NAME: Shah Sawar Khan

I.D:NO: 7712

Submitted To: Ma'am Shumaila

Date: 20-Aug-2020

Q#01:→ $\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2)$ $y(0)=0$

Sol:→ As we know $y(0)=0$

so $t=0$ and $y=0$

$dy = e^{y}.e^{-t}\sec(y)(1+t^2)dt$

$\frac{1}{e^{y}.\sec(y)}dy = (1+t^2)e^{-t}dt$

As $\cos(y) = \frac{1}{\sec(y)}$

$\int e^{-y}\cos y\,dy = \int(1+t^2)e^{-t}dt$

Using integration by parts

$e^{-y}\int\cos y\,dx - \int(\int\cos y \cdot \frac{d}{dy}e^{-y}) = (1+t^2)\int e^{-t} - \int(\int e^{-t}\frac{d}{dt}(1+t^2)) \rightarrow (1)$

Now L.H.S

$e^{-y}\int\cos y\,dx - \int(\int\cos y \cdot d/dy\, e^{-y})$

$e^{-y}\sin y - \int(\sin y . e^{-y}(-1))$

$e^{-y}\sin y + \int(\sin y . e^{-y})$

$e^{-y}\sin y + \int(e^{-y}\sin y)$

Again using integration by parts

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(\int\sin y\, d/dy\, e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(-\cos y\, \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int(\cos y e^{-y})$

Since $\int(\cos y e^{-y}) = L.H.S.$

So it is again same to the first one so L.H.S will become

$$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$$

$$2\,LHS = e^{-y}(\sin y - \cos y)$$

$$LHS = \frac{e^{-y}(\sin y - \cos y)}{2}$$

Now taking R.H.S

$$\int (1+t^2)\, e^{-t}\, dt$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \cdot \tfrac{d}{dt}(1+t^2)\right)$$

$$-(1+t^2)e^{-t} - \int\left(-e^{-t}(2t)\right)$$

$$-(1+t^2)e^{-t} + \int\left((2t)e^{-t}\right)$$

Again Using integration by parts

$$-(1+t^2)e^{-t} + \left(2t\int e^{-t} - \int\left(\int e^{-t}\, \tfrac{d}{dt}\, 2t\right)\right)$$

$$-(1+t^2)e^{-t} + \left(-2te^{-t} - \int(-e^{-t}\, 2)\right)$$

$$-(1+t^2)e^{-t} + \left(-2te^{-t} + \int(2e^{-t})\right)$$

$$-(1+t^2)e^{-t} + \left(-2te^{-t} - 2e^{-t}\right) + c$$

$$-(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c$$

$$-e^{-t} - e^{-t}t^2 - 2te^{-t} - 2e^{-t} + c$$

$$-(t^2+2t+3)\, e^{-t} + c = RHS$$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y-\cos y)}{2} = -(t^2+2t+3)e^{-t}+c$$

we know that

$t=0$ & $y=0$

Put it above

$$\frac{1}{2}(0-1) = -3+c$$

$$c = 5/2$$

Put value of c

$$\boxed{\frac{e^{-y}}{2}(\sin y-\cos y) = -(x^2+2t+3)e^{-t}+5/2}$$

Answer

Q#02:- $(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{x+y}-\sqrt{x-y})dy=0$

Sol:- $\dfrac{dy}{dx} = \dfrac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \longrightarrow (1)$

This is Homogeneous differential eq: in x & Y

to solve this put $y=vx$

$$\frac{dy}{dx} = v+x\frac{dv}{dx}$$

Thus eq (i) becomes

$$v+x\frac{dv}{dx} = \frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

Now take integration on both Sides

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

Scanned with CamScanner

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = 1/cx$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = 1/cx$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$\boxed{x+\sqrt{x^2-y^2} = c_1} \qquad \therefore \frac{1}{c} = c_1$$

Which is Required Solution

Q#03:- $(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

Sol:- $(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

$f(D)y = f(x)$

As it is non-homogenous linear eq:
So Solution will be

$y = y_c + y_p$ — (1)

Complementary Sol: y_c

$D^4+D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow \boxed{D=0}$

or $D^2+1=0 \Rightarrow D^2 = -1$

$D = \sqrt{-1} \Rightarrow \boxed{D = i}$ or $\boxed{D = 0+i}$

Roots are real & complex

$y_c = c_1 e^{0x} + e^{0x}(c_2\cos x + c_3 \sin x)$

$y_c = c_1 + c_2\cos x + c_3 \sin x$

$y_p = \frac{1}{f(D)} F(x)$

$y_p = \frac{1}{D^4+D^2}(3x^2+4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4+D^2$

at $D=0 \Rightarrow f(D)=0$

So $f'(D) = 4D^3 + 2D$

Now Also for $D=0 \Rightarrow f'(D) = 0$

again differentiating

$$f''(D) = 12D + 2$$

So for $D = 0$

$$f''(D) = 12(0) + 2 = 2$$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$y_p = \frac{x^2 \, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

Putting $D=0$ in all

so $$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So putting in eq: (i)

$$y = c_1 + c_2\cos x + c_3\cos x + \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$

THE END.