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Quiz : Differential Equation

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Question :

A yarn merchant sells brands A, B, C of yarn of each of which is a blend of Pakistani, Egyptian and American cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one kilogram of A, B, C costs 40, 50, 60 rupees. Find the cost of kilogram of each country.

Sol:-

$$x = A \quad y = B \quad z = C$$

let x, y & z be the cost/kg of Pakistani

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Egyptian and American cotton respectively.
Then according to the given condition

$$\frac{1}{4}x + \frac{1}{4}y + \frac{1}{4}z = 40 \quad \text{--- (1)}$$

$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50 \quad \text{--- (2)}$$

$$\frac{2}{4}x + \frac{2}{4}z = 60 \quad \text{--- (3)}$$

multiply '4' on both sides of eqn.

$$\therefore x + 2y + z = 160$$

$$2x + y + z = 200$$

$$2x + 2z = 240$$

using the above eqn in matrix form

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 160 \\ 200 \\ 240 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad b = \begin{pmatrix} 160 \\ 200 \\ 240 \end{pmatrix}$$

$$\boxed{AX = b}$$

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first let's find $|A|$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= 1(1-0) - 2(2-1) + 1(2-1)$$

$$= 1 - 2 - 1 = -2$$

So

$$\boxed{|A| = -2}$$

Now finding $|A_x|$, $|A_y|$ and $|A_z|$

$$|A_x| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$$

$$= 160(1-0) - 2(200-120) + 1(0-120)$$

$$= 160 - 160 - 120$$

$$\boxed{|A_x| = -120}$$

$$|A_y| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$$

$$|A_y| = 1(200-120) - 160(2-1) + 1(240-200)$$

4

$$80 - 160 + 40$$

$$\boxed{|A_y| = -40}$$

$$|A_z| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix}$$

$$= 1(120 - 0) - 2(240 - 200) + 160(0 - 1)$$

$$= 120 - 80 + (-160)$$

$$\boxed{|A_z| = -120}$$

According to Cramer's Rule

$$x = \frac{|A_x|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_y|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_z|}{|A|} = \frac{-120}{-2} = 60$$

c

⑤

OR

$$A, B, C = (60, 20, 60)$$

It means that

Pakistan blend cost/kg of cotton = 60

Egyptian " " " = 20

American " " " " = 60

