DSP ⇒ 1

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Q No (1) (Part a)

Solution:

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The Characteristics eq is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \text{ Hence}$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The Particular Solution is

$$y_p(n) = K(-1)^n u(n).$$

Substituting this eq in difference eq, we get

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2)$$

$$= (-1)^n u(n) - (-1)^{n-1} u(n-1).$$

For $n = 2$, $K(1+4+4) = 2 \Rightarrow K = \frac{2}{9}$

Total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

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from the initial Condition.

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = \frac{7}{9}.$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2$$

$$\Rightarrow C_2 = \frac{1}{3},$$

Q No 1 (b)

Solution:-

$$\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.7\lambda + 0.1) = 0$$

$$\lambda^2 - 0.5 - 0.2\lambda + 0.1 = 0$$

$$\lambda(\lambda \cdot 0.5) - 0.1(\lambda - 0.5) = 0$$

$$\lambda = 0.5 \qquad \lambda = 0.1$$

General form of solution to be Homogeneous equation is

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n.$$

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$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{2}^n$$

with $\cancel{x}\,z(n) = u(n)$ we have

$$y(0) = 2$$

$$y_1(1) - 0.7y(0) = 0 \Rightarrow$$

$$y(1) = 1.4$$

Hence $C_1 + C_2 = 2$ and

$$\frac{1}{2} C_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$C_1 + \frac{2}{5} C_2 = \frac{14}{5}$$

Thes equations yield

$$C_1 = \frac{10}{3} \qquad C_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

Step response is

$$S(n) = \sum_{K=0}^{n} h(n-K) \Rightarrow \frac{10}{3} \sum_{K=0}^{n} \left|\frac{1}{2}\right|^{n-1} - \frac{4}{3}^n \sum \left(\frac{1}{5}\right)^{n-1}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{K=0}^{n} 2^K - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{K=0}^{n} 5^K$$

$$= \frac{10}{3}\left(\left(\frac{1}{2}\right)^{n}\left(2^{n+1}-1\right)u(n) - \frac{1}{3}\left(\left(\frac{1}{5}\right)^{n}\left(5^{n+1}-1\right)u(n)\right)\right.$$

Q No 2 (a)

Sol:

$$\frac{X(z)}{z} = \frac{z^2}{(2z-1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{A_1}{2z-1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

Find A, B, C

$A = 4$

$B = 3$

$C = -1$

Hence

$$x(n) = \left[4(2)^n - 3 - n\right]u(n)$$

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Q No 2 (b)

Solution:

We have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

Where C is a circle radius greater than $|a|$. we evaluate $f(z) = z^n$. We distinguish two cases.

Case (1) $\Rightarrow$ if $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. The only pole inside C is $z = a$

Hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

Case (2) $\Rightarrow$ if $n < 0$ $f(z) = z^n$ has an nth-order pole at $z = 0$

Having Contributers from both pole

For $n=-1$ we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)} dz$$

$$= \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

if $n=-2$ we have

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)} dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a}$$

By Contining same way we can show that

$x(n)=0$ for $n<0$ thus

$$x(n) = a^n u(n)$$

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Question No # 3

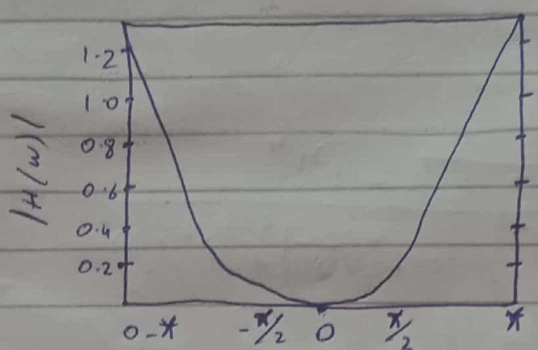
Part (a)

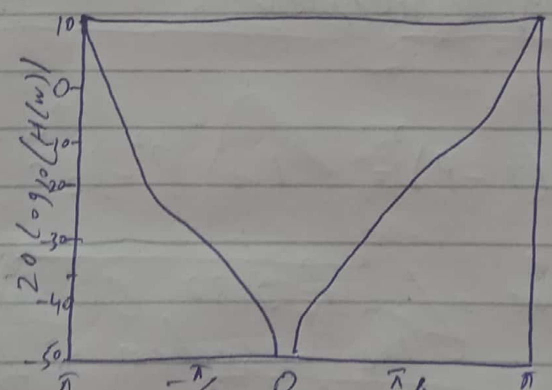
Solution:

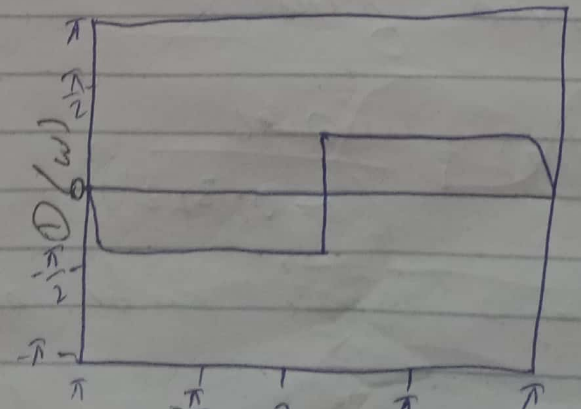
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

Hence $b_0 = (1-P)^2$







At $\omega = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{\left(1-Pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-P)^2}{\left((1-P\cos(\pi/4)) + jP\sin(\pi/4)\right)^2}$$

$$= \frac{(1-P)^2}{\left(1-P/\sqrt{2} + jP/\sqrt{2}\right)^2}$$

Hence

$$\frac{(1-P)^4}{\left[(1-P/\sqrt{2})^2 + P^2/2\right]^2}$$

or equivalently

$$\sqrt{2}(1-P)^2 = 1 + P^2 - \sqrt{2}P$$

The value of $P = 0.32$ satisfy this equation. The system fn for desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The Same Principle can be applied for Design of band Pass filter

Part (b)

Clearly. Filter must have poles at

$P_{1,2} = r e^{\pm j\pi/2}$ and zeros at $z=1$ and $z=-1$.

The System fn is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2-r^2}$$

The frequency response $H(\omega)$ of filter at $\omega = \pi/2$.

Thus we have

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determind by evaluating $H(\omega)$ at $\omega = 4\pi/9$

Thus we have

(11)

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \cdot \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

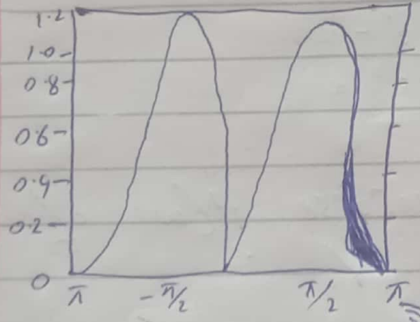
$$= \frac{1}{2}$$

or $1.94(1-r^2)^2 = 1-1.88r^2+r^4$

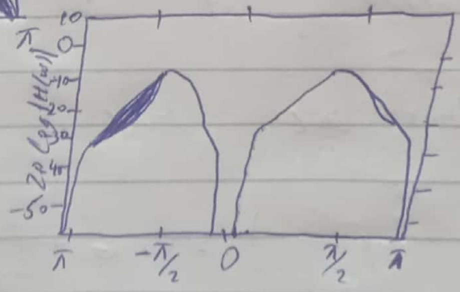
Value of $r^2 = 0$ satisfy equation.

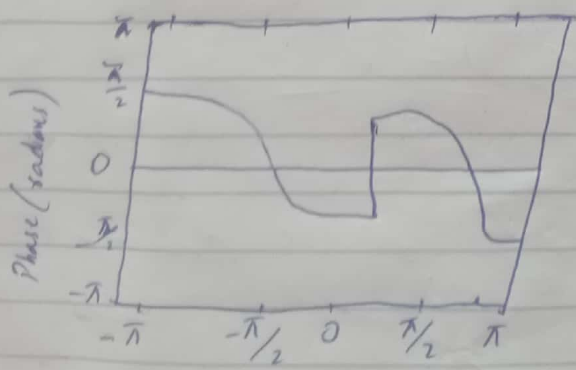
f/n for r for desired

filter is $H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$



$|H(w)|$





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Q No # 4 (a)

Sol:- The fourier transform of the sequence is

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1 - e^{-jwL}}{1 - e^{-jw}}$$

$$= \frac{(\sin wL/2)}{\sin(w/2)} e^{-jw(L-1)/2}$$

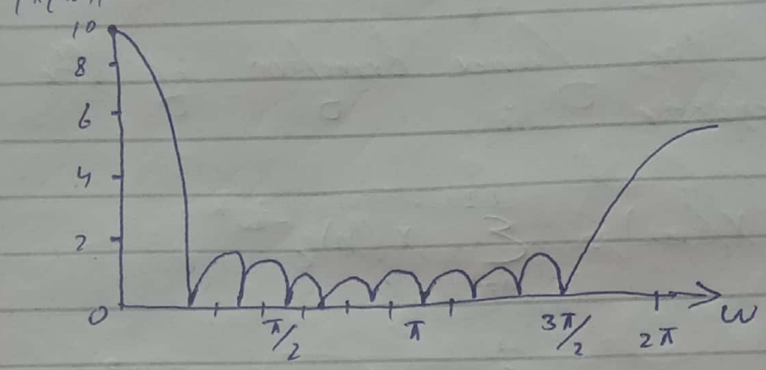
The magnitude and phase of $X(w)$ are illustrated in for $L = 10$ The N-Point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequencies

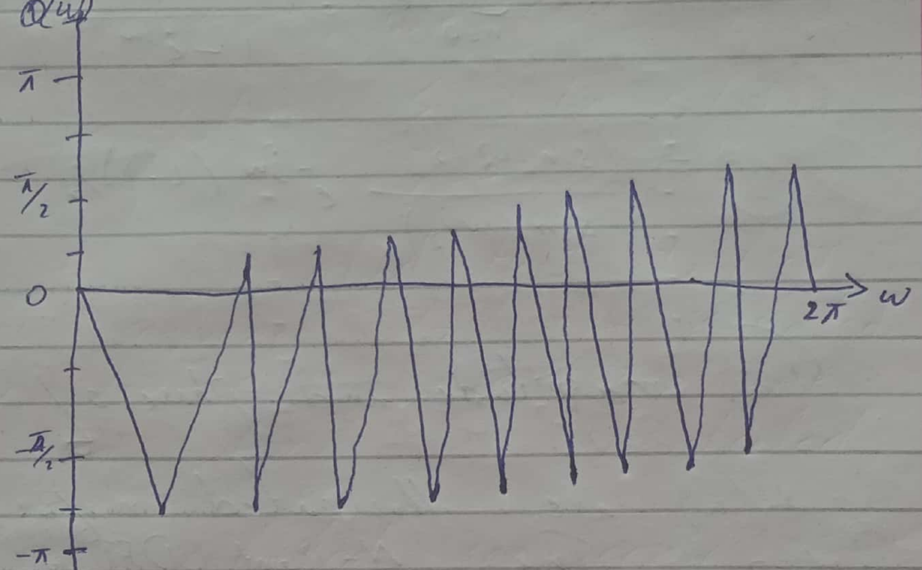
$$W_k = 2\pi K/N \quad K = 0, 1, 2 - N-1$$

Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \quad K = 0, 1, - N-1$$

$$= \frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





if N is sellected such that $N = L$ then DFT becomes

$$X(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2, 3, \ldots L-1 \end{cases}$$

Thus there is only one non zero value in DFT. This is apparent from $X(w)$. Since $X(w) = 0$ at frequencies $w_K = 2\pi K/L$ $K \neq 0$. The reader should verify that

$x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT.

Q No # 4 (b)

Solution:

Each Sequence Consists of four non-zero points. For the purposes of illustrating the operation involved in Circular convolution, it is desirable to graph each sequence as points on a circle.

Thus Sequence $x_1(n)$ and $x_2(n)$ are graphed as illustrated in figure. We note that, The sequences are graphed in counter clock direction o circle. No $X_3(m)$ is obtained by Circulating convolving $X_1(n)$ and $x_2(n)$ as specified by by begining with $m = 0$.

We have

$$x_3(0) = \sum_{n=0}^{3} V_1(m)\, x_2((l-n))_N \quad m=0$$

$x_2((l-n))_4$ is simply the Sequence $x_2(n)$ followed and graphed on Circle.

(15)

The Product Sequence is obtained by multiplying $x_1(n)$ with $x_2((1-n))_4$. Pointed in figure.

Finaly we add values to obtain.

$$x_3(0) = 14$$

for $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

it is verified that $x_2((1-n))_4$ is sequence $x_2((-n))_4$ rotated counter clock wise by one unit time as illustrated.

The rotated seqeuence multiplies $x_1(n)$ to yield product sequence

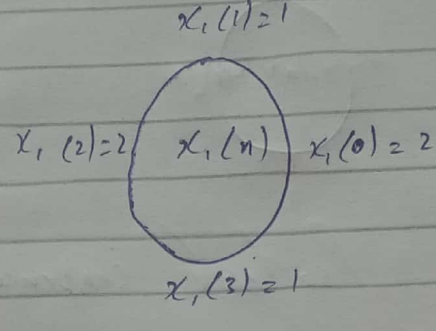
$$x_3(1) = 16$$

$m=2$ we have

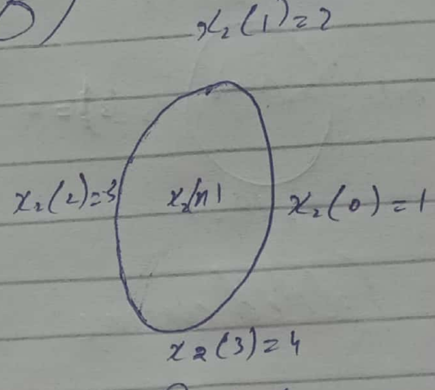
$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence in fig. Rotated two units of time in counter clock wise direction

The Resultant sequence is illustrated in figure

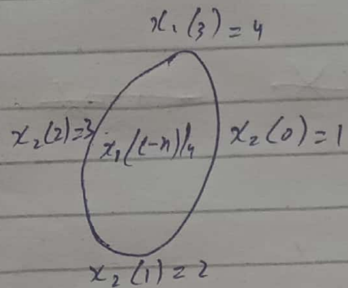


Folded sequence

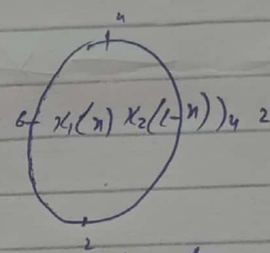


Product seq

a

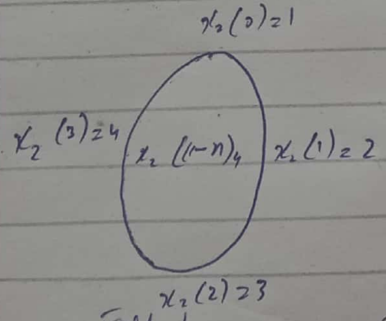


Folded



Product

b



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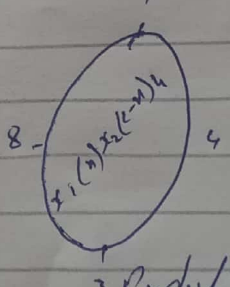
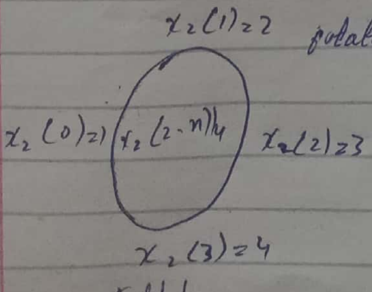


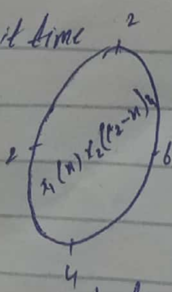
Product

c

$x_2(1)=2$ rotated one unit time

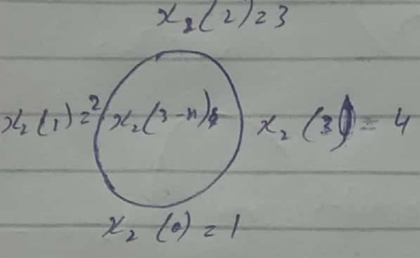


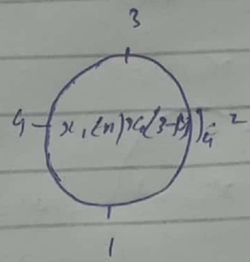
Folded



Product.

(d)





Folded sequence   Product sequence

Rotated by 3 units in time