Assignment No. 01

Subject : Differential Equation

TOPIC : Application of ODE and PDE in Engineering.

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Odinary Differential Equation:

An equation contain only ordinary derivatives of one or more dependent variables of a single independent variable.

e.g

$$\frac{dy}{dx} + 5y = e^x, \quad \frac{(dx)}{dt} + \frac{(dy)}{dt} = 2x + y$$

Application's:

Modeling with First order equation

1. Newton's law of cooling.
2. Electrical circuits.

Modeling Free mechanical oscillations.

1. No damping.
2. Light damping
3. Heavy damping.

→ modelling forced mechanical oscillations.

→ Computer excercise or activity.
→ Newton's Law of Cooling
→ Beam
→ Radio Active elements.
→ physical Application.

Physical Application of ODE:

(i) Its velocity $(v) = \frac{dx}{dy}$

(ii) Its acceleration $(a)\ \frac{dv}{dt}$ or $d x^2/dt^2$ or $v\frac{dv}{dx}$.

If, however the body be moving along a curve then.

Its velocity $(v) = \frac{ds}{dt}$ or

$v\frac{dv}{ds}$ or d^2s/dt^2.

Newton's Second Law

The sate of change in momentum encountered By a moving object is equal to the net force applied to it.
In mathematical Terms.

$$F = \frac{d(mv)}{dt} \Rightarrow m\frac{dv}{dt} + v\cdot\frac{dm}{dt} \Rightarrow F = m\frac{dv}{dt}.$$

$$\boxed{F = ma}$$

Newton's Law of Cooling:

The rate of change of temp of an object is proportional to the difference b/w its own temperature and the temperature of its surrunding.

There force
$d\theta/dt = EA(\theta - \theta_r)$: E·A
Constant that depends upon the object. A - surface area.
θ - A certain.

Radio-Active Half-Life:

The rate of decay is dependent upon the number of molecules / atoms that are there.

$$dN/dt = KN$$

Partial differential equation (PDE).

An equation contains partial derivates of one or more dependent variables of two or more independent variables.

e.g

$$\frac{\partial u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 2\frac{\partial u}{\partial t} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

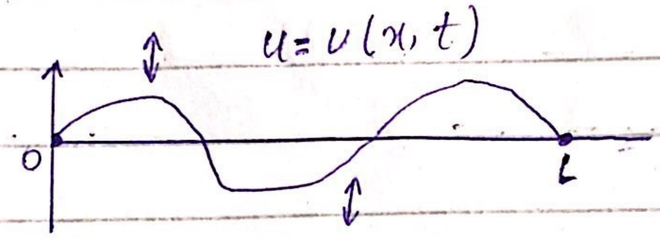
Application's:

PDE's are used to model many systems in many different field of science and engineering.

→ Laplace equation.
→ Heat equation.
→ wave equation.

wave equation:

The simplest situation to give rise to the one-dimensional wave equation is the motion of a stretched string espicially transverse vibration.



It can be shown by applying Newtons law of motion to a small segment of the string statisfies the (PDE).

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Laplace's equation:

The two dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = K\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

it is clear that if the heat flow is steady i.e time dependent then $\frac{\partial u}{\partial t} = 0$ so the temperature $u(x, y)$ is a solution.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Transverse vibrations equation.

$$a^2 \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0$$

For a homogeneous rod: where $u(x, t)$ is the displacement at the time (t) of the cross section through (x).