"Final Exam"

NAME:

MUHAMMAD ILYAS

ID :

7956

SECTION:

'B"

SEMESTER:

4th

SUBJECT :

STRUCTURAL ANALYSIS - I

INSTRUCTOR:

ENGR. Sir Amjad Islam

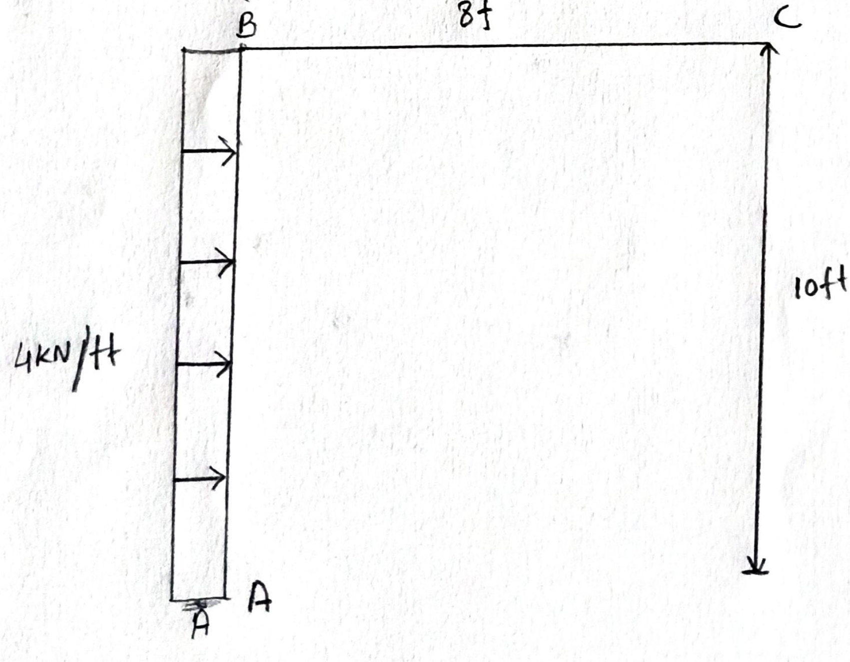
Q:No:01

Determine the vertical displacement of free end point C on the frame frame shown in figure. Take $E = 29(10^3)$ ksi and $I = 600 \text{ in}^4$ for both members. Use method of virtual work.

Given data:

$$E = 29 \times 10^3 \text{ ksi}$$

$$I = 600 \text{ in}^4$$



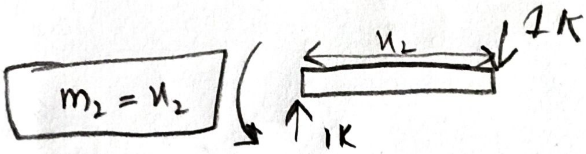
To find:

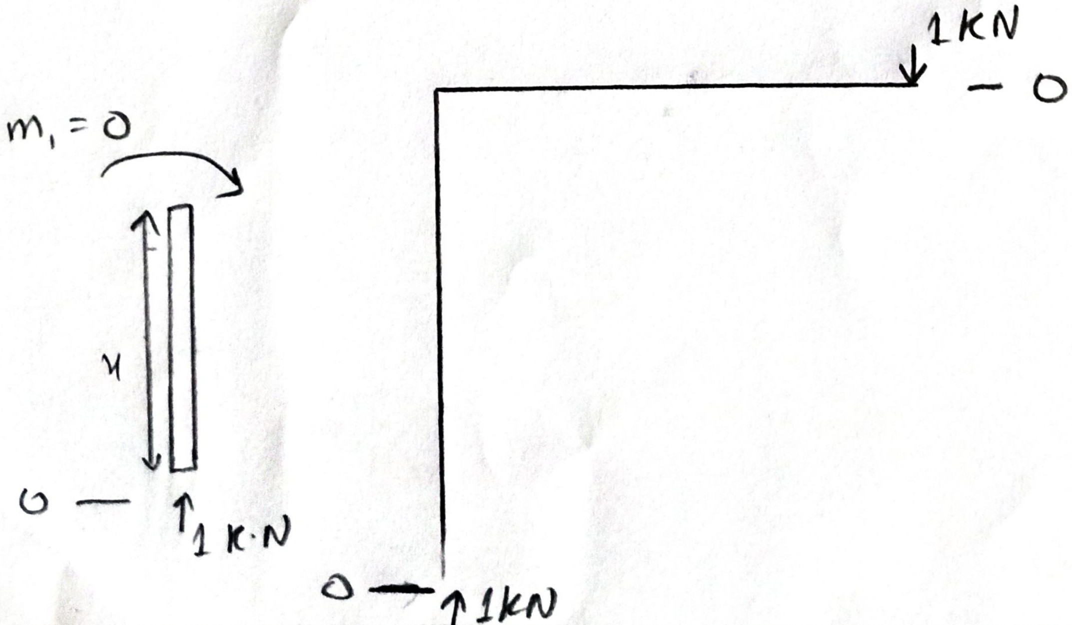
Vertical displacement of free end.

Solution :

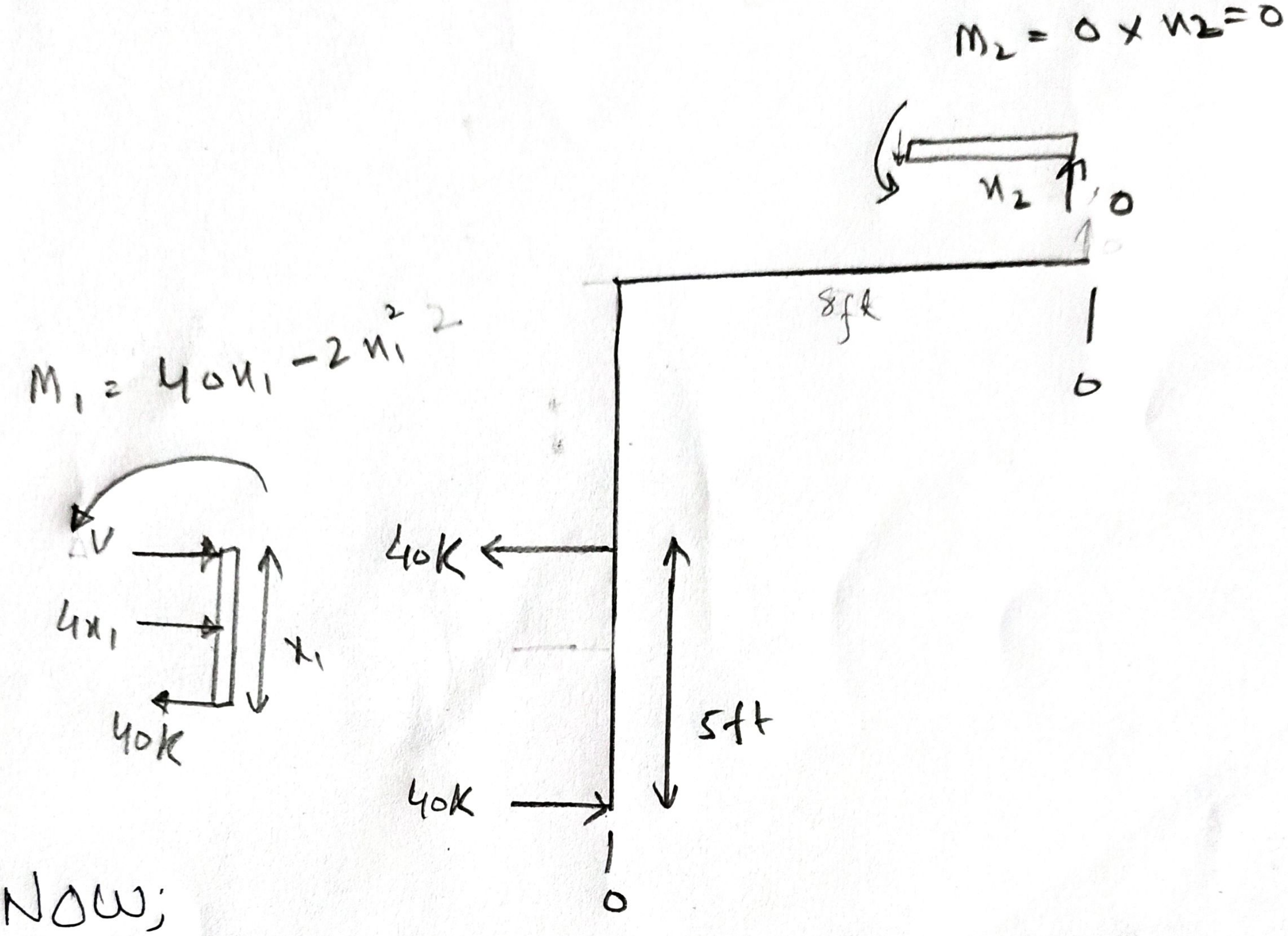
Virtual moment :

For convenience the co-ordinates x_1 and x_2 as shown in fig. will be used. A vertical unit load is applied at C.





Real moments:



NOW;

Using virtual work equation.

$$\Delta_c = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_c = \int_0^{10} \frac{m_1 M_1\,dx_1}{EI} + \int_0^{8} \frac{m_2 M_2}{EI}\,dx_2$$

$$= \int_0^{10} \left(\frac{0x \times 40x_1 - 2x_1^2}{EI} \right) dx_1 +$$

$$\int_0^{8} \left(\frac{1 x_2 \times (0)}{EI} \right) dx_2$$

$$\Delta_c = 0 + 0$$

$$\boxed{\Delta_c = 0}$$

→ As the "C" is free ended and no support lies there and there is no vertical load applying on frame structure. Hence no verticle displacement occured at point "C".

Q:NO:02

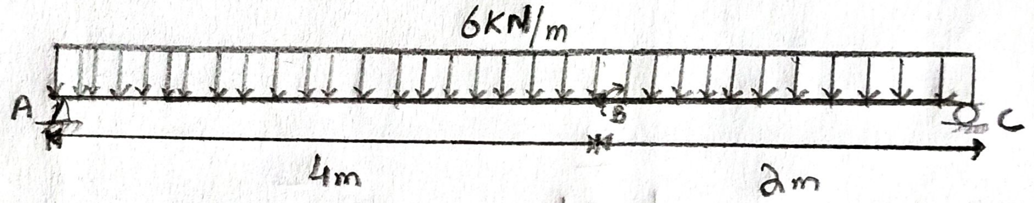
Determine the slop and displacement at point B. Assume the support at A is a pin and C is a roller. Take $E = 200$ GPa, $I = 60(10^6)mm^4$

Use Castigliano's theorm.

Given data:

$E = 200$ GPa

$I = 60 \times 10^6\ mm^4$



Requird/ Determine:

θ at point B=?

Δ at point B=?

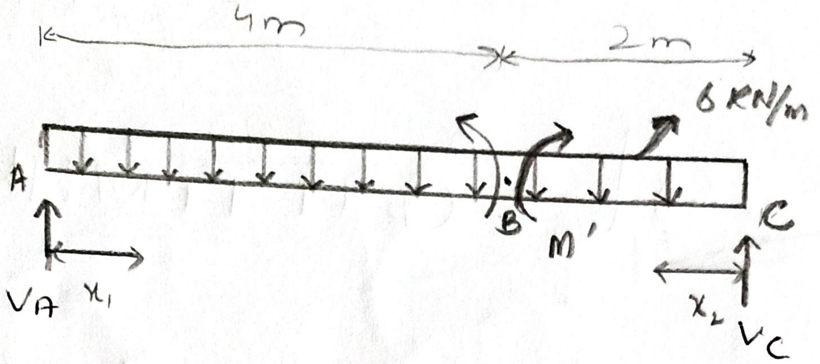
Solution:

Slop θ:

External couple moment M':

Since the slop at point b is to be determind, an external couple M' is placed on the beam at this point as shown in the fig.

Internal moments:



$V_A = 18 + 0.1667 M'$, $V_C = 18 - 0.1667 M'$

As $\curvearrowright + M_C = 0$

$$V_A \times 6 - (6 \times 6) \times 3 - M' = 0$$

$$V_A = \frac{108 + M'}{6}$$

$$V_A = 18 + 0.1667 M'$$

Similarly; $V_C = 18 - 0.1667 M'$

<u>Now</u> <u>for</u> <u>x_1</u>

$+\circlearrowleft \Sigma M = 0$

$$M_1 = (18 + 0.1667 M') x_1 - 3 x_1^2$$

$$\frac{DM_1}{DM'} = 0.1667 x_1$$

Now for x_2;

$\curvearrowright \Sigma M = 0$

$$M_2 = (18 - 0.1667 M') x_2 - 3 x_2^2$$

$$\frac{dM_2}{dM'} = -0.1667 x_2$$

For slop;

Using Castigliano's equation

$$\theta_B = \int_0^L M \frac{\partial M}{\partial M'} \frac{dx}{EI}$$

$$\theta_B = \int_0^4 \frac{(18x_1 - 3x_1^2) \times 0.1667 x_1 \, dx_1}{EI} +$$

$$\int_0^2 \frac{(18x_2 - 3x_2^2) \times -0.1667 x_2 \, dx_2}{EI}$$

$$= \int_0^4 \frac{(3x^2 - 0.5x^3)}{EI}\, dx_1 + \int_0^2 \frac{(-3x^2 + 0.5x^3)}{EI}\, dx_2$$

$$= \left| \frac{\frac{3x^3}{3} - \frac{0.5x^4}{4}}{EI} \right|_0^4 + \left| \frac{\frac{-3x^3}{3} + \frac{0.5x^4}{4}}{EI} \right|_0^2$$

$$= \frac{32}{EI} \quad \frac{-6}{EI}$$

$$= \frac{26}{EI} \text{ kN-m}^2$$

$$\theta_B = \frac{26}{200 \times 10^6 \times \frac{60 \times 10^6}{1000^4}}$$

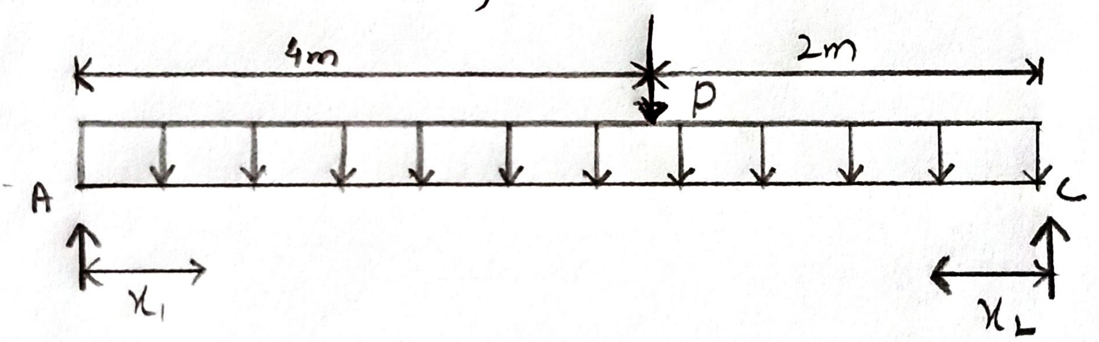
$$\boxed{\theta_B = 0.00216 \text{ rad}}$$

For displacement :

External force:

A vertical force is applied/ placed on the beam at B as shown in fig.

Internal moment;



$18+0.333P$ $18+0.666P$

For x_1;

$\circlearrowleft + \Sigma M_1 = 0$

$$M_1 = (18+0.333P) \times x_1 - 3x_1^2$$

$$\boxed{\frac{\partial M}{\partial P} = 0.33x_1}$$ → eq (i)

For x_2

$\circlearrowright + \sum M = 0$

$$M_2 = (18 + 0.6667P)\,x_2 - 3x_2^2$$

Now

$$\frac{dM_2}{dp} = 0.6667\,P x_2 \rightarrow eq^{(ii)}$$

Put $P = 0$ in eq (i) and (ii)

$$M_1 = 18x_1 - 3x_1^2 \;\; kN\text{-}m^2$$

$$M_2 = 18x_2 - 3x_2^2 \;\; kN\text{-}m^2$$

Now for displacement

Using Castigliano's equation

$$\Delta_B = \int_0^L M \left(\frac{\partial M}{\partial P}\right) \frac{dx}{EI}$$

$$\Delta_B = \int_0^4 \frac{18x_1 - 3x_1^2 \times 0.33x_1 \, dx_1}{EI} +$$

$$\int_0^2 \frac{(18x_2 + 3x_2^2) \times 0.667x_2}{EI}$$

$$\Delta_B = \int_0^4 \frac{5.94x_1^2 - x^3}{EI} dx_1 +$$

$$\int_0^2 \frac{12x_2^2 - 2x_2^3}{EI} dx_2$$

$$\Delta_B = \left| \frac{\frac{5.94x^3}{3} - \frac{x^4}{4}}{EI} \right|_0^4 + \left| \frac{\frac{12x^3}{3} - \frac{2x^4}{4}}{EI} \right|_0^2$$

$$\Delta_B = \frac{62.72 + 24}{EI}$$

$$\Delta_B = \frac{86.72}{EI} \text{ KN-m3}$$

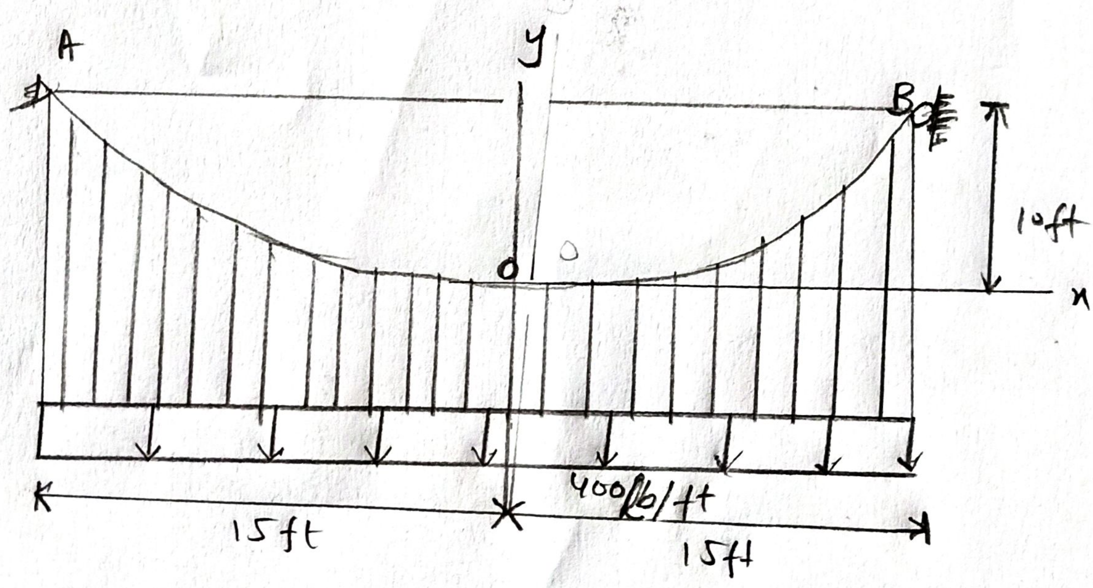
$$= \frac{86.72}{200\times10^{6}\times\frac{60\times10^{6}}{10^{12}}}$$

$\boxed{\Delta_B = \cancel{0}\,0.0072\,m}$ or $\boxed{\Delta_B = 7.22\times10^{-3}\,m}$

Q: No: 03

The cable is subjected to the uniform loading. If the slop of the cable at point O is zero. Determine the equation of curve and the force in the cable at O and B.

Given:



Solution:

As we know that

$$y = \frac{h}{L^2} x^2$$

$$y = \frac{16}{15^2} x^2$$

$$y = \cancel{0.0356 x^2} \quad 0.0444 x^2$$

We also know that

$$T_0 = F_H = \frac{w_0 L}{2h} = \frac{400 \times 15^2}{2 \times 16}$$

$$F_H = T_0 = 4500 \text{ lb} \Rightarrow 4.5k$$

We also know that

1st method:

$$T_B = T_{max} = \sqrt{F_H^2 + (w \cdot L)^2}$$

$$T_B = (4500)^2 + (400 \times 15)^2$$

$$T_B = \sqrt{56250000}$$

$T_B = 7500\ lb$

or

$\boxed{T_B = 7.500k}$

2nd Method

$T_B = T_{max} = \omega \cdot L \sqrt{1 + \left(L/2h\right)^2}$

$= (400 \times 15) \times \sqrt{\left(1 + \frac{15}{2 \times 10}\right)^2}$

$T_B = 7500\ lb$

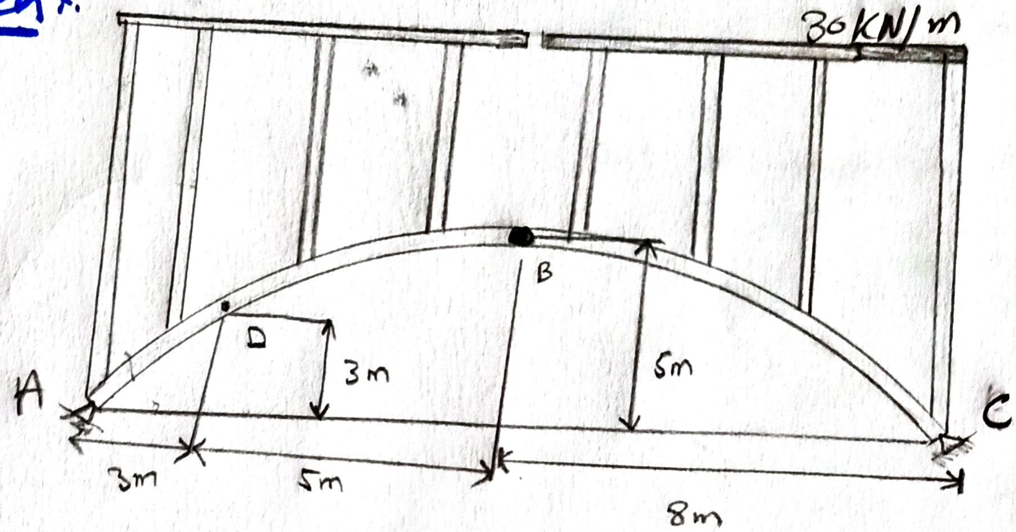
or

$\boxed{T_B = 7.5k}$

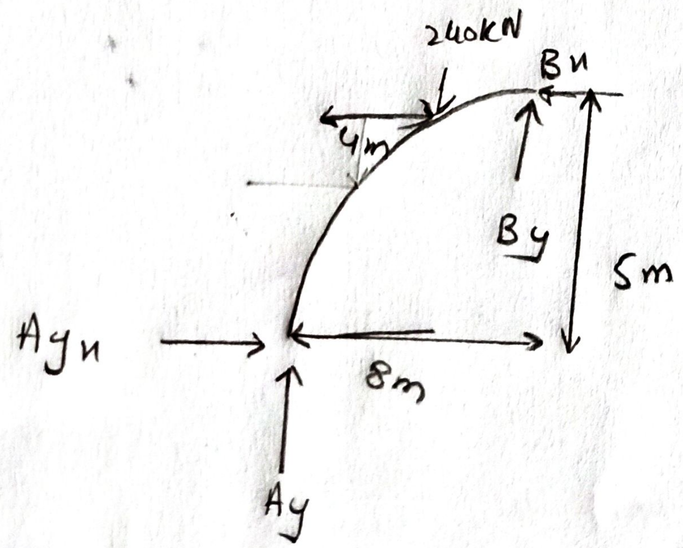
Q:No:04

The three-hinged spanded arch is subjected to the uniform load of 30kN/m. Determine the internal moment in the arch at point D.

Given:-



Solution:

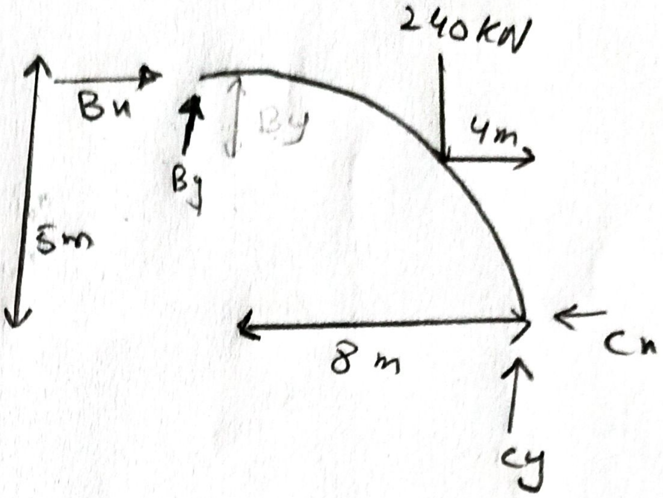


$+ \circlearrowright \Sigma M_A = 0$

$$Bx \times 5 + By \times 8 - (30 \times 8 \times 8/2)$$

$$5bx + 8by = 960 \rightarrow \text{eq(i)}$$

ii) Member BC :-



$\circlearrowright + \Sigma M_c = 0$

$-Bx \times 5 + By \times 8 + 30 \times 8 \times 8/2 = 0$

$-5Bx + 8by = -960$ eq (ii)

Subtracting eq (ii) from eq (i)

$5Bx + 8By = 960$

$\underline{\overset{-}{+}5Bx \pm \cancel{8By} = \overset{-}{+}960}$

$10 Bx = 1920$

$\boxed{Bx = 192 \text{ kN}}$

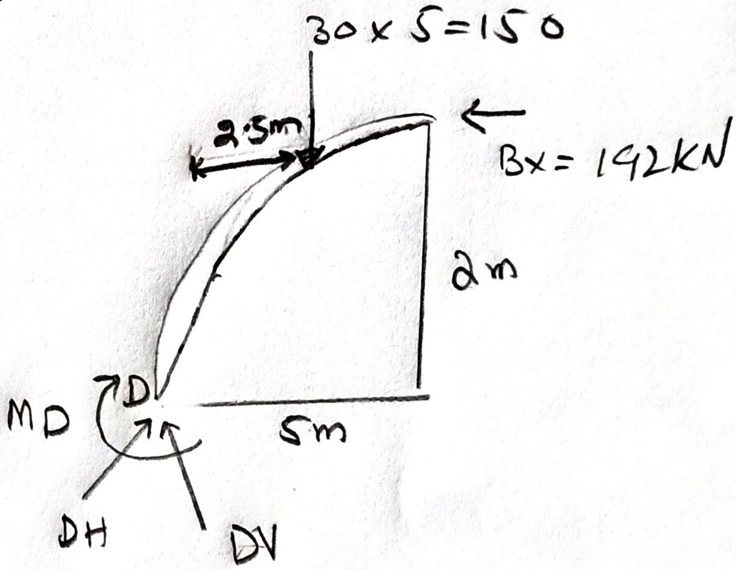
From eq(i)

$$5(192) + 8B_y = 960$$

$$8B_y = 960 - 960$$

$$\boxed{B_y = 0\,KN}$$

Segment DB:-



$$\circlearrowleft + \Sigma M_D = 0$$

$$-M_D + 192 \times 2 - (30 \times 5) \times 2.5 = 0$$

$$\boxed{M_D = +9\,KN\text{-}m}$$