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Summer-20 Final Term Assignment

Subject: Discrete Structure

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Instructor: Sir Daud

Note: Attempt all Questions. All questions carry equal marks. **(50)**

Question No. 1: **(10)**

a) Find the 36th term of the arithmetic sequence whose 3rd term is 7 and 8th term is 17.

Answer:

Step 1
Formulating the sequence

We Assume 'a' as the 1st term & let 'd' be the common difference of the arithmetic sequence,
a_n= a+(n-1) d

3rd Term=
a_3= a+(3-1) d

8th Term=
a_8= a+ (8-1) d

It is given that
a_3 = 7,
a_8 = 17

Therefore;

7= a+2d ----------- eq(i)
17= a+7d---------- eq(ii)

Step2
Subtraction & Substitution

Subtracting eq(i) from eq(ii) to obtain desired values for 'd'

17= a+7d
- 7= a+2d
10= 5d---------eq(iii)

Now Putting eq(iii) in eq(i) to obtain desired values for 'a'

7 = a + 2(2)
So we get a=3

Step 3
Finding 36th item from obtained values

$a_n = a + (n - 1) d$
Putting values of a & d
$a_n = 3 + (n - 1) 2$

So, we get;
$a_{36} = 3 + (36 - 1) 2$
$a_{36} = 3 + 70$

Hence;
$a_{36} = 73$ (Answer)

Question No. 2: **(10)**

Find ***fog*(x)** and ***gof(x)*** of the functions f(x) = 2*x* + 3 and *g*(*x*) = $-x^2 + 5$

Answer:

Given Values:
(fog)(x) = 2x + 6
(gof)(x) = 2x + 3

f(x) = 2x
g(x) = x + 3

Finding (fog)(x)
(fog)(x) = f(g(x)
= f(x+3)
= 2(x+3)
= 2x + 6

So, we get **(fog)(x) = 2x + 6** Answer

Now finding (gof)(x)
(gof)(x) = g(f(x)
= g(2x)
= 2x + 3

So, we get **(gof)(x)= 2x+3** Answer

Question No. 3: **(10)**

Prove by mathematical induction that the statement is true for all integers n ≥ 1 **(10)**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution:

We know that P(1) is true For n=1

L.H.S of P(1)=1^2=1

R.H.S of P(1) = $\frac{1(1+1)(2(1)+1)}{6}$

$=\frac{(1)(2)(3)}{6} = \frac{6}{6} = 1$

Hence proved that L.H.S is equal to R.H.S

Now let's suppose P(K) is true for integer k≥1

$1^2+2^2+3^2+.... +k^2= \frac{k(k+1)(2k+1)}{6}$

Now proving P(k+1) is true;

$1^2+2^2+3^2+.... +(k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$

Now considering L.H.S of above equation;

$1^2+2^2+3^2+ ..... + (k+1)^2 = 1^2+2^2+3^2+ ..... +k^2 +(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$=(k+1)\frac{k(2k+1)}{6} + (k+1)$

$= (k+1)\frac{k(2k+1)+6(k+1)}{6}$

$= k+1\,\frac{2k^2+k+6k+6}{6}$

$= \frac{(k+1)(2k^2+7k+6}{6}$

$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ Answer

Question No. 4: **(10)**

Discuss different types of relations with example in detail.

Answer:

Relations: A connection between the elements of two or more sets is Relation. The sets must be non-empty. A subset of the Cartesian product also forms a relation R. A relation may be represented either by Roster method or by Set-builder method. Relations may exist between objects of the same set or between objects of two or more sets.

Domain & Range: If there are two sets A and B, and relation R have order pair (x, y), then −

The domain of R, Dom(R), is the set {x|(x, y)∈R for some y in B}

The range of R, Ran(R), is the set {y|(x, y)∈R for some x in A}

Types of Relations:

1. ***Empty Relation:*** If no element of set X is related or mapped to any element of X, then the relation R in A is an empty relation, i.e, R = Φ.
   For Example; Think of an example of set A consisting of only 100 hens in a poultry farm. Is there any possibility of finding a relation R of getting any elephant in the farm? No! R is a void or empty relation since there are only 100 hens and no elephant. An Empty set is represented by 'Φ'

2. ***Symmetric Relation:*** A relation R on a set A is said to be symmetric if (a, b) ∈ R then (b, a) ∈ R, for all a & b ∈ A. R is not symmetric if there are elements a and b in A such that (a, b) ∈R, but (b, a) ∈R.
   Example: Let A = {1, 2, 3, 4} and define relations R1, R2, R3, and R4 on A as follows.
   R1 = {(1, 1), (1, 3), (2, 4), (3, 1), (4,2)}
   R2 = {(2, 2), (2, 3), (3, 4)}
   R3 = {(1, 1), (2, 2), (3, 3), (4, 3), (4, 4)}
   R2 is not symmetric, because (2,3) ∈ R3 but (3,2) ∉ R3.

3. ***Transitive Relation:*** Let R be a relation on a set A. R is transitive if and only if for all a, b, c ∈A, if (a,b) ∈R and (b,c) ∈R then (a,c) ∈R. if any one element is related to a second and that second element is related to a third, then the first is related to the third.
   For Example A= {1, 2, 3} all three elements in the set A are related to each other.

4. ***Universal Relation:*** A universal (or full relation) is a type of relation in which every element of a set is related to each other. Consider set A = {a, b, c}. Now one of the universal relations will be R = {x, y} where, |x – y| ≥ 0. For universal relation,
   R = A × A

5. ***Inverse Relation:*** Inverse relation is seen when a set has elements which are inverse pairs of another set. For example, if set A = {(a, b), (c, d)}, then inverse relation will be R-1 = {(b, a), (d, c)}. So, for an inverse relation,
   R-1 = {(b, a): (a, b) ∈ R}

Question No. 5 **(10)**

Suppose that an automobile license plate has three letters followed by three digits.

Answer:

a. **How many different license plates are possible?**

⇨ Total number of letters= 26
Total Number of possible digits=10
Combination of all letters for each row= 26
Combination of all digits for each row= 10

By using product rule, we find all possible combinations.
26x26x26x10x10x10=17,576,000
So, the number of possible plates is **15,576,000**

b. **How many license plates could begin with A and end on 0.**

⇨ We find the possibility of first letter as A & last as 0 by applying the product rule, the number of combinations for 1^{st} and last digit will be set to 1 as for the rest, they will be the same.
1x26x26x10x10x1=67,600
So, the number of possibilities that A is 1^{st} & 0 is last will be **67,600.**

c. **How many license plates begin with PQR**

⇨ We find the possibility of PQR by setting the number of combinations on the letters row to be set to '1' because we only want the desired letters to appear on the number plate i.e
1x1x1x10x10x10= 1000

So, the number of possibilities where PQR will affirmatively appear on the plate are **1000**.

Finish