Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	Sir Pir Meher	**Total Marks:**	30

Student Details

Name:	Bakht Zaman Gohar	**Student ID:**	13678

<table>
<tr><td rowspan="2">Q1.</td><td>(a)</td><td>Consider the following analog signal

$$x_a(t) = 3cos100\pi t + 4sin200\pi t$$
i. Determine the minimum sampling rate required to avoid aliasing.
ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.
iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?</td><td>Marks 5
CLO 1</td></tr>
<tr><td>(b)</td><td>Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n , & n \geq 0 \\ 0, & n < 0 \end{cases}$$
This is signal is sampled at the rate $F_s = 2Hz$.

i. Draw the sampled signal.
ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .
iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.</td><td>Marks 5
CLO 1</td></tr>
<tr><td>Q2.</td><td>(a)</td><td>Determine the response of the system to the following input signal with given impulse response

$$x[n] = \left\{ 2, \underset{\uparrow}{1}, -2, 3, -4 \right\} \quad , h[n] = \left\{ \underset{\uparrow}{3}, 1, 2, 1, 4 \right\}$$</td><td>Marks 5
CLO 2</td></tr>
</table>

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

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I.D:- 13678

SEMESTER:- 6th

Paper:- DSP

★ MID TERM ★

Q1:- (Part - a)

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

i) ★ Solution:-

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

we know that

$$F = \omega / 2\pi$$

$$\text{So, } F_1 = \frac{100\pi}{2\pi} = \frac{100}{2}$$

$$F_1 = 50Hz$$

$$F_2 = \frac{200\pi}{2\pi} = \frac{200}{2}$$

$$F_2 = 100Hz$$

As F_2 is greater so we need F_{max} to find to put in niquest criteria

So, $F_s \geq 2F_{max}$

then, $F_s \geq 2 \times 100 = 200hz$

So, we required $F_s \geq 200hz$ to avoid aliasing.

ii) ★ Solution:-

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

If the given signal is sampled by $F_s = 100Hz$

then,

$$x(n) = x_a(nT)$$

So, when we sampled the signal at given rate it becomes

$$x(n) = 3\cos\left(\frac{100\pi}{100}\right)n + 4\sin\left(\frac{200\pi}{100}\right)n$$

$$x(n) = 3\cos n\pi + 4\sin 2\pi n$$

$$x(n) = 3\cos n\pi + 4\sin n2\pi$$

OR $x(n) = 3\cos(2\pi)(\tfrac{1}{2})n + 4\sin 2\pi n$

iii) ★ Solution:-

the original signal without sampling is $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

& the sampled signal is

$$x(n) = 3\cos 2\pi(\tfrac{1}{2})n + 4\sin 2\pi n$$

And we know that

Folding Frequency $= \frac{F_s}{2} = \frac{100}{2} = 50Hz = F_1$

& $\frac{200}{2} = 100Hz = F_2$

Now to reconstruct the signal

$\omega_1 = 2\pi f_1$ $\qquad$ $\omega_2 = 2\pi f_2$

$\omega_1 = 2\pi(50)$ $\qquad$ $\omega_2 = 2\pi(100)$

$\omega_1 = 100\pi$ $\qquad$ $\omega_2 = 200\pi$

As we know from

$$= A\cos\omega_1 t + A\sin\omega_2 t$$

then

$$= 3\cos 100\pi t + 4\sin 200\pi t$$

So, the reconstruct signal is

$$y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

Q1:- (Part-b)

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

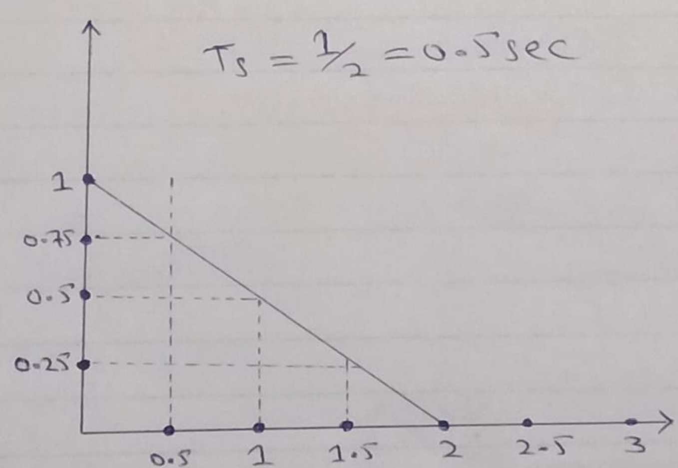
i) **Solution:-**

To draw the sampled signal

$F_s = 2\text{Hz}$

$T_s = \frac{1}{F_s}$

$T_s = 1/2 = 0.5\text{sec}$



ii) ★ Solution:-

From give question we know that $n=3$ bits per sample

So, $L = 2^n$

∴ where "L" is quantization level

$L = 2^3$

$L = 8$ levels

To draw the quantization graph we need to find the quantization level or resolution which is Δ

we also need the dynamic range which is represent as $x_{max} - x_{min}$

So, $$\Delta = \frac{x_{max} - x_{min}}{L}$$

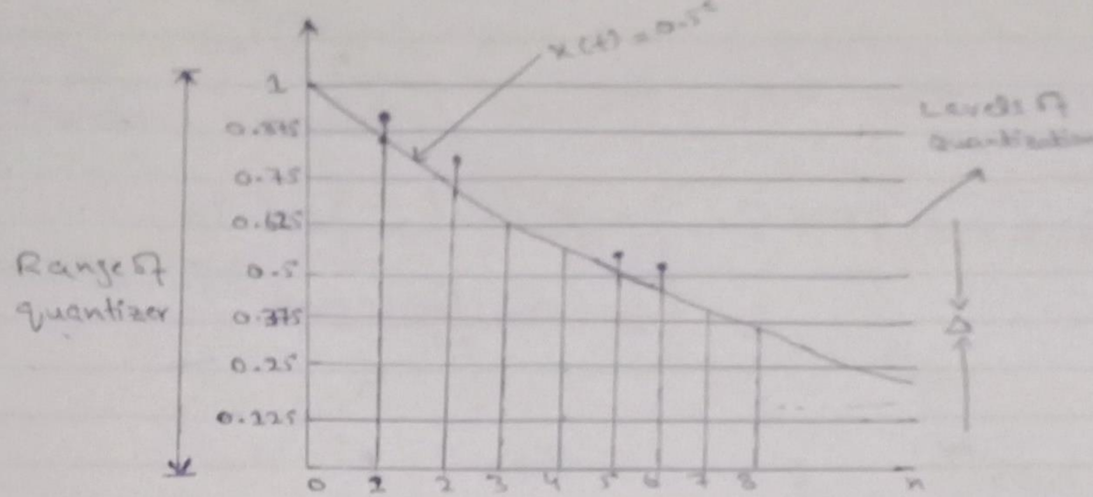
$x_{max} = 1$, $x_{min} = 0$ ∴ wher L is the number of quantization level

$$\Delta = \frac{1-0}{L} \qquad \Delta = \frac{1}{8} = \frac{1}{8}$$

Thus, $\Delta = 0.125$ which is our quantization ~~lev~~ levels range

As 0.5^n where $n \geq 0$

~~So, 0.5^1~~



iii) ★ **Solution**:-

★ Tabular form ★

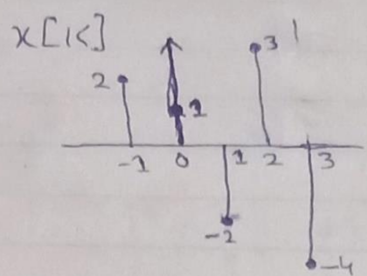
n	$x(n)$ Disvete time signal	$x_q(n)$ Truncation	$x_q(n)$ Rounding	$e_q(n) = x_q(n) - x(n)$ Rounding
0	1	1.0	1.0	0.0
1	0.875	0.8	0.9	-0.1
2	0.75	0.7	0.8	-0.1
3	0.625	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.375	0.3	0.4	-0.1
6	0.25	0.2	0.3	-0.1
7	0.125	0.1	0.1	0.0

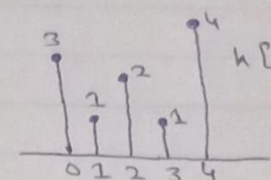
Q2:- (Part-a) Determine the response of the following input signal with given impulse response.

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

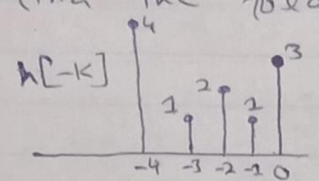
★ Solution:-

$$y[n] = \sum_{k=\infty}^{\infty} x[k]\, h[n-k]$$





Now we find the folded signal $h[-k]$



For $n = 0$

$$y(0) = \sum_{k=-1}^{0} x[-1]\, h[-1] + x(0)\, h(0)$$

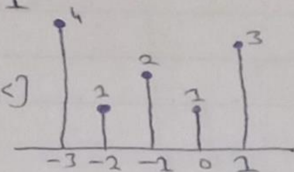
$$= 2 \times 1 + 1 \times 3$$

$$= 2 + 5$$

$$y(0) = 5$$

For $n = 1$

$h[1-k]$



$$y[1] = \sum_{k=-1}^{1} x[n]h[1-k]$$

$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$

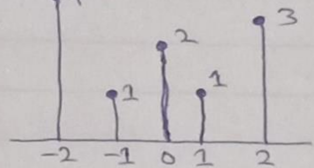
$= (2)(2) + (1)(1) + 3(-2)$

$= 4 + 1 - 6$

$= -1$

$y[1] = -1$

For $n = 2$



By putting in formula

$$y[2] = \sum_{k=-1}^{2} x[n]h[2-k]$$

$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

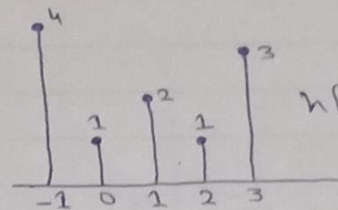
$= 2(1) + (1)(2) + (-2)(1) + 3(3)$

$= 2 + 2 - 2 + 9$

$= 11$

$y[2] = 11$

For $n=3$



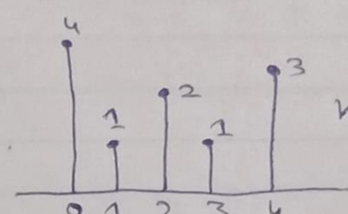
$$y[3] = \sum_{k=-1}^{3} x[n]\, h[3-k]$$

$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 2\times 4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$
$$= 8 + 1 - 4 + 3 - 12$$
$$= 9 + 3 - 16$$
$$= -4$$

$y[3] = -4$

For $n=4$

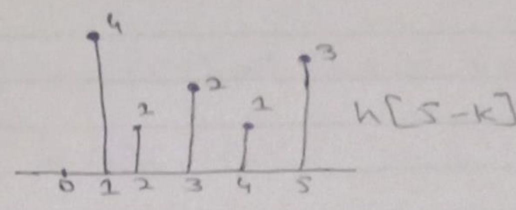


$$y[4] = \sum_{k=0}^{3} x(n)\, h(4-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + 0$$
$$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$$
$$= 4 - 2 + 6 - 4 \quad = 8 - 4 \quad = 4$$

So, $y[4] = 4$

For $n=5$



$$y[5] = \sum_{k=1}^{3} x(n)h(5-k)$$

$$= x(1)h(1) + x(2)h(2) + 3(3)h(3) + 0 + 0$$

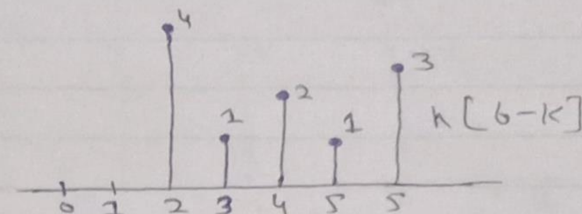
$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$= -8 + 3 - 8$$

$$= -13$$

$$y[5] = -13$$

For $n=6$



$$y[6] = \sum_{k=2}^{3} x(n)h(6-k)$$

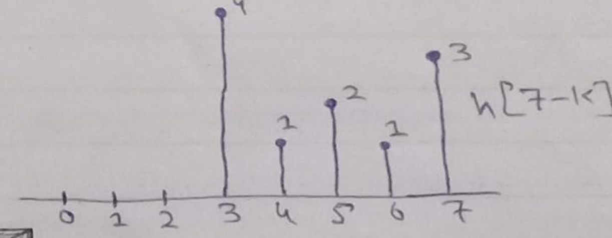
$$= x(2)h(2) + x(3)h(3) + 0 + 0$$

$$= (3)(4) + (1)(-4)$$

$$= 12 - 4$$

$$y[6] = 8$$

For $n=7$



then,

$$y[7] = x(3)\,h(3) + 0 + 0$$

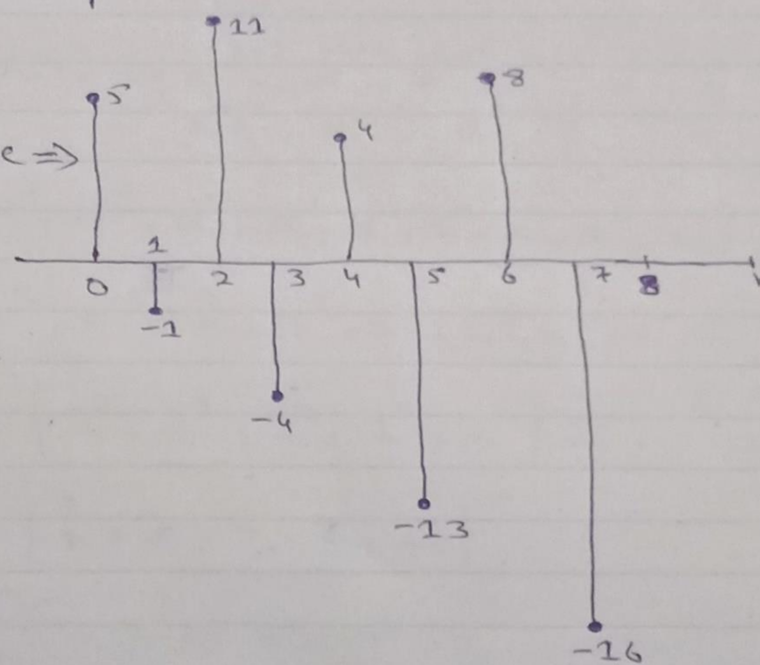
$$y[7] = (4)(-4)$$

$$y[7] = -16$$

As for $n = 8$ there is no overlap

So, ~~{6,~~ $\{\underset{\uparrow}{5}, -1, 11, -4, 4, -13, 8, -16\}$

Response ⇒



Q2:- (Part-b) Compute the convolution $y(n)$ of the following signal.

$$x(n)\begin{cases}\alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere}\end{cases}$$

$$h(n)\begin{cases}2^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere}\end{cases}$$

★ Solution:-

$$y(n) = \sum_{K=0}^{4} h(K)x(n-K)$$

$$x(n) = \{\alpha^{-3+1}, \alpha^{-2+1}, \alpha^{-1+1}, \alpha^{0+1}, \alpha^{1+1}, \alpha^{2+1}, \alpha^{3+1}, \alpha^{4+1}, \alpha^{5+1}\}$$

$$x(n) = \{\alpha^{-2}, \alpha^{-1}, \alpha^{0}, \alpha^{1}, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$$

$$x(n) = \{\alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha^{1}, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$$

$$h(n) = \{2^0, 2^1, 2^2, 2^3, 2^4\}$$

$$h(n) = \{\underset{\uparrow}{1}, 2, 4, 8, 16\}$$

$$Y[n] = \sum_{K=0}^{4} x(n-K), \quad -3 \le n \le 9$$

As $y[n] = x[n] * h[n]$

	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5	α^6
1	α^{-3}	α^{-1}	1	α	α^2	α^3	α^4	α^5	α^6
2	$2\alpha^{-2}$	$2\alpha^{-1}$	2	2α	$2\alpha^2$	$2\alpha^3$	$2\alpha^4$	$2\alpha^5$	$2\alpha^6$
4	$4\alpha^{-2}$	$4\alpha^{-1}$	4	4α	$4\alpha^2$	$4\alpha^3$	$4\alpha^4$	$4\alpha^5$	$4\alpha^6$
8	$8\alpha^{-2}$	$8\alpha^{-1}$	8	8α	$8\alpha^2$	$8\alpha^3$	$8\alpha^4$	$8\alpha^5$	$8\alpha^6$
16	$16\alpha^{-2}$	$16\alpha^{-1}$	16	16α	$16\alpha^2$	$16\alpha^3$	$16\alpha^4$	$16\alpha^5$	$16\alpha^6$

$$y(n) = \{\alpha^{-2},\ 2\alpha^{-2}+\alpha^{-1},\ 4\alpha^{-2}+2\alpha^{-1}+1,\ 8\alpha^{-2}+4\alpha^{-1}+2+\alpha,\ 16\alpha^{-2}+8\alpha^{-1}+4+2\alpha+\alpha^2,\ 16\alpha^{-1}+8+4\alpha+2\alpha^2+\alpha^3,\ 16+8\alpha+4\alpha^2+2\alpha^3+\alpha^4,\ 16\alpha+8\alpha^2+4\alpha^3+2\alpha^4+\alpha^5,\ 16\alpha^2+8\alpha^3+4\alpha^4+2\alpha^5+\alpha^6,\ 16\alpha^3+8\alpha^4+4\alpha^5+2\alpha^6,\ 16\alpha^4+8\alpha^5+4\alpha^6,\ 16\alpha^5+8\alpha^6,\ 16\alpha^6\}$$

Q3:- Determine the z-transform of following signals & also sketch its Region of convergence (ROC)

i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n , & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} , & n < 0 \end{cases}$

★ Solution:-

As we know that

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

So the given question becomes

$$X(z) = \sum_{n=0}^{\infty} \left(\tfrac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{0} \left(\tfrac{1}{3}\right)^{n} z^{-n} - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\tfrac{1}{3}\right)^n z^{n} - 1$$

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\frac{z}{3}+1-\frac{1}{4z} - \left[1-\frac{1}{4z}-\frac{z}{3}+\frac{z}{12z}\right]}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$=$

$$= \frac{2 - \frac{7}{3} - \frac{1}{4z} - 1 + \frac{1}{4z} + \frac{7}{3} - \frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

$$= \frac{2 - 1 - \frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

$$= \frac{1 - \frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

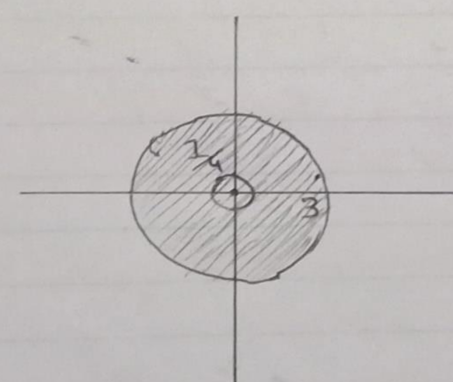
So, $$X(z) = \frac{1 - \frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

★ **Region of Convergence :-**

thus the ROC will be

$\Rightarrow \frac{1}{4} < |z| < 3$

★ **Sketch :-**



ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

★ **Solution:-**

Apply z-transform

$$X(z) = \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} (3)^n z^{-1}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{\left(1-\frac{3}{z}\right) - \left(1-\frac{1}{2z}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$= \frac{1-\frac{3}{z} - 1 + \frac{1}{2z}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$= \frac{\frac{1}{2z} - \frac{3}{z}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$= \frac{\frac{1-6}{2z}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$= \frac{-\frac{5}{2} z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

So, $$X(z) = \frac{-\frac{5}{2}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

★ **Region of Convergence:-**

$$\frac{1}{2} < |z|$$

$$\text{or } |z| > \frac{1}{2}$$

$$3 < |z|$$

$$\text{or } |z| > 3$$

Here $z > \frac{1}{2}$ & $|z| > 3$

So, ROC becomes

$$\Rightarrow |z| > 3$$

★ **Sketch:-**