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7229

In Daily life:
These are use in the basic laws of physics for example newton laws of linear motion, and Einstein equation in general relativity.

→ As a civil engineering mostly commercial with building structure and Geometrical shape so any work revolved around modeling structure, fluids, and more can be modeled using differential. If you have any complicated geometrices, which most realistic problem you will likely to use the said differential equation.

→ Mechanical Engineering:-
There are different order of partial deriatio describing the rate of changes representing real physical qualities.

→ The use of the sperration of variables techn-ique to solve partial differential eq relating to real conduction in solids and vibration of solids in multidimensional system.

→ Economic Field:-
In field of economics we use partial diff equation to check what happen to other variables while keeping one variable const.

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Application of Partial differential equations-

Many engineering problems are governed by different types of partial differential eqn and some of the more important types are given below.

- Laplace equation:-
- Poisons equation
- Helmholtz equation
- wave equation
- Seperat differential equation.

For equation which can be expressed in seperable form as show below, the solution can be obtained easily as,

$$\frac{dy}{dx} = f(x,y) \quad \frac{dy}{x(y)} = f(x)\,dx \quad \int \frac{dy}{x(y)}$$

$$= \int f(x)\,dx + c$$

then $\int M(x)\,dx = -\int N(y)\,dy + c$

separable diff equation:-
The equation which can be expressed as seper-able form is shown below.

The solution can be obtained easly as

$$\frac{dy}{dx} = x^3 + (y^2+1) \Rightarrow \frac{dy}{y^2+1} = x^3 dx$$

$$\int \frac{dy}{y^2+1} = \int x^3 dx + c \Rightarrow \tan^{-1} y = \frac{1}{4} x^4 + c$$

$$\Rightarrow y \ \tan(\frac{1}{4} x^4 + c).$$

Based of the boundary condition $c = 3$ hence $y^2 - 2y$

$$y^2 - 2y = x^3 + 2x^2 + 2x\ 3.$$

This quanratic eq in y^2 can be solved with two solution by the quadratic eq as

$y = 1 / \sqrt{x^3 + 2x^2 + 2x + y}$ and $y = 1 + \sqrt{x^3 + 2x^2 + 2x + y}$

variation of paramoter:-

For the following eq form it is possible to solve it by variation of paramoter.

$$\frac{dy}{dx} = P(x)\, y + Q(x)$$

put $y = c(x) e^{\int p(x)\, dx}$ by differentiating it can be given

$$\frac{dy}{dx} = \frac{d(x)}{dx} = \int P(x)\, dx + c(x) P(x) e^{\int p(x)} \phi \over P(x)\, y$$

substitu it to the original ODE.

$$\frac{d(x)}{dx} = Q(x) = \int P(x)\, dx \text{ comparing}$$

the term it given

$$c(x) = \int Q(x) e^{\int P(x)\, dx}\, dx + C$$

The Bernoulli's Eq^n:-

The Bernoulli eq is an imp equation type which can be solved in a simple way by variation of parameters.

Consider the following form of eq.

$$\frac{dy}{dx} = P(x)\,y + Q(x)\,y^n$$

step 1:- put $z = y^{1-n}$

step 2:- $\frac{dz}{dx} = (1-n)\,y^{-n}\,\frac{dx}{dx}$

$$\frac{dz}{dx} = (1-n)\,P(x)\,z + (1-n)\,Q(x)$$

The non-linear ODE's now becomes linear ODE. It can be solved by formula.

Step 3:-

$n = -1, z = y^{?}$ inverting z to get y

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}.$$

$$\frac{dz}{dx} = \frac{1}{x} z + x^2$$

$$z = e^{\int \frac{1}{x} dx} \left(\int x^2 e^{-\int \frac{1}{x} dx} dx + c \right)$$

$$= cx + \frac{1}{2} x^3.$$

Back substitution of $z = y^2$ of $z = y$

$$y^2 = cx + \frac{1}{2} x^3$$

Homogenous Equations:-

For equation of the following type, where all the coefficient are constt, it can be evaluated according to different.

Laplace Equations:-
It can form an important governing condition for many problems. some of the more common ~~an~~ forms are given by.

Three dimension laplace equation

$$u_{xx} + u_{yy} + u_{zz} = 0.$$

Two dimensional heat conduction:-

$$\alpha^2 (u_{xx} + u_{yy}) u_t$$

Two dimensional seepage problems

$$(k_x u_{xx} + k_y u_{yy}) = 0.$$

There are two major types of Boundary condition to this problem.

Direct Problem:-

Boundary condition prescrible as ϕ it.