Differential Equation
Sessional Assignment

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Question 1:

Use any of methods for solving the ordinary Differential equation. as given below.

Solve and graph the solution. show the details of your work.

$x^2y'' - 4xy' + 6y = 0 \qquad y(1) = 0.4, \quad y'(1) = 0$

Solution:

$$x^2y'' - 4xy' + 6y = 0 \quad \text{—— (i)}$$

Put $y = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

Put $y'' = m(m-1)x^{m-2}$ in equ (i)

$$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0$$

$$\Rightarrow \cancel{x^2} m(m-1)x^m \cancel{x^{-2}} - 4\cancel{x}m\cancel{x}^m . \cancel{x^{-1}} + 6x^m = 0$$

$$x^m \{m(m-1) - 4m + 6\} = 0$$

Dropping x^m

$$\Rightarrow \quad m(m-1) - 4m + 6 = 0$$

$$m^2 - 5m + 6 = 0$$

Finding roots [redacted]

$$m^2 - 5m + 6$$

$$\Rightarrow \quad m = \frac{5 \pm \sqrt{(-5)^2 + 4(6)}}{2a}$$

$$m = \frac{5+1}{2} \qquad m = \frac{5-1}{2}$$

$$m = \frac{\cancel{6}\,3}{\cancel{2}\,1} \qquad m = 2$$

$$m = 3 \quad \text{and} \quad m = 2$$

So

$$y' = x^3 \qquad y^2 = x^2$$

[redacted]

General solution is

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1 x^3 + c_2 x^2$$

(y')

$$y' = 3c_1 x^2 + 2c_2 x$$

Putting values of y_0 and y'

$$\Rightarrow \begin{cases} 0.4 = c_1 I^3 + c_2 I^2 \\ 0 = 3c_1 I^3 + 2c_2 I^2 \end{cases}$$

$$\Rightarrow \begin{cases} 0.4 = c_1 + c_2 \\ 0 = 3c_1 + 2c_2 \end{cases}$$

$$\Rightarrow \begin{cases} 0.4 - c_2 = c_1 \\ 0 = 3(0.4 - c_2) + 2c_2 \end{cases}$$

$$\Rightarrow \begin{cases} 0.4 - c_2 = c_1 \\ 1.2 = c_2 \end{cases}$$

$$\Rightarrow \begin{cases} 0.4 - 1.2 = c_1 \\ c_2 = 1.2 \end{cases}$$

$$\begin{cases} c_1 = 0.4 - 1.2 \\ c_2 = 1.2 \end{cases}$$

$$\begin{cases} c_1 = -0.8 \\ c_2 = 1.2 \end{cases}$$

Solution Become

$$y = (-0.8x^3) + 1.2x^2$$ Ans.

Question 13

$$x^2 y'' + 3xy' + 0.75y = 0$$

$y(1) = 1$
$y'(1) = 1.5$

Solution : [redacted]

$$x^2 y'' + 3xy' + 0.75 y = 0 \quad \text{——— (1)}$$

Putting $y = x^m$

$$y = x^m$$

$$\cancel{y' = m(m-1)x}$$

$$y' = mx^{m-1}$$

$$y'' = (m-1)x^{m-2}$$

Putting values in equation 1

$$x^2 m(m-1)x^{m-2} + 3x\,mx^{m-1} + 0.75\,x^m = 0$$

$$\Rightarrow \cancel{x^2} m(m-1)x^m \cancel{x^{-2}} + 3\cancel{x}\,mx^m.\cancel{x^{-1}} + 0.75x^m = 0$$

$$\Rightarrow x^m[m(m-1) + 3m + 0.75)] = 0$$

Dropping common

$$m^2 + 2m + 0.75 = 0$$

Finding roots

$m^2 + 2m + 0.75 = 0$

$m_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (0.75}}{2}$

$m_{1/2} = \frac{-2 \pm 1}{2}$

$m_1 = \frac{-2+1}{2}$ $m_2 = \frac{-2-1}{2}$

$= \frac{-1}{2}$ $\wedge$ $m_2 = -\frac{3}{2}$

General Solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^{0.5} + c_2 x^{-1.5}$

$y' = 0.5 c_1 x^{-1.5} - 1.5 c_2 x^{-2.5}$

c_1 and c_2

Putting values

$$\begin{cases} y(1) = 1 = c_1 \cdot 1^{-0.5} + c_2 \cdot 1^{-1.5} \\ 1.5 = y'(1) = 0.5 c_1 \cdot 1^{1.5} - 1.5 c_2 \, 1^{-2.5} \end{cases}$$

$$\begin{Bmatrix} 1 = c_1 + c_2 \\ 1.5 = 0.5c_1 - 1.5c_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 1 = c_1 + c_2 \\ 3 = c_1 + 3c_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 1 - c_2 = c_1 \\ 3 = 1 - c_2 + 3c_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 1 - c_2 = c_1 \\ 2 = 2c_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 1 - c_2 = c \\ 1 = c_2 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 = c_1 \\ 1 = c_2 \end{Bmatrix}$$

$$\begin{Bmatrix} c_1 = 0 \\ c_2 = 1 \end{Bmatrix}$$

Solution for I.V.P is

$y = x^{-1.5}$ Ans.

Question 14

$$x^2y''+xy'+9y=0$$

$$y(1)=0$$

$$y'(1)=2.5$$

Solution

$$x^2y''+xy'+9y=0$$

Put $y=x^m$

$$y=x^m$$

$$y'=mx^{m-1}$$

$$y''=(m-1)mx^{x-2}$$

Now

$$\cancel{x^2}\,m(m-1)x^m\cancel{x^{-2}}+mx^m+9x^m=0$$

Dropping common factor

$$m(m-1)+m+9=0$$

$$m^2-m+m-9=0$$

$$m^2+9=0$$

$$\cancel{m^2=-9}$$

$$m^2-(3i)^2=0$$

$$(m-3i)(m+3i)=0$$

$$m_1=3i \wedge m_2=-3i$$

Putting values

$x^{m_1} = x^{3i} = (e^{\ln x})^{3i} = e^{3i\ln x}$

$x^{m_2} = x^{-3i} = (e^{\ln x})^{-3i} = e^{-3i\ln x}$

Recall that:

$e^x = e^{a+ib} = e^a(\cos b + i\sin b)$

$\Rightarrow e^{3i\ln x} = e^0[\cos(3\ln x) + i\sin(3\ln x)]$

$= \cos(3\ln x) + i\sin(3\ln x)$

$\rightarrow$

$e^{-3i\ln x} = e^0[\cos(3\ln x) - i\sin(3\ln x)] = \cos(3\ln x) - i\sin(3\ln x)$

Now

$x^{m_1} = \cos(3\ln x) + i\sin(3\ln x)$

$x^{m_2} = \cos(3\ln x) - i\sin(3\ln x)$

Adding:

$x^{m_1} - x^{m_2} = \cancel{\cos(3\ln x)} + i\sin(3\ln x) - \cancel{\cos(3\ln x)} + \sin 3(\ln x)$

$\frac{xm^1 - xm^2}{2i} = \frac{\cancel{2i}\sin(3\ln x)}{\cancel{2i}}$

$= \sin 3\ln x$

Solution will be

$y_1 = \cos(3\ln x)$ $\quad y_2 = \sin 3\ln x$

$y = c_1\cos(3\ln x) + c_2\sin(3\ln x)$

$y_1 = c_1\overset{1}{\cancel{\cos(3\ln 1)}} + c_2\overset{0}{\cancel{\sin(3\ln 1)}} = 0 \Rightarrow c_1 = 0$

$y'(1) = -3c_1\overset{0}{\underline{\sin(3\ln 1)}} + 3c_2\overset{1}{\cancel{\cos(3\ln 1)}}$

$= 2.5$

$c_1 = 0$

$c_2 = \frac{5}{6}$

$y = \frac{5}{6}\sin 3\ln x$ Ans.

Question 15

$$x^2y'' + 3xy' + y = 0 \qquad y(1) = 3.6$$
$$y'(1) = 0.4$$

Solution:

$$x^2 y'' + 3xy' + y = 0$$

Put

$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)x^{m-2}$$

$$\Rightarrow \cancel{x^2}\, m(m-1)x^m \cdot \cancel{x^{-2}} + 3mx^m + x^m = 0$$

By dropping factor

$$m(m-1) + 3m + 1 = 0$$
$$m^2 + 2m + 1 = 0$$
$$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0$$
$$m = 1$$

∴ It becomes

$$y_1 = x^m = x^{-1} = \frac{1}{x}$$

Standard form is

$$y'' + \frac{3}{x} \cdot y' + \frac{1}{x^2} \cdot y = 0$$

$p(x) = 3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = 3\ln x$

Put

$y_2 = u y_1$

where

$u = \int v\,dx \quad \wedge \quad v = \frac{1}{y_1^2} e^{-\int p dx}$

v?

$e^{-\int p dx} = e^{-3\ln|x|} = \left(e^{\ln|x|}\right)^{-3} = x^{-3}$

$v = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$

By integration:

$u = \int \frac{dx}{x} = \ln|x|$

So

$y_2 = u y_1 = y_1 \ln x = \frac{1}{x} \cdot \ln x$

Since

$y = c_1 y_1 + c_2 y_2$

$= c_1 \cdot \frac{1}{x} + c_2 \cdot \frac{1}{x} \cdot \ln x$

$= \frac{1}{x} \cdot (c_1 + c_2 \ln x)$

P. Rule

$\Rightarrow \quad y' = (x^{-1})'(c_1 + c_2 \ln x) + x^{-1}(c_1 + c_2 \ln x)'$

$= -x^{-2}(c_1 + c_2 \ln x) + \frac{1}{x} c_2 \cdot \frac{1}{x}$

$= \frac{1}{x^2}(-c_1 - c_2 \ln x + c_2)$

Values of C

$$\begin{cases} 3.6 = y(1) = \frac{1}{1}(c_1 + c_2 \ln 1) \\ 0.4 = y'(1) = \frac{1}{1^2}(-c_1 - c_2 \ln 1 + c_2) \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 0.4 = c_1 + c_2 \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 0.4 = -3.6 + c_2 \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 4.0 = c_2 \end{cases}$$

Solution

$$y = (3.6 + 4.0 \ln x) \cdot \frac{1}{x}$$

Question No 16

$$(x^2 D^2 - 3xD + 4I)y = 0$$

$y(1) = -\pi,$
$y'(1) = 2\pi$

Solution:

$$(x^2 D^2 - 3xD + 4I)y = 0$$

$$\Rightarrow x^2 D(Dy) - 3x Dy + 4y$$

$$\Rightarrow x^2 y'' - 3xy' + 4y = 0$$

Put $y = x^m$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$$x^2 m(m-1) x^{m-2} - 3x m x^{m-1} + 4x^m = 0$$

$$\cancel{x^2} m(m-1) x^m \cdot \cancel{x^{-2}} - 3\cancel{x} m x^m \cancel{x^{-1}} + 4xm = 0$$

$$\Rightarrow m(m-1) - 3m + 4 = 0$$

$$\Rightarrow m^2 - 4m + 4 = 0.$$

$$\Rightarrow (m-2)^2 = 0$$

$(m-2)^2=0$

$m=2$

Now

$y_1 = x^m = x^2$

It becomes

$$y'' - \frac{3}{x} \cdot y' + \frac{4}{x^2} \cdot y = 0$$

$$P(x) = -3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = -3\ln(x)$$

Put $y_2 = u y_1$

where

$$u = \int v\,dx \quad \& \quad v = \frac{1}{y_1^2} e^{-\int p dx}$$

Lets find v

$$e^{-\int p dx} = e^{3\ln|x|} = (e^{\ln|x|})^3 = x^3$$

$$v = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}$$

By '$\int$'

$$u = \int \frac{dx}{x} = \ln|x|$$

So

$$y_2 = u y_1 = y_1 \ln x = x^2 \ln x$$

General solution is

$$y = c_1 y_1 + c_2 y_2$$
$$= c_1 x^2 + x^2 \ln x$$
$$= x^2 (c_1 + c_2 \ln x)$$

Product rule:

$$y' = (x^2)'(c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$$
$$= 2x (c_1 + c_2 \ln x) + c_2 x^2 \cdot \frac{1}{x}$$

$$= 2c_1 x + 2c_2 x \ln x + c_2 x$$
$$= 2c_1 x + c_2 x (2 \ln x + 1)$$

Finding c

$$\begin{cases} -\pi = y(1) = 1^2 (c_1 + c_2 \ln 1) \\ 2\pi = y'(1) = 2c_1 + c_2 (2 \ln 1 + 1) \end{cases}$$

$$\begin{cases} -\pi = c_1 \\ 2\pi = 2c_1 + c_2 \end{cases}$$

$$\begin{cases} -\pi = c_1 \\ 4\pi = c_2 \end{cases}$$

Particular solution of IVP

$$y = x^2 (-\pi + 4\pi \ln x)$$

Question 17

$(x^2D^2 + xD + I)y = 0$ $y(1) = 1$

$y'(1) = 1$

Solution:

$x^2D^2 + xD + I\ y = 0$

$y(1) = 1$

$y'(1) = 1$

$= x^2D(Dy) + xDy + y$

$= \cancel{y}\ x^2 y'' + xy' + y$

Now

$x^2y'' + xy' + y = 0$

Out $y = x^m$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

Putting values.

$x^2 m(m-1)x^{m-2} + xmx^{m-1} + x^m = 0$

$x^2 m(m-1)x^m \cdot x^{-2} + x\,m\,x^m x^{-1} + x^m = 0$

Dropping common

$m(m-1) + m + 1 = 0$

$m^2 + m - m + 1 = 0$

$m^2 + 1 = 0$

$m^2 - i^2 = 0$

$(m-i)(m+i)=0$

$m_1 = i$ $\quad$ $m_2 = -i$

Using fact that

$$x = e^{\ln x}$$

$x^{m_1} = x^{i} = (e^{\ln x})^{i} = e^{i \ln x}$

$x^{m_2} = x^{-i} = (e^{\ln x})^{-i} = e^{-i \ln x}$

Recalling

$$e^{z} = e^{a+ib} = e^{a}(\cos b + i \sin b) \quad z \in \mathbb{C}$$

so we have

$e^{i \ln x} = e^{0}\{\cos(\ln x) + i \sin(\ln x)\}$

$= \cos(\ln x) + i \sin(\ln x)$

that gives

$x^{m_1} = \cos(\ln x) + i \sin(\ln x)$

$x^{m_2} = \cos(\ln x) - i \sin(\ln x)$

Adding Both

$x^{m_1} + x^{m_2} = \cos(\ln x) + i \sin(\ln x) + \cos(\ln x)$

$- i \sin(\ln$

$= 2\cos(\ln x)$

Divide by 2

$$\frac{x^{m_1}+x^{m_2}}{2} = \frac{\cancel{2}\cos(\ln x)}{\cancel{2}} = \cos(\ln x)$$

Subtract the second formula from the first and divide it by to $2i$ after that

$$x^{m_1}+x^{m_2} = \cos(\ln x) + i\sin(\ln x) - \cos(\ln x) + i\sin\ln x]$$
$$= 2i\sin(\ln x)$$

$\div\ 2i$ it becomes

$$\frac{x^{m_1, m_2}}{2i} = \sin(\ln x)$$

Now

$y_1 = \cos\ln x \quad \wedge \quad y_2 = \sin(\ln x)$

General Solution is

$y = c_1 y_1 + c_2 y_2 = (c_1\cos(\ln x) + c_2\sin(\ln x$

Using Chain Rule:

$$y' = -c_1\sin(\ln x)\cdot(\ln x)' + c_2\cos(\ln x)\cdot(\ln x)'$$
$$= -\frac{c_1}{x}\sin(\ln x) + \frac{c_2}{x}\cos(\ln x)$$

Value of C

$$\begin{cases} 1 = y(1) = c_1 \cos(\ln 1) + c_2 \sin(\ln 1) \\ 1 = y'(1) = -c_1 \sin(\ln 1) + 3 c_2 \cos(\ln 1) \end{cases}$$

$$\begin{cases} 1 = c_1 \cos(0) + c_2 \sin(0) \\ 1 = -c_1 \sin(0) + c_2 \cos(0) \end{cases}$$

$$\begin{cases} 1 = c_1 \\ 1 = c_2 \end{cases}$$

Answer:

$$y = \sin(\ln x) + \cos(\ln x)$$

Question No 18

$$(9x^2 D^2 + 3xD + I)y = 0$$

$y(1) = 1$

$y'(1) = 0$

Solution:

$$9x^2 D^2 y + 3x Dy + Iy = 9x^2 D(Dy) + 3x Dy + y$$

$$= 9x^2 y'' + 3xy' + y$$

Put $y = x^m$

$y = x^m$

$y' = m x^{m-1}$

$y'' = (m-1) m \, x^{m-2}$

Equation Becomes

$9x^2 m(m-1) x^{m-2} + 3x \, m x^{m-1} + x^m = 0$

$9\cancel{x^2} m(m-1) x^m \cancel{x^{-2}} + 3 \cancel{x} m \, x^m \cancel{x^{-1}} + x^m = 0$

$9m(m-1) + 3m + 1 = 0$

$9m^2 - 9m + 3m + 1 = 0$

$9m^2 - 6m + 1 = 0$

$m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$9m^2 - 6m + 1 = 0$

$m_{1/2} = \dfrac{6 \pm \sqrt{6^2 - 4 \cdot 9}}{18}$

$= \dfrac{6}{18}$

$m_{1/2} = \dfrac{1}{3}$

Now

$m = \dfrac{1}{3}$

Since

$y_1 = x^m = x^{1/3}$

Standard form:

$$y'' + \frac{1}{3x} \cdot y' + \frac{1}{9x^2} \cdot y = 0$$

We can see that

$$p(x) = \frac{1}{3} \cdot \frac{1}{x} \Rightarrow \int p\, dx = \frac{1}{3} \ln|x|$$

put

$y = u y_1$

$$u = \int U\, dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int p\, dx}$$

Finding U

$$e^{-\int p\, dx} = e^{-1/3 \ln|x|} = \left(e^{\ln|x|}\right)^{-1/3} = x^{-1/3}$$

$$U = x^{-1/3} \cdot \frac{1}{(x^{-1/3})^2} = x^{-1/3 - 2/3} = x^{-1} = \frac{1}{x}$$

By '∫'

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^{1/3} + x^{1/3} \ln x$

$= x^{1/3} (c_1 + c_2 \ln x)$

Product rule:

$y' = (x^{1/3})' (c_1 + c_2 \ln x) + x^{1/3} (c_1 + c_2 \ln x)'$

$= \frac{1}{3} \cdot x^{2/3} ((c_1 + c_2 \ln x) + x^{1/2} c_2 \cancel{x} \cdot \frac{1}{\cancel{x}}$

$= \frac{1}{3} \cdot x^{\frac{2}{3}} (c_1 + c_2 \ln x) + x^{1/3} c_2$

Now

$$\begin{cases} 1 = y(1) = 1^{1/3} (c_1 + c_2 \ln 1) \\ 0 = y'(1) = \frac{1}{3} \cdot 1^{-2/3} (c_1 + c_2 \ln 1) + 1^{1/3} c_2 \end{cases}$$

$$\begin{cases} 1 = c_1 \\ 0 = \frac{c_1}{3} + c_2 \end{cases}$$

$$\begin{cases} 1 = c_1 \\ c_2 = -\frac{1}{3} \end{cases}$$

Solution: $y = x^{1/3} (1 - \frac{1}{3} \ln x)$

Question 19

$(x^2D^2 - xD - 15I)y = 0$ $\quad y(1) = 0.1$

$y'(1) = -4.5$

Solution:

$x^2D^2y - xDy - 15Iy = x^2D(Dy) - xDy - 15y$

$= x^2y'' - xy' - 15y$

$\Rightarrow \quad x^2y'' - xy' - 15y = 0$

Put $y = x^m$

$y = x^m$

$y' = mx^{m-1}$

$y'' = (m-1)m\,x^{m-2}$

Now

$x^2m(m-1)x^{m-2} - xmx^{m-1} - 15x^m = 0$

$\cancel{x^2}\,m(m-1)x^m \cdot \cancel{x^{-2}} - \cancel{x}mx^m\cancel{x^{-1}} - 15x^m = 0$

Dropping common factor

$m^2 - 2m - 15 = 0$

$m_{1/2} = \dfrac{2 \pm \sqrt{(-2)^2 + 4\cdot 15}}{2}$

$m_{1/2} = \dfrac{2 \pm 8}{2}$

So it has distinct real roots.

$m_1 = 5 \quad \wedge \quad m_2 = -3$

Now

$y_1 = x^{m_1} = x^5$ & $y_2 = x^{m_2} = x^{-3}$

General solution:

$y = c_1 y_1 + c_2 y_2 = c_1 x^5 + c_2 x^{-3}$

$y' = 5c_1 x^4 - 3c_2 x^{-4}$

Finding c

$\begin{cases} 0.1 = y(1) = c_1 \cdot 1^5 + c_2 \cdot 1^{-3} \\ -4.5 = y'(1) = 5c_1 \cdot 1^4 - 3c_2 \cdot 1^{-4} \end{cases}$

$\begin{cases} 0.1 = c_1 + c_2 \\ -4.5 = 5c_1 - 5c_2 \end{cases}$

$\begin{cases} 0.1 - c_2 = c_1 \\ -4.5 = 5(0.1 - c_2) - 3c_2 \end{cases}$

$\begin{cases} 0.1 - c_2 = c_1 \\ 5 = 8c_2 \quad / : 8 \end{cases}$

$\begin{cases} 0.1 - c_2 = c_1 \\ 0.625 = c_2 \end{cases}$

$\begin{cases} 0.1 - 0.625 = c_1 \\ 0.625 = c_2 \end{cases}$

$\begin{cases} -0.525 = c_1 \\ 0.625 = c_2 \end{cases} \Rightarrow y = -0.525x^5 + 0.625x^{-3}$

Ans

Question 2

(i) (a) $x' = \sqrt{x}$

Solution:

$$x' = \sqrt{x}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{x}$$

$$\Rightarrow \frac{dx}{\sqrt{x}} = 1\,dt$$

$$\Rightarrow \frac{1}{\sqrt{x}}\,dx = dt$$

$\int$ on b/s

$$\int \frac{1}{\sqrt{x}}\,dx = \int dt$$

$$2\sqrt{x} + C_1 = t + C_2$$

$$2\sqrt{x} = t + c$$

Taking square

$$4(x) = (t+c)^2$$

$$x = \left(\frac{t+c}{4}\right)^2 \text{ Ans.}$$

$x' = e^{-2x}$

solution:

$$x' = e^{-2x}$$

$$\frac{dx}{dt} = e^{-2x}$$

$$\frac{dx}{e^{-2x}} = dt$$

$$\int \frac{1}{e^{-2x}}\, dx = \int dt$$

$$\frac{e^{2x}}{2} = t + c$$

$$e^{2x} = 2(t+c)$$

$$2x = \ln 2(t+c)$$

$$x = \frac{\ln 2(t+c)}{2}$$

Ans.

c $y' = 1+y^2$

Solution:

$$\frac{dy}{dt} = 1+y^2 \Rightarrow \frac{dy}{1+y^2} = dt$$

$$\int \frac{1}{1+y^2}\, dt = \int dt$$

$$y = \tan(t+c).$$

$$u' = \frac{1}{5-2u}$$

Solution:

$$\frac{dy}{dt} = \frac{1}{5-2u} \Rightarrow \frac{dy}{5-2u} = dt \quad (1)$$

$$\int \frac{dy}{5-2u} = \int dt$$

$$= -(u-5)u + c_1 = t + c_2$$

$$= -(u-5)u = t + c$$

$$= u(1) = \frac{t+c}{-(u-5)} \text{ Ans.}$$

e $x' = au + b, \; a, b > 0$

Solution

(?)

f $Q' = \frac{Q}{4 + 4Q^2}$

Solution:

$$\frac{dQ}{dt} = \frac{Q}{4 + Q^2}$$

$$\frac{Q^2 \, dQ}{Q} = dt$$

$$\int \frac{Q^2}{Q} = \int dt$$

$$3 \ln |Q| + \frac{Q^2}{2} + C$$

$\cancel{x}' = e^{\cancel{x}^2}$

Solution:

$$\frac{d\cancel{x}}{dt} = e^{x^2}$$

$$\frac{dx}{dt} = dt$$

$$\frac{dx}{e^{\cancel{2}x^2}} = dt$$

$$\int \frac{1}{e^{x^2}}\, du = \int dt$$

$y' = r(a-y).$

Solution:

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)\cancel{dt}}\, dy = \int dt$$

$$\frac{(a-y)u}{r} = t+c$$

$$= u(t) = \frac{r(t+c)}{a-y}$$ ~~Ans~~.

Q2 Solve $y' = r(a-y)$ where r and a are constants.

Solution:

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\, dy = \int dt$$

$$a-y = k(r)^{1/2}$$

Q3 (a)

Solution:

Put $x(0) = 1$

$$x(t) = \frac{(t+c)^2}{4}$$

$x(0) = 1$ $x(t) = x(0) = 1$

$$1 = \frac{(0+c)^2}{4}$$

$$\sqrt{c^2} = \sqrt{4}$$

$$c = 2$$

b Solution:

$$x(t) = \frac{\ln 2(t) + c}{2}$$

$x(0) = 1$

$1 = \ln 2(0) + c$

$1 = \ln + c$

$2 = \ln c^2$ Ans.

Question 4 (a)

$$x' = \frac{2x}{t+1}$$

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\frac{dx}{2x} = \frac{1}{t+1} dt$$

$$\int \frac{dx}{2x} = \int \frac{1}{t+1} dt$$

$$\frac{x^2}{4} = \ln(t+1) + c$$ Ans.

$\theta' = t\sqrt{t^2+1}\ \sec\theta$

solution:

$$\frac{d\theta}{dt} = t\sqrt{t^2+1}\ \sec\theta$$

$$\frac{d\theta}{\sec\theta} = t\,(t^2+1)\ dt.$$

$$\int \frac{d\theta}{\sec\theta} = \int t\sqrt{t^2+1}\ dt$$

$$= \sin\theta = \frac{(t^2+1)^{3/2}}{3} + c$$

Ans.