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Q NO 1

Part 1:

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\frac{\partial w^3}{\partial t^2} = \frac{c^2 \partial w}{\partial x^2}$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + (-\sin(2x+2ct)) + 2c$$

$$\frac{\partial w^2}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+ct) + 4c^2$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct$$

$$= (-\sin(x+ct) - 4(\cos(x+2ct))$$

Q NO 1
Part b:

$$w = \sin(x+ct) + \cos(2x+2ct)$$

Given:

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} \quad \text{— (1)}$$

Now $\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct)+\cos(2x+2ct)\right]$

$$= \frac{\partial}{\partial t}\left(\sin(x+ct)\right) + \frac{\partial}{\partial t}\left(\cos(2x+2ct)\right)$$

$$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

Now $\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right]$

$$\Rightarrow \frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)$$

Now, $\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct)+\cos(2x+2ct)\right]$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

(1) $-c^2\sin(x+ct) - 4c^2\cos(2x+2ct) =$
$c^2\left[\sin(x-ct) - 4\cos(x+t)\right]$

$-c^2\cancel{\sin}(x+ct) - 4c^2\cos(2x+2ct) = -c\sin(2t + ct) - 4c^2\cos x(x+t)$

$\boxed{0=0}$ Satisfied.

(ii) $w = \tan(2x + ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

&

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$$

$$2c^2 \sec^2(2x+ct)\tan(2x+ct)$$

Now

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$$

$$\Rightarrow 4c^2 \cancel{\sec^2}(2x+ct)\tan(2x+cl) = 4c^2\cancel{\sec^2}(2x+ct)\tan(2x+ct$$

$\boxed{0=0}$ Satisfied

Q NO 2:

Given function is

$$f(x)\begin{cases} x; & -\pi < x \le 0 \\ 2x: & 0 \le x \le \pi \end{cases}$$

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2} \longrightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (x\cos nx\,dx) + \frac{1}{\pi}\int_0^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1 - (-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

$$\text{So, } a_n = \begin{cases} \frac{-2}{\pi n^2} & : \text{ if } n \text{ is odd} \\ 0 & : \text{ if } n \text{ is even} \end{cases} \longrightarrow (2)$$

(6)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int^{\pi}$$

$$\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

(3) $$b_n = \frac{1}{\pi}\left[\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{\pi\cos n\pi}{n}\right] = \frac{3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

$$F(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

Q3

$y' - 4y'' + 13y = 8\sin 3x \quad y(0) = 1$ and $y(0) = 2$

Solution:

$= y' - 4y'' + 13y = 0$

Change ② into Auxiliary equation

Put $y = m$ in ②

$m^2 - 4m + 13 = 0$

Use Quadratic formula

$a = 1, \quad b = -4, \quad c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{\cdot 3i}}{2}$$

$$= \frac{4 + 6i}{2}$$

$$= 2 \pm 3i$$

$m_1 = 2 + 3i$

$m_2 = 2 - 3i$

$y_c = e^2(c_1 \cos 3x + c_2 \sin 3x) \rightarrow$ Ⓐ

let

$y_p = A\cos 3x + B\sin 3x$

Diff w.r.t 'x'

$y_p' = -3A\sin 3x + 3B\cos 3x$

Diff w.r.t 'x'

$y_p'' = -9A\cos 3x + 9B\sin 3x$

Put in (1)

$(-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x)$
$+ 13(A\cos 3x + B\sin 3x) - B\sin 3x$

$-9A\cos 3x - 12B\cos 3x + 13A\cos 3x$
$-9B\sin 3x + 12A\sin 3x + 13B\sin 3x$
$- 8\sin 3x.$

$= (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

comparing coefficients.

$\sin 3x \Rightarrow \quad 4B + 12A = 8 \text{ — (a)}$

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow 4A = 12B$

$A = 3B \text{ — (b)}$

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$B = 1/5$

$\Rightarrow A = 3/5$

$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$

$y = y_c + y_p$

$y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (C)$

Now we need to find the values ~~of the c~~

of c_1 & c_2 for this

Put $x=0$ & $y=1$ in (C)

$1 = e^{2(0)}(c_1\cos 3(0) + c_2\sin 3(0)) + \frac{3}{5}(0)$

$1 = c_1 + 3/5$

$c_1 = 1 - 3/5$

$c_1 = 2/5$

Diff (C) w.r.t 'x'

$c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$
$- 6/5 \sin 3x + 3/5 \cos 3x \rightarrow (D)$

Put $y' = 2$, $x = 0$ in (D)

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$
$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$
$- 6/5 \sin 3x + 3/5 \cos 3x$

Put $y = 2$, $x = 0$

$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0))$
$+ c_2(2e^{2(0)}\sin(0) + 3e^{2(0)}\cos(0))$
$- 6/5 \sin 3(0) + 3/5 \cos 3(0)$

$2 = c_1(2) + c_2(3) - 0 + 3/5$

$2 = 4/5 + 3c_2 + 3/5$

$2 = 7/5 + 3c_2$

$3c_2 = 3/5$

$y = e^{2x}(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$

Required solution

Q NO 4

Sol:

It is already in symbolic form

$(D^2 - DD')z = \cos x \cos 2y$ —— (a)

Put A.E $D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m$ i-e $D = m, D' = 1$

$m^2 - m = 0$

$m = 0, 1$

Therefore C.f $= f(y) + f_2(y-x)$
from eq (a)

$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$

$= \frac{1}{2} \frac{1}{D^2 - DD'} 2\cos x \cos 2y$

As

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$C.f = f_1(y-x) + x f_2(y-x)$

$P.I = \frac{1}{D^2 + 2DD' + D'} [2(y-x) + \sin(x-y)]$

$= \frac{1}{(D+D')^2} (2(y-x) + \sin(x-y))$

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By General Method

$m = -1, \; y = x = c$

$$\frac{1}{D+D'}\left[2c + \sin(-c)\right] dx$$

$$\frac{1}{D+D'}\left[2cx - (\sin c)x\right]$$

Replacing c by y-x

Again put $y-x = c$

$$\int (2xc - x\sin c)\, dx \Rightarrow cx^2 - \frac{x^2}{2}\sin c$$

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}\sin(x-y)$$

Hence the Required solution is

$$z = C.f + P.I = f_1(y-x) + x f_2(y-x) + x^2 y - x^3 + \tfrac{1}{2}x^2\sin(x-y)$$