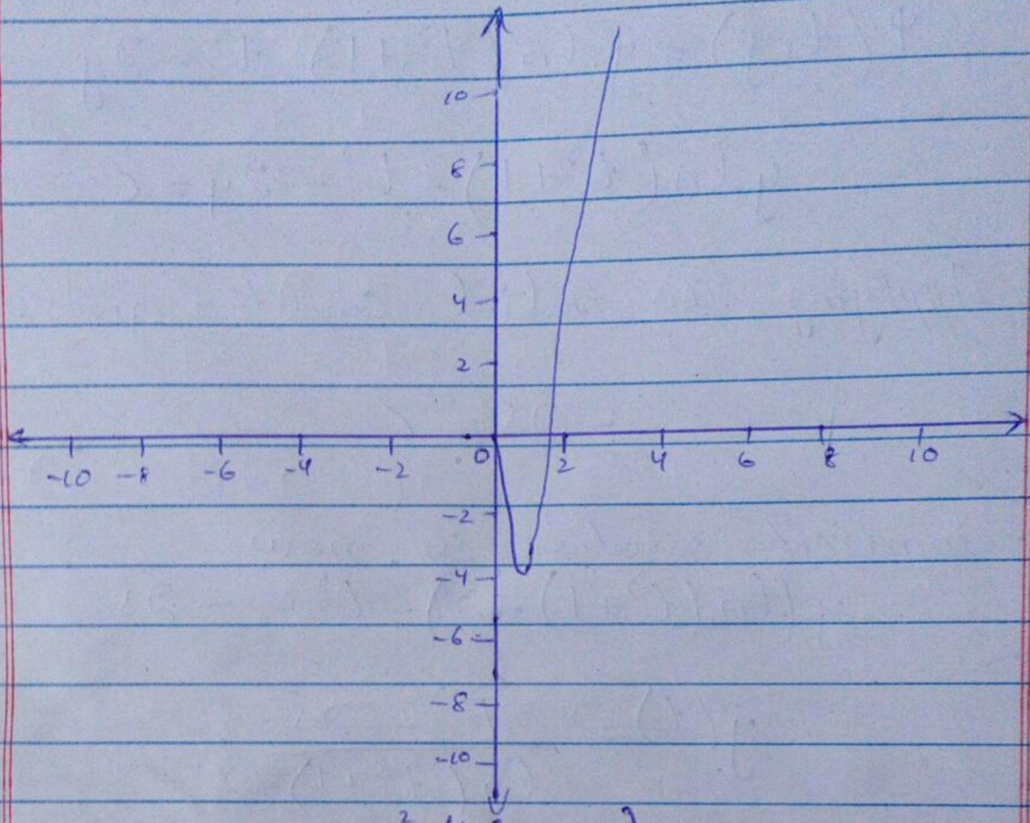
Now, all we need to do is to determine C_1 and C_2 from IVP.

$$\begin{cases} -\pi = y(1) = 1^2 (C_1 + C_2 \ln 1) \\ 2\pi = y'(1) = 2C_1 + C_2(2\ln 1 + 1) \end{cases}$$

$$\Rightarrow \begin{cases} -\pi = C_1 \\ 2\pi = 2C_1 + C_2 \end{cases} \Rightarrow \begin{cases} -\pi = C_1 \\ 4\pi = C_2 \end{cases}$$

The particular solution of the IVP

$$\boxed{y = x^2(-\pi + 4\pi \ln x)}$$



$y = x^2\pi(4\ln x - 1)$

Q(6) $(x^2D^2 + xD + I)y = 0, \quad y(1) = 0, \quad y'(1) = 1$

Solution:-

(1) let $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

(2) Substitute into the differential equation

$x^2 m(m-1)x^{m-2} + x m x^{m-1} + 1x^m = 0$

$\Rightarrow m(m-1)x^m + mx^m + 1x^m = 0$

$\Rightarrow (m(m-1) + m + 1)\,x^m = 0$

Hence the auxillary equation is

$m^2 + 1 = 0 \Rightarrow m = \pm i$

(3) The general solution is of the form

$y = C_1\cos(\ln x) + C_2\sin(\ln x)$

$y' = -C_1\dfrac{\sin(\ln x)}{x} + C_2\dfrac{\cos(\ln x)}{x}$

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Applying

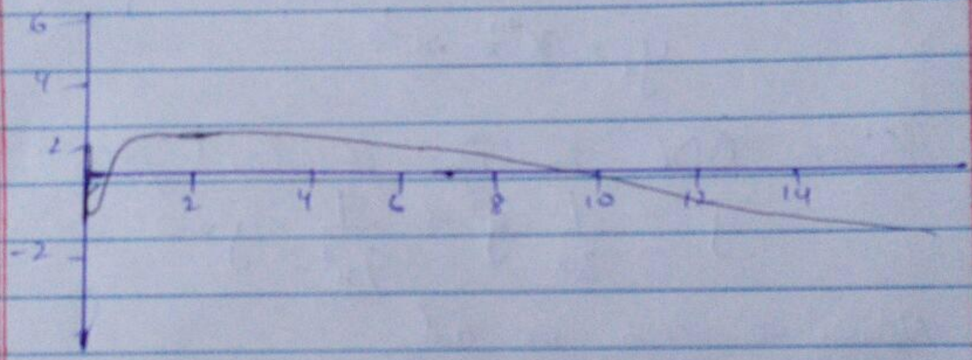
$$y(1) = C_1 \cos(\cancel{\ln 1})^{1} + C_2 \sin(\cancel{\ln 1})^{0}$$

$$\Rightarrow C_1 = 1$$

$$y'(1) = -C_1 \frac{\sin(\cancel{\ln 1})^{0}}{\cancel{1}} + C_2 \frac{\cos(\cancel{\ln 1})^{1}}{\cancel{1}}$$

$$C_2 = 1$$

Hence $y = \cos(\ln x) + \sin(\ln x)$



Q(7) $(9x^2D^2 + 3xD + I)y = 0,\ y(1) = 1,\ y'(1) = 0$

Solution:-

(1) $9x^2D^2y + 3xDy + Iy = 9x^2D(Dy) + 3xDy + y$

$= 9x^2y'' + 3xy' + y$

$$9x^2y'' + 3xy + y = 0$$

substitute,

$$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$$

$$9x^2m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$$

$$9\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} + 3\cancel{x}mx^m \cdot \cancel{x^{-1}} = 0$$

$$9m(m-1) + 3m = 0$$

$$9m^2 - 9m + 3m + 1 = 0 \Leftrightarrow 9m^2 - 6m + 1 = 0$$

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Let's find the root of equation

$$m^2-4m+4=0 \Leftrightarrow (m-2)^2=0$$

$$9m^2-6m+1=0 \Leftrightarrow m_{1/2}=\frac{6\pm\sqrt{(6)^2-4\cdot 9}}{18}$$

$$\Leftrightarrow m_{1/2}=\frac{6}{18}$$

$$\Leftrightarrow m_{1/2}=\frac{1}{3}$$

$$m=\frac{1}{3}$$

$$y_1=x^m=x^{1/3}$$

(2) Given ODE in the standard form

$$y''+\frac{1}{3x}\cdot y'+\frac{1}{9x^2}\cdot y=0$$

Now, we can see that

$$p(x)=\frac{1}{3}\cdot\frac{1}{x} \Leftrightarrow \int p\,dx=\frac{1}{3}\ln|x|$$

Put: $y_2=uy_1$

where

$$u=\int U\,dx \wedge U=\frac{1}{y_1^2}e^{-\int p\,dx}$$

$$e^{-\int p\,dx}=e^{-\frac{1}{3}\ln|x|}=\left(e^{\ln|x|}\right)^{-\frac{1}{3}}=x^{-\frac{1}{3}}$$

$$\Rightarrow U=x^{-\frac{1}{3}}\cdot\frac{1}{(x^{1/3})^2}=x^{-\frac{1}{3}-\frac{2}{3}}=x^{-1}$$

$$=\frac{1}{x}$$

By integration, we have

$$u = \int \frac{dx}{x} = \ln|x|$$

So, $y_2 = uy_1 = y_1 \ln x = x^{\frac{1}{3}} \ln x$

(3) So, The general solution is

$$y = C_1 y_1 + C_2 y_2$$
$$= C_1 x^{1/3} + x^{1/3} \ln x$$
$$= \boxed{x^{\frac{1}{3}}(C_1 + C_2 \ln x)}$$

$$\Rightarrow y' = (x^{1/3})'(C_1 + C_2 \ln x) + x^{1/3}(C_1 + C_2 \ln x)'$$
$$= \frac{1}{3} \cdot x^{-2/3}(C_1 + C_2 \ln x) + x^{1/3} C_2 \cancel{x} \cdot \frac{1}{\cancel{x}}$$
$$= \frac{1}{3} \cdot x^{-2/3}(C_1 + C_2 \ln x) + x^{\frac{1}{3}} C_2$$

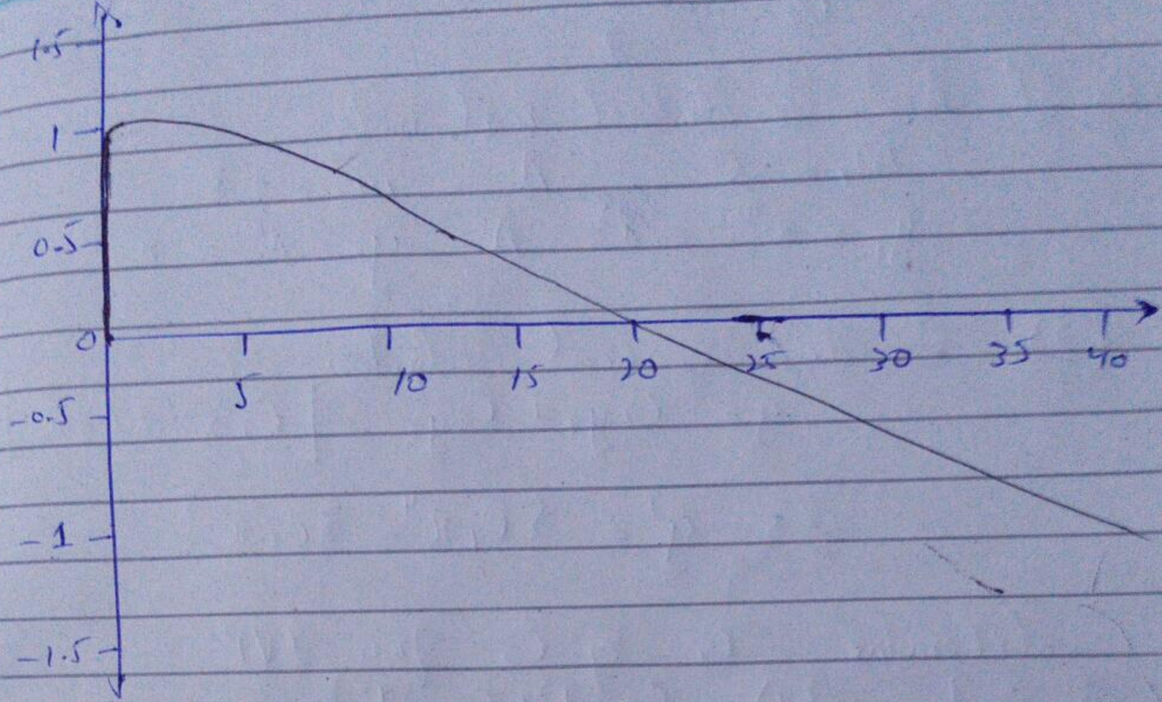
Determine C_1 & C_2 from IVP

$$\begin{cases} 1 = y(1) = 1^{1/3}(C_1 + C_2 \ln 1) \\ 0 = y'(1) = \frac{1}{3} \cdot 1^{-2/3}(C_1 + C_2 \ln 1) + 1^{1/3} C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 \\ 0 = \frac{C_1}{3} + C_2 \end{cases}$$

$$= \begin{cases} 1 = C_1 \\ -\frac{1}{3} = C_2 \end{cases}$$

$$\boxed{y = x^{1/3}\left(1 - \frac{1}{3} \ln x\right)}$$



Q(8) $(x^2D^2 - xD - 15I)y = 0$, $y(1)=0$, $y'(1) = -4.5$

Solution:

(1) $x^2D^2y - xDy - 15Iy = x^2D(Dy) - 15y$

$= x^2y'' - xy' - 15y$

let's solve the equation:

$$x^2y'' - xy' - 15y = 0$$

Let's substitute:

$$y = x^m, \; y' = mx^{m-1}, \; y'' = m(m-1)x^{m-2}$$

into the given ODE

$$x^2m(m-1)x^{m-2} - xmx^{m-1} - 15x^m = 0$$

$$\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} - \cancel{x}mx^m\cancel{x^{-1}} - 15x^m = 0$$

$$m(m-1) - m - 15 = 0 \iff m^2 - 2m - 15 = 0$$

Let's find the root of equation

$$m^2 - 2m - 15 = 0 \iff m_{1/2} = \frac{2 \pm \sqrt{(-2)^2 + 4.15}}{2}$$

$\Leftrightarrow m_{1/2} = \frac{2 \pm 8}{2}$

So, it has two distinct real roots:

$m_1 = 5 \wedge m_2 = -3$

$y_1 = x^{m_1} = x^5 \wedge y_2 = x^{m_2} = x^{-3}$

(2) So, the general solution is:

$y = C_1 y_1 + C_2 y_2 = \boxed{C_1 x^5 + C_2 x^{-3}}$

$\Rightarrow y' = 5C_1 x^4 - 3C_2 x^{-4}$

determine C_1 & C_2 from IVP

$$\begin{cases} 0.1 = y(1) = C_1 \cdot 1^5 + C_2 \cdot 1^{-3} \\ -4.5 = y'(1) = 5C_1 \cdot 1^4 - 3C_2 \cdot 1^{-4} \end{cases}$$

$$= \begin{cases} 0.1 = C_1 + C_2 \\ -4.5 = 5C_1 - 3C_2 \end{cases}$$

$$= \begin{cases} 0.1 - C_2 = C_1 \\ -4.5 = 5(0.1 - C_2) - 3C_2 \end{cases}$$

$$= \begin{cases} 0.1 - C_2 = C_1 \\ 5 = 8C_2 \quad / : 8 \end{cases}$$

$$= \begin{cases} 0.1 - C_2 = C_1 \\ 0.625 = C_2 \end{cases}$$

$$= \begin{cases} 0.1 - 0.625 = C_1 \\ 0.625 = C_2 \end{cases} \Rightarrow \begin{cases} -0.525 = C_1 \\ 0.625 = C_2 \end{cases}$$

$$\boxed{y = -0.525x^3 + 0.625x^{-3}}$$