Q No 01

① The order of Matrix AB is $m \times n$.

m = No of Rows
n = No of Columns.

② The number of non-zero rows in an Echelon form is rank of the matrix

③ If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = 8$.

proof: $\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} \Rightarrow a - 4 \times 2 \Rightarrow a - 8 = 0$

$a = 8$

④ If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = ?$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix} \Rightarrow -2i^2 - i \times i$

$\Rightarrow -2(-1) - i^2 \quad \because$ as $i^2 = -1$

$\Rightarrow +2 - (-1)$

$\Rightarrow 3$

(5) The Matrix $A=\begin{bmatrix}9&0\\0&9\end{bmatrix}$ is Scalar Matrix.

Reason : As the diagonal elements $\begin{bmatrix}*&*\\ *&*\end{bmatrix}$ are same "9" and non diagonal element are zero. that's why it is a scalar matrix.

(6) Solution of $\frac{dy}{dx}+2xy=y$?

Solution : $\frac{dy}{dx}+2xy=y$

$$\frac{dy}{dx}=y-2xy$$

$$\frac{dy}{dx}=y(1-2x)$$

$$\frac{dy}{y}=(1-2x)dx$$

$$\frac{1}{y}dy=(1-2x)dx$$

take integration.

$$\int\frac{1}{y}dy=\int(1-2x)dx$$

$$\ln y=\int 1\,dx-\int 2x\,dx$$

$$\ln y=x-\frac{\cancel{2}x^2}{\cancel{2}}+c$$

$$\ln y=x-x^2+c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x-x^2+c}$$

$$\boxed{y = e^{x(1-x)+c}}$$

Ans.

7. The order and degree of given differential equation is

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

order = 1 one.

Degree = 3 three.

8. The order and degree of given differential equation is

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$

order = two "2"

Degree = one "1"

9. The differential equation $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0) = 5$ is

Solution: $2y' + x^2y = x^2+3$, $y(0) = 5$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$$ Divided 2 both sides

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

$$\mu = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6}\left(\frac{x^2}{2}\right) y = \frac{1}{2} e^{x^3/6}(x^2+3)$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + c}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$$ Ans

---

(10) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ Expand by C_1

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - b^2c) - 1(ac^2 - a^2c) + 1(ab^2 - a^2b)$$

$$\Rightarrow bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$$

$$\Rightarrow ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$\Rightarrow ab^2 - a^2b - a^2c - cb^2 + bc^2 - ac^2$$

$$a^2(c-b) + b^2(a-c) + c^2(b-a)$$

Ans

Q No 02

Part A

Given: Express the Determinent.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c.$$

(a)

Common abc from eq (a)

$$\Rightarrow abc\left(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b\right)$$

$$\Rightarrow \boxed{abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right]}$$

Ans

QNo 2 : ← Part B →

Given :-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution :-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Charateristic eq → $|A-\lambda I| = 0 \longrightarrow A$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A-\lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Expand by R1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow B$$

Expand by R1

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow 3-\lambda \left[(3-\lambda)(2-\lambda) - (-1)(-1) \right] + 1 \left[(-1)(2-\lambda) - (-1)(-1) \right]$$
$$-1 \left[(-1)(-1) - (-1)(3-\lambda) \right]$$

$$\Rightarrow 3-\lambda \left(6 - 3\lambda - 2\lambda + \lambda^2 - 1 \right) + \left(-2 + \lambda - 1 \right) - \left(1 + 3 - \lambda \right)$$

$$\Rightarrow 3-\lambda \left(\lambda^2 - 5\lambda + 5 \right) + \left(\lambda - 3 \right) - \left(4 - \lambda \right)$$

$$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda + \lambda - 3 - 4 + \lambda$$

$$\boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \longrightarrow a$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -1(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1\left[(3-\lambda)(2-\lambda)-(-1)(-1)\right] +1\left[(-1)(2-\lambda)-(-1)(-1)\right]$$

$$\Rightarrow -1\left(6-3\lambda-2\lambda+\lambda^2-1\right)+\left(-2+\lambda-1\right)$$

$$\Rightarrow -6+3\lambda+2\lambda-\lambda^2+1+\lambda-3$$

$$\Rightarrow \boxed{\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -(1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$-\left((-1)(2-\lambda)-(-1)(-1)\right)+\left((3-\lambda)(2-\lambda)-(-1)(-1)\right)$$

$$-\left(-2+\lambda-1\right)+\left(6-3\lambda-2\lambda+\lambda^2\right)-1$$

$$-\left(-3+\lambda+6-5\lambda+\lambda^2-1\right)$$

$$\Rightarrow 3-\lambda-6+5\lambda-\lambda^2+1$$

$$\boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

* put eq a, b and c in eq B

$$2-\lambda\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-\lambda^2+6\lambda-8-8$$
$$-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By Synthetic Division we get

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$
$$(\lambda=0)$$
$$\lambda-2=0 \quad \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization

$$\lambda^2-4\lambda-4\lambda+16$$
$$\lambda(\lambda-4)-4(\lambda-4)$$
$$(\lambda-4)(\lambda-4)=0$$
$$\lambda=4 \quad , \lambda=4$$

$$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4\ \lambda_4=4}$$

Ans

Q No 3

$$(x^2+3y^2)dx - 2xy\,dy = 0$$
$$x=2,\ y=6$$

Solution: $(x^2+3y^2)dx - 2xy\,dy = 0$

$$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$$

Dividing both sides by $2xydx$

We
$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$= \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow A$$

Let $y = vx$
$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— ⓐ}$$

$$dy = v\,dx + x\,dv$$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

Put eq ⓐ in A

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

$\times$ both sides by 2.

$$2\left(v + x\frac{dv}{dx}\right) = \cancel{2} \times \frac{1}{\cancel{2}}\left[\frac{1}{v} + 3v\right]$$

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$\times$ both sides by dx

$$\left(2x\frac{dv}{\cancel{dx}}\right) \times \cancel{dx} = \frac{1+v^2}{v}$$

$$2x\,dv = \left(\frac{1+v^2}{v}\right)dx$$

We Mutiplying both sides by $\frac{v}{x(1+v^2)}$

$$\frac{v}{1+v^2}dv = \frac{1}{x}dx$$

take integration both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + C$$

$$\ln|1+v^2| = \ln x + \ln c$$

take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$\therefore$ Rul $a^x = a^8$
$x = 8$

$$1+v^2 = xc$$

put $v = y/x$

$$1+\left(y/x\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \quad \text{eq - (b)}$$

put $x=2,\ y=6$ in eq (b)

$$(2)^2+(6)^2 = (2)^3c$$

$$4+36 = 8c$$

$$8c = 40$$

$$\boxed{c = 5}$$ put in eq (b)

$x^2+y^2=5x^3$

$y^2=5x^3-x^2$

$y^2=x^2(5x-1)$

Taking square root both sides

$\sqrt{x^2}\cdot\sqrt{5x-1}$

$y=\pm x\sqrt{5x-1}$

$$\boxed{y=\pm x\sqrt{5x-1}}$$

Ans.