NAME :- M. Junaid

ID :- 16027

Subject :- linear Algebra

Instructor :- Sir Shakeel

Section : A

Q1

Determine if the following system is Consistent or not

$$x_1 - (3^{rd})^{10} x_2 + x_3 = 0$$

$$-2x_2 \, 8x_3 = 8$$

$$5x_1 - 5x_3 = 10$$

ID = 16027

Solution :-

ID = 16027

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & 5 & 10 \end{array}\right] \quad R_3 - 5R_1$$

$$-5 - 5 = 1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & -10 & 10 \end{array}\right] \quad R_2 | 4$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -1 & 1 \end{array}\right] \quad R_3 | 10$$

P.T.O

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

becous

Consitict bending g

$$-1 y_3 = 1$$
$$y_3 = 1$$

$$x_2 - 4x_3 = 4$$
$$x_2 = 4 + 4y_3$$
$$x_2 = -8$$

$$x_1 - 18x_2 + y_3 = 0$$
$$x_1 = 8y_2 - x_3$$

$$x_1 = 8(8) - 1$$

$$x_1 = 64 - 1$$
$$x_1 = 63$$

Q12

Find the inverse of

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4\text{th}-1D \\ 5 & -2 & 7 \end{bmatrix}$$

by adjoint Method.

Solution

we know that

$$A^{-1} = \frac{Adj\,A^{-1}}{|A|}$$

Now for $|A|$

$$|A| \begin{vmatrix} 3. & 4 & 5 \\ 2 & -1 & 4\text{th } 1D \\ 5 & -2 & 7 \end{vmatrix} = 7-2 = 5 \neq 0$$

he

p.t.o

p.t.o

So the Adjoint of A
is possible

$$\begin{vmatrix} 7 & 4 & 5 \\ 2 & -1 & 4th \\ 5 & -2 & 3 \end{vmatrix}$$

Now for A^{-1}

we know that

$$A^{-} = \frac{AdjA}{|A|}$$

put values

$$\begin{vmatrix} 7 & 4 & 5 \\ 2 & -1 & 4th \\ 5 & -2 & 3 \end{vmatrix} |A|$$

$$\overline{\quad 5 \quad}$$

$$\begin{vmatrix} 7/5 & 4/5 & 5/5 \\ 2/5 & -1/5 & 4/4 \\ 5/5 & -2/5 & 3/5 \end{vmatrix}$$

unnecessary

P.T.O

$$A^{-1}\begin{vmatrix} 1.4 & 0.8 & 1 \\ 0.4 & -0.2 & \frac{41D}{5} \\ 1 & -0.2 & 6.6 \end{vmatrix}$$

Now changing the values

1D 4th = 2

$$2R_1 - R_2 \begin{vmatrix} 1.4 & 0.8 & 1 \\ 1 & 2 & 2 \\ 1 & -2.0 & 0.6 \end{vmatrix}$$

Dignol

$$1.4/2R_2 - R_1 \begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 2 \\ 1 & -2.0 & 0.6 \end{vmatrix}$$

$$2R_3 - R_1 \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \end{vmatrix}$$

$$R_3 - R_1 \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \end{vmatrix}$$

Adjeent method proced

Becous there o and

I Exist

Q3 Solove the following System of linear equation by Guass-jordan Method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Salution

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right] \times (1/2)$$

$$R_1/2 \rightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 4 \end{array}\right] \times (-1)$$

$$R_2 - 1 \times R_1 \rightarrow R_2$$

P.T.O

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right] \times (-3)$$

$$R_3 \quad 3 \times R_1 \to R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right) \times (1/2)$$

$$R_{2/2} \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right] \times (1)$$

$$R_3 - (-1) \times R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & 11 \end{array}\right] \times (-1/9)$$

$R_3/-9 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right] \times (-2)$$

$R_1 - 2 \times R_3 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 59/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right] \times (-1)$$

$R_1 \; 1 \times R_2 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

$$\left\{\begin{array}{l} x = 41/9 \\ y = 2 \\ z = 11/9 \end{array}\right\}$$

General Solution $x \begin{bmatrix} 41/9 \\ 2 \\ 11/9 \end{bmatrix}$

Ans

Q.4

Show that this Matrix is Diagonsible ?

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Solution

Matrix A is Digonsiable if A CDC^{-1}

let $(A - \lambda I_3) = 0$

$$A - \lambda I_3 \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 11 & 1-\lambda \end{bmatrix}$$

p.t.o

$$\Rightarrow 4-\lambda \begin{vmatrix} 3\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 2 \\ -2 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 4-\lambda \left((2-\lambda)(1-\lambda)-8\right) - 2\left(-5(1-\lambda)+4\right) - 2\left(-20+2(3-\lambda)\right) = 0$$

$$\Rightarrow 4-\lambda \left[3-3\lambda\ \lambda+\lambda^2-8\right] - 2\left(-5+5\lambda+4\right) - 2\left(-20+6-2\lambda\right) = 0$$

$$\Rightarrow 4-\lambda\left[\lambda^2-4\lambda-5\right] - 2\left[5\lambda-1\right] - 2\left[14-2\lambda\right] = 0$$

$$\Rightarrow 4\lambda^2+16\lambda-20-\lambda^3+4\lambda^2+5-10\lambda+2+28+4\lambda = 0$$

$$\Rightarrow -\lambda^3+8\lambda^2+15\lambda+10=0$$

$$\lambda = 9.65$$
$$\lambda = -0.82$$
$$\lambda = -0.829$$

for $\lambda = -965$

$$A-\lambda I_3 = \begin{bmatrix} 565 & 2 & -2 \\ -5 & -665 & 2 \\ -2 & 4 & -865 \end{bmatrix}$$

$A = -0.82$ is repeated

take 1

Value

for $\lambda = -0.82$

$$A - \lambda I_3 \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 3.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

In end or by solving only 2 eigenspaces or 2 bills vector in total

So Matrix A is not Digonalizable.

~ * ~ * ~ * ~

Q 5 ::

Delermine if the following homogeneous has non trival Salution Describe the Salution it.

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 + 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Slove The homogeneous System of linear equation.

$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 25x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

P.T.O

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So we have a Solution of

$$x = \begin{bmatrix} \frac{4}{3} 8 \\ 0 \\ 8 \end{bmatrix} = 8 \begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix}$$

~ ⋆ ~ ⋆ ~ ⋆ ~

Q.6

Reduce the Matrix to normal form and find its Rank
$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Solution :-

Maximum posible Rnk for Matrix if $|A| = 0$

$$|A| = \begin{vmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{vmatrix}$$

oR

Rnk = No of non-Zero rows

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 0 & 3 & 14 & 0 \end{bmatrix}$$

$R_3 - R_1$

$1 - 1 = 0$

$3 - 3 = 0$

$4 - 4 = 0$

$0 - 3 = -3$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

p.t.o

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad R_2/3$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$R_2 - R_1$

$1 - 1 = 0$

$3 - 3 = 0$

$4 - 4 = 0$

$3 - 3 = 0$

Rank $= 2$

~ ☆ ~ ☆ ~ ☆ ~ ☆ ~