

Explain in detail types of stirrups with figures & Also explain ACI Codes for shear design.

Ans:-

Stirrup:- stirrups are closed-loop bars tied at regular intervals in beam reinforcement to hold the bars in position.

Types of stirrups:-

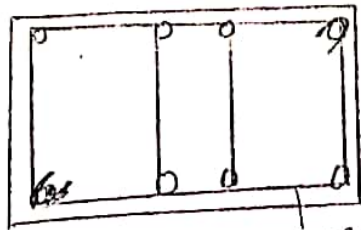
i. single legged stirrup:- The single-leg stirrups have rarely been used because they are mostly used when binding only two rods.

ii. Two legged stirrup:- It is most commonly & widely used stirrup. Minimum 4 bars are required for providing this stirrup.



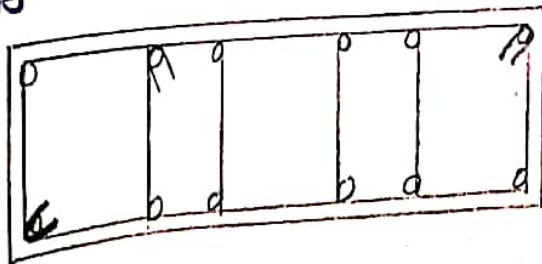
2 legged stirrup.

iii. Four legged stirrup:- These stirrups are used in case of web reinforcement.



4 legged stirrup.

iv. Six legged stirrup:-



ACI codes for Shear Design of a Beam:-

According to ACI-318, following are the formulas used for the shear design of a beam.

1- Critical section:- Critical section occurs at 45° & is at distance $(d)''$ from the face of support which is equal to effective depth.

2- Shear strength capacity of concrete is

$$V_c = 2 \times \sqrt{f'_c} \times b_w \times d$$

3- Minimum Web Reinforcement:-

If $V_u \leq \phi V_c$, then theoretically no web reinforcement is required. However ACI code require provision of atleast a minimum area of web reinforcement equal to.

$$\phi = 0.75 \rightarrow \text{For shear design.}$$

($\because V_u = \text{Total factored shear applied at a given section}$)

For minimum Reinforcement Area:-

$$A_{\text{min}} = 0.75 \times \frac{\sqrt{f'_c} \times b_w \times s}{f_y} \quad \text{or} \quad \frac{S_o \times b_w \times s}{f_y} \rightarrow \left[\begin{array}{l} \text{highest} \\ \text{value is} \\ \text{selected} \end{array} \right]$$

By interchanging the above formulas, we can obtain the formula for maximum spacing.

$$S_{\text{max}} = \frac{A_u \times f_y}{0.75 \times \sqrt{f'_c} \times b_w} \quad \text{or} \quad \frac{A_u \times f_y}{S_o \times b_w} \left[\begin{array}{l} \text{lesser value} \\ \text{is selected} \end{array} \right]$$

4- No web-reinforcement is required if

$$V_u < \frac{1}{9} \phi V_c$$

\Rightarrow Between critical section " V_u " & " ϕV_c ", spacing b/w web reinforcement can be find by

$$S = \frac{\phi \times A_u \times f_y \times d}{V_u - \phi V_c}$$

5- If $V_s \leq 4 \times \sqrt{f_c} \times b_w \times d$, then max spacing for stirrups will be the smallest of the following.

i- 24"

ii- $d/9$

iii- $S_{max} = \frac{A_u \times f_y}{0.75 \times \sqrt{f_c} \times b_w}$

iv- $S_{max} = \frac{A_u \times f_y}{50 \times b_w}$

$\therefore (V_s = \text{shear force carried by web reinforcement})$

\Rightarrow If $V_s > 4 \times \sqrt{f_c} \times b_w \times d$

\downarrow
Max. spacing will be halved

\Rightarrow If $V_s > 8 \times \sqrt{f_y} \times b_w \times d$

\downarrow
Then either increase cross-sectional dimensions or increase f_c .

Q. No 2

4

Given data:-

Breadth of web of beam (b_w) = 14"

Effective depth (d) = 29"

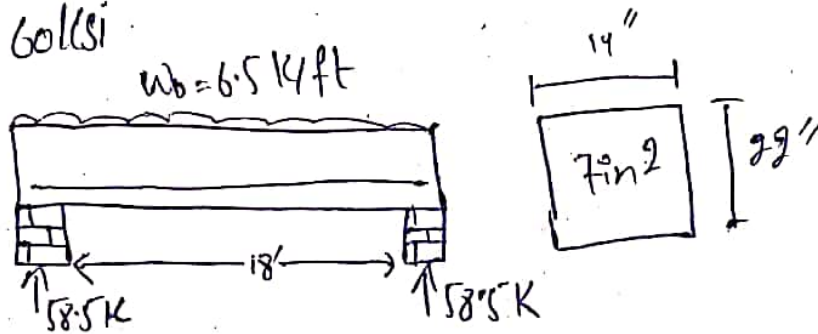
Given load = 6.5 k/ft

Steel Area = ~~6~~ 7 in²

f'_c = 4 ksi

f_y = 60 ksi

Sol:-



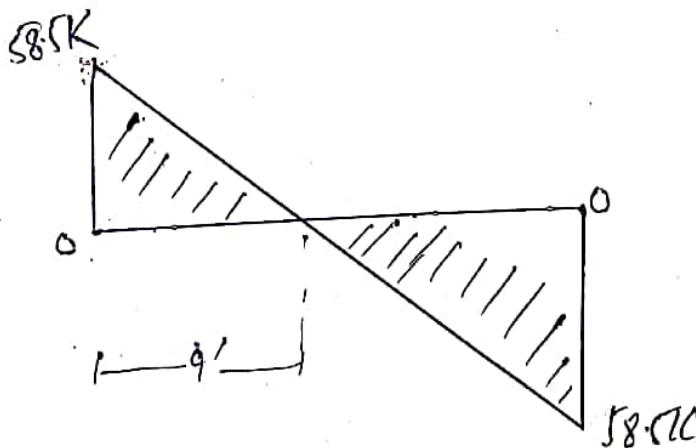
Step # 01:- Reactions on supports

Finding the reactions due to applied load

$$\text{Total Load} = \frac{6.5 \times 18}{2} = 58.5 \text{ kips}$$

Step # 02:- Shear Force Diagram

The required shear diagram will be



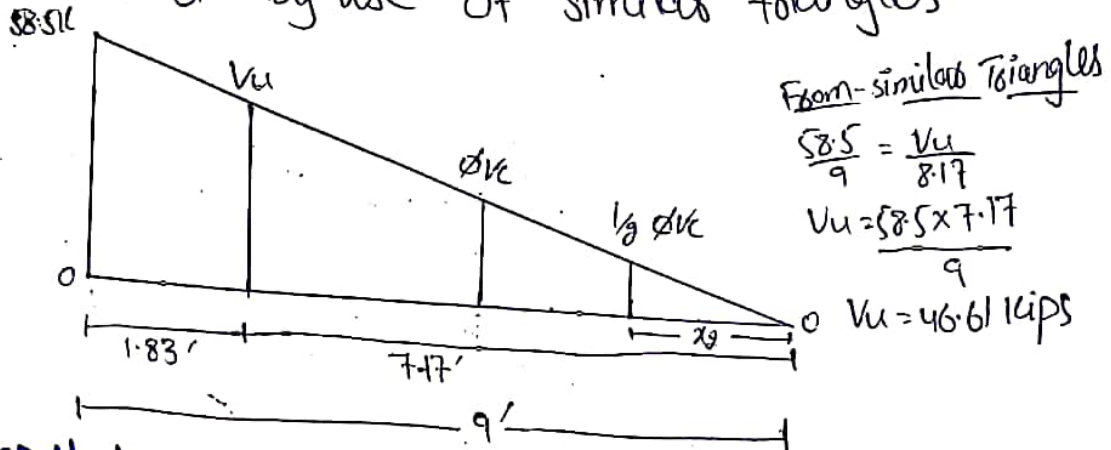
Step #03:-

Finding the value of critical shear ' V_u ' & its location.

As

we know that critical shear is located at distance ' d ' from face to support (d) = 29" = 1.83'

\Rightarrow we will find the values of critical shear at distance ' d ' by use of similar triangles.



Step #04:-

Finding the value of ' ϕV_c ' & ' $\frac{1}{2} \phi V_c$ ' & also its distances from zero shear to right side.

By formula,

$$\begin{aligned} \Rightarrow \phi V_c &= \phi \times 2 \times \sqrt{f_c} \times b_w \times d \\ &= 0.75 \times 2 \times \sqrt{4000} \times 14 \times 29 \\ &= 29219 \text{ lbs} \\ &= 29.219 \text{ kips} \end{aligned}$$

\Rightarrow location of ϕV_c by similar triangles,

$$\frac{58.5}{9} = \frac{\phi V_c}{x_1} \Rightarrow \frac{58.5}{9} = \frac{29.21}{x_1}$$

$$\Rightarrow x_1 = 4.49'$$

Similarly,

$$\frac{1}{2} \phi V_c = \phi V_c / 2 \Rightarrow 29.21 / 2 = 14.60 \text{ kips}$$

\Rightarrow location of $\frac{1}{2} \phi V_c$ will be

$$\frac{58.5}{9} = \frac{14.60}{x_2} \Rightarrow x_2 = 2.94'$$

Step # 05:-

Finding the value of ϕV_s

By formula, $V_u = \phi V_s + \phi V_c$

$$\Rightarrow \phi V_s = V_u - \phi V_c$$
$$= 46.61 - 29.21$$

$$\phi V_s = 17.4 \text{ kips}$$

Step # 06:-

check on section adequacy,

By formula,

$$= \phi \times 8 \times \sqrt{f_c} \times b_w \times d$$

$$= 0.75 \times 8 \times \sqrt{4000} \times 14 \times 22 = 116877 \text{ lb}$$
$$= 116.87 \text{ kips}$$

As $\phi \times 8 \times \sqrt{f_c} \times b_w \times d > \phi V_s$

So section is adequate!

Step # 07:-

check on Maximum ^{spacing} f_{os} stirrups,

$$= \phi \times 4 \times \sqrt{f_c} \times b_w \times d$$

$$= 0.75 \times 4 \times \sqrt{4000} \times 14 \times 22$$

$$= 58438 \text{ lbs}$$

$$= 58.43 \text{ kips}$$

As $\phi \times 4 \times \sqrt{f_c} \times b_w \times d > \phi V_s$

So maximum will be selected from the following 4 conditions.

1- $S_{max} = 24"$

2- $d/8 = 22/2 = 11"$

3- $S_{max} = \frac{A_u \times f_y}{0.75 \times \sqrt{f_c} \times b_w}$

Here we are using #3 stirrup,
dia = $(3/8)" = 0.375"$

So Area = $\frac{\pi}{4} (0.375)^2 = 0.11 \text{ in}^2$

for 2-legged stirrup

$$\Rightarrow \text{Area} \times 2$$

$$\Rightarrow 0.11 \times 2 = 0.22 \text{ in}^2$$

$$S_{max} = \frac{0.99 \times 60000}{0.75 \times 14000 \times 14} = 19.87''$$

$$4-S_{max} = \frac{A_u \times f_y}{50 \times b_w} = \frac{0.99 \times 60000}{50 \times 14} = 18.85''$$

From above 4 conditions, least value of spacing for # 3, 2 legged stirrup will be selected as,

$$S_{max} = 11''$$

Step # 8:-

stirrups spacing from/at critical section will be

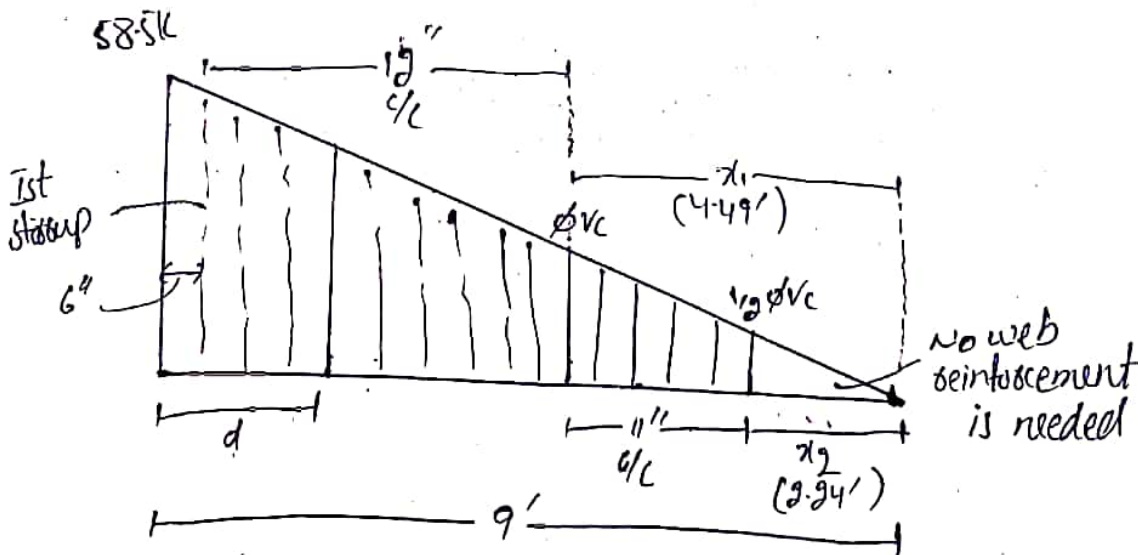
$$S = \frac{\phi \times A_u \times f_y \times d}{V_u - \phi V_c} = \frac{0.75 \times 0.99 \times 60 \times 22}{46.61 - 29.21}$$

$$S = 12.5'' \approx 12''$$

So 12" c/c

Step # 09:-

Final sketch will be

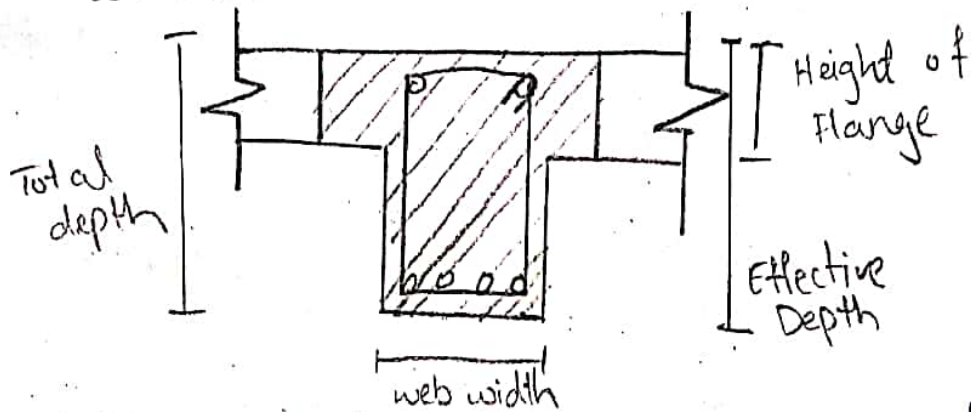


As first stirrup from face of support

$$S/2 = 12/2 = 6''$$

T-Beam:-

⇒ In most of the reinforced concrete structures, concrete slabs are cast monolithically with the slab so, in this case the beam that act as an intermediate beam are called T-Beams



⇒ Because of their T-shape, these beams are called T-Beams

⇒ It is provided at the center of the slab to resist the loads.

⇒ The upper most area of the beam attached to the slab is called Flange.

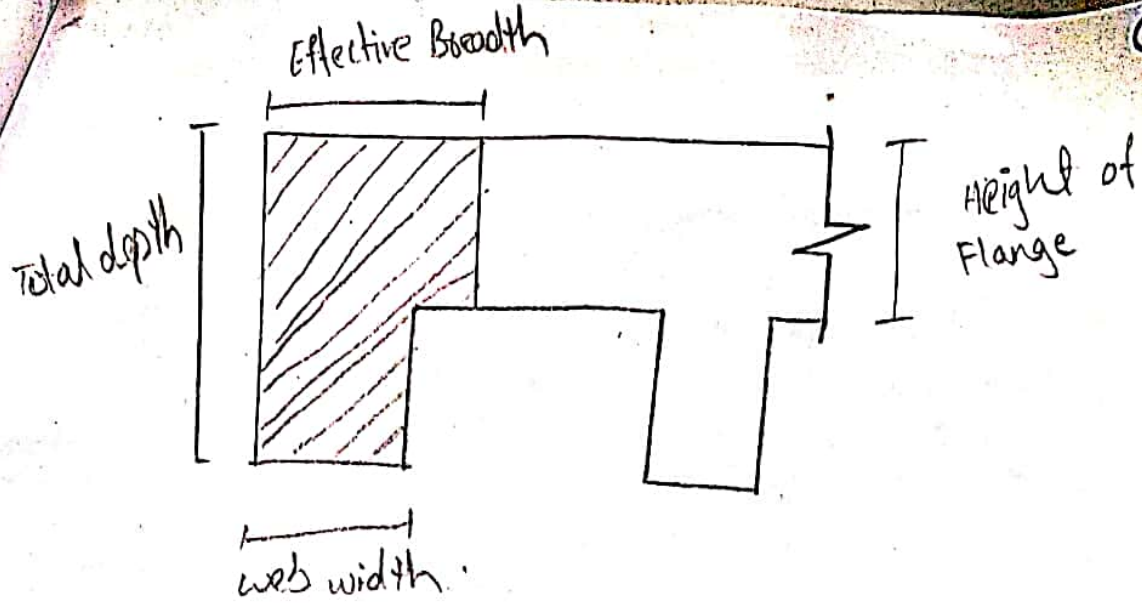
⇒ The bottom rectangular portion of the beam is called web of the beam.

L-Beam:- L-shaped structure that is in contact with the slab & present at the corner of the floor is called L-Beam.

⇒ L-Beams are also called Edge Beams

⇒ It is always provided at the corner of the slab.

⇒ L-Beams are typically floor beams because of their reduced overall structural depth, the beams are in prestressed or reinforced concrete.



Flexural Analysis of T-Beam:-

Flexural Analysis of T-Beam consists of the following steps:-

1- For finding the ultimate factored moment, we use the following formula,

$$M_u = \frac{W_u \times L^2}{8}$$

(W_u = Total factored load
 L = Total span of the beam)

2- Effective width (b_e) for T-Beam is calculated as:-

- i- $16(h_f) + b_w$
- ii- c/c distance
- iii- span/4
- iv- $\frac{c_t s}{2} + b_w$

\therefore (h_f = height of flange
 $c_t s$ = clear transverse span)

- we have to select the least value from above formula.

- If c/c distance is given, then there is no need to of " $\frac{c_t s}{2} + b_w$ "

3- Checking whether Rectangular or T-Beam Analysis is required:-

- i- If $a > h_f \rightarrow$ special Analysis is required
- ii- If $a < h_f \rightarrow$ Rectangular beam Analysis is required.

where
 $(a = \text{Depth of compression block})$
 $(h_f = \text{Height of flange})$

4- For finding Area of steel, we have to use

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)}$$

$\phi = \text{strength reduction factor}$
 $d = \text{effective depth}$
 $a = \text{compression block depth}$
 $b_w = \text{web width of beam}$

5- For checking the range of Reinforcement ratio,

$$J_{max} = 0.85 \times B \times \frac{f'_c}{f_y} \times \left(\frac{E_u}{E_u + E_y} \right)$$

$$J_{min} = \frac{200}{f_y}$$

$$J = \frac{A_{st}}{b \times d}$$

6- Formula for finding no. of bars required is

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

7- for checking minimum width for bars accommodation

$$b_{min} = 2(\text{clear cover}) + 2(\text{dia of stirrup}) + \text{No. of bars} \times (\text{dia of bars}) + \text{spacing b/w bars} (\text{dia of bars})$$

8- Design Moment of given by

$$M_d = \phi \times f_y \times A_{st} \times (d - a/2) \rightarrow \text{if } a < h_f$$

$$M_d = \phi \times [A_s \times f_y \times (d - h_f/2) + (A_s - A_{st}) \times f_y \times (d - a/2)]$$

if $a > h_f$

Q.No 4

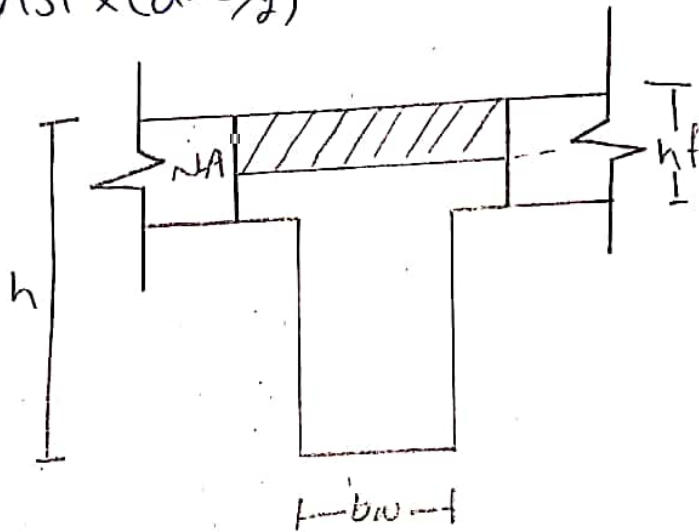
What is the difference b/w CASE-1 & CASE-2 in the design of T-Beam?

CASE:-01

From the Figure $a < h_f$
So in this case, Rectangular Beam Analysis is required.

So, the design Moment formula will be

$$M_d = \phi \times f_y \times A_{st} \times (d - a/2)$$

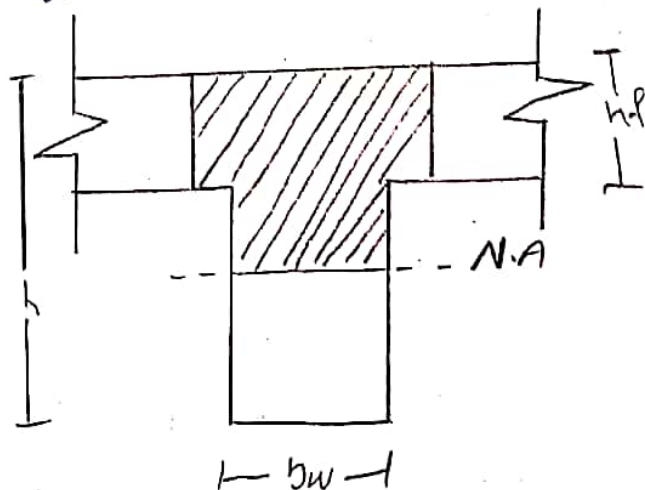


CASE 2:-

From the Figure $a > h_f$
So in this, special beam Analysis i.e., T-Beam Analysis is required.

So, the required Design Moment ^{formula} will be

$$M_d = \phi \times [A_s \times f_y \times (d - \frac{h_f}{2}) + (A_s - A_{sf}) \times f_y \times (d - a_g)]$$



Q. No 5

A floor system consists of 3.5" concrete slab supported by 16' simple span spaced at 9' c/c the beam having a web width of 10" & effective depth of 18" & total height is 23". Calculate the necessary flexural reinforcement if the factored applied moment is 5800 kip-inch. Use $f_c' = 3 \text{ ksi}$ & $f_y = 60 \text{ ksi}$

Given:-

Height of flange $(h_f) = 3.5''$

c/c distance = 9'

length/span of the beam = 16'

web width $(b_w) = 10''$

Height $(h) = 23''$

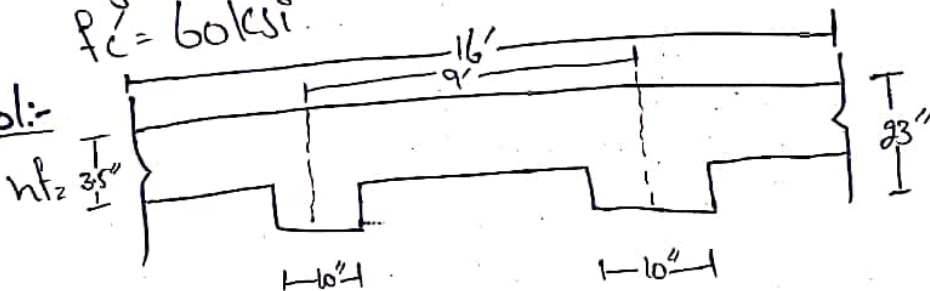
Effective depth $(d) = 18''$

Total factored moment $(M_u) = 5800 \text{ kip-inch}$

$f_y = 60 \text{ ksi}$

$f_c' = 3 \text{ ksi}$

Sol:-



Step # 01:-

Calculate the effective (b_e) for T-beam

$$1 - 16(h_f) + b_w = 16(3.5) + 10 = 66''$$

$$2 - \text{span}/4 = \frac{16}{4} \times 12 = 48''$$

$$3 - \text{c/c distance} = 9 \times 12 = 108''$$

Selecting the least value of b_e as,

$$b_e = 48''$$

Step # 02:-

check whether Rectangular or T-beam Analysis is required

Trial #01:- Let $a = hf = 3.5''$

$$A_{st} = \frac{Mu}{\phi \times f_y \times (d - a_f)} = \frac{5800}{0.90 \times 60 \times (18 - 3.5/2)} = 6.61 \text{ in}^2$$

Trial #02:-

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b \times e}$$

$$a = \frac{6.61 \times 60}{0.85 \times 3 \times 48} = 3.2''$$

$$\epsilon_t = 6.55 \text{ in}^2 \Rightarrow 3.2'' < 3.5''$$

So Rectangular Beam Design is required

Trial #03:-

$$a = 3.21''$$

$$\epsilon_t = \frac{5800}{0.90 \times 60 \times (18 - 3.21/2)} = 6.55 \text{ in}^2$$

So Area of steel is 6.55 in^2

Step # 03:-

check s_{max} & s_{min}

$$\Rightarrow s_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \left(\frac{\epsilon_u}{\epsilon_u + \epsilon_t} \right)$$

$$= 0.85 \times 0.85 \times \frac{3}{60} \left(\frac{0.003}{0.003 + 0.005} \right) = 0.0013$$

$$\Rightarrow s_{min} = \frac{200}{f_y} = \frac{200}{60000} = 0.003$$

$$\Rightarrow s = \frac{A_{st}}{b \times d} = \frac{6.55}{10 \times 18} = 0.036$$

$$s_{min} < s < s_{max}$$

$$0.003 < 0.036 < 0.013$$

As the value of ρ_{max} is less than ρ , so we have to design it as "doubly reinforced beam"
2) First we have to find the Area of steel against ρ_{max} .

$$\rho_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = \rho_{max} \times (b \times d)$$

$$A_{st} = 0.013 \times (10 \times 18)$$

$$A_{st} = 2.34 \text{ in}^2$$

Step # 04:-

Finding the value of M_{ug} :-

By formula

$$M_{ug} = \phi \times A_{st} \times f_y \times (d - a/2)$$

First Finding the value of 'a'

$$\Rightarrow a = \frac{A_{st} \times f_y}{0.85 \times f_c \times b} = \frac{2.43 \times 60}{0.85 \times 3 \times 10}$$

$$a = 5.79 \text{''}$$

$$\Rightarrow M_{ug} = 0.90 \times 2.43 \times 60 \times (18 - 5.79/2)$$

$$\Rightarrow M_{ug} = 1986.67 \text{ kip-inch}$$

$$\text{As } M_{ug} < M_u$$

$$1986.67 < 5800$$

So we have to design the beam in such way that it can resist more bending moment than the applied external moment.

Step # 05:-

Finding difference in moments & Area of steel.

$$M_{u1} = M_u - M_{ug}$$

$$= 5800 - 1986.67$$

$$M_{u1} = 3813.33 \text{ kip-inch}$$

By formula,

$$A_{st} = \frac{Mu}{\phi \times f_y \times (d - d')} = \frac{3813.33}{0.90 \times 60 \times (18 - 2.5)}$$

$$A_{st} = 4.56 \text{ in}^2$$

Step #06:-

Finding total steel Area

$$A_s = A_{st} + A_{s'} \\ = 2.43 + 4.56 = 6.99 \text{ in}^2$$

Step #07:-

Selection of Bar:-

In tension zone

Let we use #8 bars

$$\text{dia } (8/8) = 1" \text{ , Area} = \frac{\pi}{4} (1)^2 = 0.785 \text{ in}^2$$

By formula

$$\text{no. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}} = \frac{6.99}{0.785} = 8.9 \approx 9$$

So 9 #8 bars

In compression zone:-

Let we use #7 bars

$$\text{dia } (7/8)" \text{ , Area} = \frac{\pi}{4} (7/8)^2 = 0.601 \text{ in}^2$$

By formula

$$\text{no. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}} = \frac{4.56}{0.601} = 7.58 \approx 8$$

So 8 #7 bars.

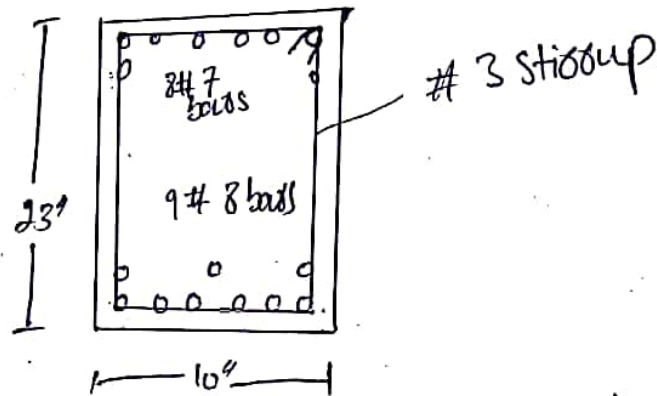


Step # 08:-

Min width for Accomodation of bars.

$$D_{min} = (2 \times 1.5) + (2 \times 3/8) + 9(8/8) + 8(8/8)$$
$$= 20.75''$$

As $20.75'' > 10''$
So, the bars will be placed in multiple layers.



$$\text{Effective depth } (d) = 23 - 1.5 + \frac{3}{8} + \frac{8}{8} + \frac{1}{9}(8/8) = 19.6''$$

$$\text{Effective cover } (d') = 1.5 + \frac{3}{8} + \frac{7}{8} + \frac{1}{9}(7/8) = 3.18''$$

Step # 09:-

Finding the Design Moment

$$M_d = \phi [A_s \times f_y \times (d - d') + (A_{st} - A_{s't}) \times f_y \times (d - a/9)]$$

$$\text{First } a = \frac{(A_s - A_{s't}) \times f_y}{0.85 \times f_c \times b} = \frac{(9 \times 0.785 - 8 \times 0.601) \times 60}{0.85 \times 3 \times 10} = 5.31''$$

$$\Rightarrow M_d = 0.90 [(8 \times 0.601) \times 60 \times (19.6 - 3.18) + (9 \times 0.785 - 8 \times 0.601) \times 60 \times (19.6 - \frac{5.31}{9})]$$

$$M_d = 6398.38$$

$$\text{As } 6398.38 > 5800$$

Design is perfect.

Q.No 6

A beam is revised to developed & ultimate moment of 6000-kip-inches limited to 14x26 inch size, use f'_c is 4 ksi & f_y is 60 ksi. Determine flexural reinforcement assume two rows of tensile reinforcement & effective depth of beam is 22 inch.

Sol:-

concrete compression strength (f'_c) = 4 ksi

steel Tensile strength (f_y) = 60 ksi

Breadth (b) = 14"

Height (h) = 26"

Ultimate Factored Moment (M_u) = 6000 kip-inch

Effective depth of beam (d) = 22"

Assume effective cover (d') = 2.5"

Step # 01 :- (Reinforcement ratio)

By formula

$$s_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \left(\frac{\epsilon_u}{\epsilon_u + \epsilon_t} \right)$$

$$= 0.85 \times 0.85 \times \frac{4}{60} \left(\frac{0.003}{0.003 + 0.001} \right)$$

$$s_{max} = 0.0180$$

Step # 02 :- Area of steel

As we know that

$$s_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = s_{max} \times (b \times d)$$

$$A_{st} = 0.0180 \times (14 \times 22)$$

$$A_{st} = 5.54 \text{ in}^2$$

Step # 03 Design Moment

By using formula

$$M_{u2} = \phi \times A_{st} \times f_y \times (d - a/2)$$

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b} = \frac{5.54 \times 60}{0.85 \times 4 \times 14} = 6.98''$$

$$a = 6.98''$$

$$\begin{aligned} \text{So } M_{u2} &= 0.90 \times 5.54 \times 60 \left(22 - \frac{6.98}{2} \right) \\ &= 5537.4 \text{ kip-inch} \end{aligned}$$

$$A_s; \quad 5537.4 < 6000$$

So we have to design a section as doubly reinforced

Step # 04 (Difference in Moments)

$$\begin{aligned} M_{u1} &= M_u - M_{u2} \\ &= 6000 - 5537.4 \end{aligned}$$

$$M_{u1} = 462.6 \text{ kip-inches}$$

Step # 05: Area of steel

$$M_{u1} = \phi \times A_{st}' \times f_y \times (d - d')$$

compression zone area,

$$A_{st}' = \frac{M_{u1}}{\phi \times f_y \times (d - d')} = \frac{462.6}{0.90 \times 60 \times (22 - 2.5)}$$

$$A_{st}' = 0.44 \text{ in}^2$$

Step # 06 Total steel Area

$$A_s = A_{st} + A_{st}'$$

$$= 5.54 + 0.44$$

$$A_s = 5.98 \text{ in}^2$$

Step # of selection of bars & no. of bars

① Steel in tension zone :-

we use #7 bars

$$\text{dia } (7/8)^{\text{th}} = 0.875$$

$$\text{Area} = \frac{\pi}{4} (0.875)^2$$

$$A = 0.601 \text{ in}^2$$

$$\text{So, No. of bars} = \frac{A_s}{\text{Area of single bar}}$$
$$= \frac{5.98}{0.601} = 9.9 \approx 10 \text{ bars}$$

So, 10 #7 bars

② steel in compression zone :-

we use #5 bars

$$\text{dia} = (5/8)^{\text{th}} = 0.625$$

$$\text{Area} = \frac{\pi}{4} (0.625)^2$$

$$A = 0.306 \text{ in}^2$$

$$\text{No. of bars} = \frac{A_s}{\text{Area of single bar}}$$

$$= \frac{0.44}{0.306} = 1.43 \approx 2 \text{ bars}$$

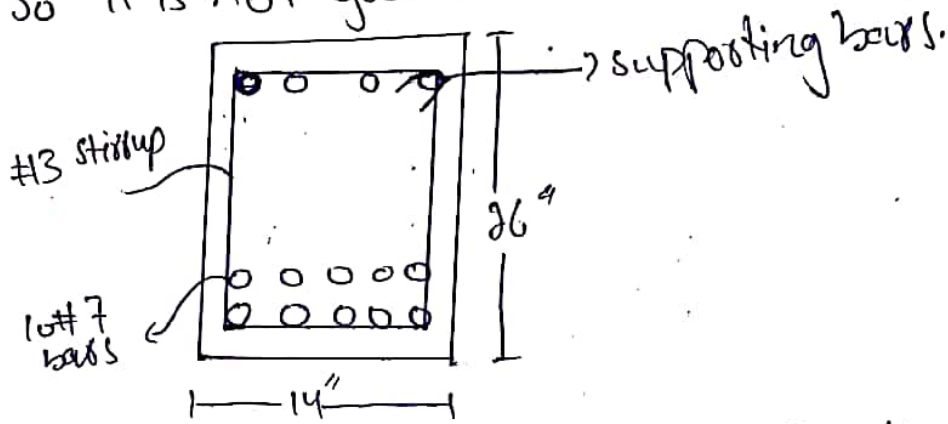
So, 2 #5 bars

Step # 8:- Minimum width of beam

$$b_{min} = 2(1.5) + 2(3/8) + 10(7/8) + 9(7/8)$$

$$b_{min} = 20.39 > 14$$

So it is not good in one layer:



$$\Rightarrow \text{effective depth } (d) = 26 - 1.5 - 3/8 - 7/8 - 1/2 (7/8)$$

$$= 22.82''$$

$$\Rightarrow \text{effective cover } (d') = 1.5 + \frac{3}{8} + 1/2 (5/8)$$

$$= 2.18$$

Step # 9:- Design Moment

$$M_d = \phi \times [A_{st} \times f_y \times (d - d') + (A_{st} - A_{st}') \times f_y \times (d - a/2)]$$

$$a = \frac{(A_{st} - A_{st}') \times f_y}{0.85 \times f_c' \times b}$$

$$= \frac{(10 \times 0.601 - 2 \times 0.306) \times 60}{0.85 \times 4 \times 14} = 6.80''$$

$$d = 0.90 [(2 \times 0.306) \times 60 \times (22.82 - 2.18) + (10 \times 0.601 - 2 \times 0.306) \times 60 \times (22.82 - 6.80/2)]$$

$$M_d = 7047.6 \text{ kip-inch}$$

$$= 7047.6 > 6000$$

Design is OK.

Assignment 1-

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FS30

Section A

Senior