Final Exam Summer

Course Name: Operation Research

Submitted By:

Abdul Razzaq (12938)

BS (SE-8) Section: A

Submitted To:

Sir Saifullah Jan

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Department of Computer Science,
IQRA National University, Peshawar Pakistan

IQRA NATIONAL UNIVERSITY

Computer Science Department

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Instructor: Saifullah Jan

Q 1: Using **simplex** method, solve the following linear programming problem.

$$5x_1 + 4x_2 + 3x_3 = 8$$

$$2x_1 + 7x_2 + 5x_3 = 5$$

$$4x_1 + 4x_2 + 2x_3 = 4.$$

Answer:

Q 1) Solution :-

*) Step 1. Select the variable farthest to the right, x_3.

*) Step 2. Multiply the last equation by 1/2 :

$$\left|\begin{array}{l} 5x_1 + 4x_2 + 3x_3 = 8 \\ 2x_1 + 7x_2 + 5x_3 = -5 \\ 2x_1 + 2x_2 + x_3 = 2 \end{array}\right.$$

*) Step 3. Eliminate x_3 from all equation above it. Multiply the third equation by five (5) and Subtract from 2^{nd}.

$$\left|\begin{array}{l} 5x_1 + 4x_2 + 3x_3 = 8 \\ -8x_1 - 3x_2 = -5 \\ 2x_1 + 2x_2 + x_3 = 2 \end{array}\right.$$

*) Mutiply the 3^{rd} equation by three and Substract from the first;

$$\left|\begin{array}{l} -x_1 - 2x_1 = 2 \\ -8x_1 - 3x_2 = -5 \\ 2x_1 + 2x_2 + x_3 = 2 \end{array}\right.$$

*) Step 4. Temporarily strike out the Last equation and the last variable.

$$\begin{cases} -x_1 - 2x_2 = 2 \\ -8x_1 - 3x_2 = -5 \\ 2x_1 + 2x_2 + x_3 = 2 \end{cases}$$

Now proceed by Steps 1-3 to deal with the smaller system. × the second equation by 1/3

$$\begin{cases} -x_1 - 2x_2 = 2 \\ +8/3x_1 + x_2 = 5/3 \\ 2x_1 + 2x_2 + x_3 = 2 \end{cases}$$

Multiply the second equation by 2, and add to the first

$$\begin{cases} 13/3x_1 = 16/3 \\ 8/3x_1 + x_2 = 5/3 \\ 2x_1 + 2x_2 + x_3 = 2 \end{cases}$$

*) Step 5. Strike out the second equation

$$\begin{cases} 13/3x_1 = 16/3 \\ 8/3x_1 + x_2 = 5/3 \\ 2x_1 + 2x_2 + x_3 = 2 \end{cases}$$

Multiply the first equation by 3/13

$$\begin{cases} x_1 = (16/13) \\ (8/3x_1) + x_2 = (5/3) \\ 2x_1 + 2x_2 + x_3 = 2 \end{cases}$$

Step 6. Eliminate all coefficients below the main diagonal. Mutiply the first equation by 8/3 and Subtract from 2nd eq.

$$\left|\begin{array}{ll} x_1 & = (16/13) \\ x_2 & = (-21/13) \\ 2x_1 + 2x_2 + x_3 & = 2 \end{array}\right.$$

Multiply the first eq by 2, Subtract from third.

$$\left|\begin{array}{ll} x_1 & = (16/13) \\ x_2 & = (-21/13) \\ 2x_2 + x_3 & = (-6/13) \end{array}\right.$$

multiply the Second eq 2, Subtract from third.

$$\left|\begin{array}{ll} x_1 & = (16/13) \\ x_2 & = (-21/13) \\ x_3 & = (36/13) \end{array}\right.$$

Solution is complete.

Q 2: Use **Vogel's approximation** method, to solve the following.

Origin	**Destination**			**Supply**
	1	2	3	
1	50	100	100	110
2	200	300	200	160
3	100	200	300	150
Demand	140	200	80	

Answer:

Q2 : ANSWER :-

If VAM is applied to this problem one obtains the accompanying vogel numbers

Row
1) $100(C_{12}) - 50(C_{11}) = 50$
2) $200(C_{23}) - 200(C_{21}) = 0$
3) $200(C_{32}) - 100(C_{31}) = 100$

Column
1) $100(C_{31}) - (50)(C_{11}) = 50$
2) $200(C_{32}) - 100(C_{12}) = 100$
3) $200(C_{23}) - 100(C_{13}) = 100$

Since three vogel numbers are tied for the maximum of 100, arbitrarily select the first in the list and assign 140 trucks to x_{31}. Note that subscript "31" is based on the lowest cost in Row 3, or C_{31}. The value of "140" is determined using the equation.

$X_{ij} = \min(S_i, d_j)$; where S_i; supply from source i and d_j; demand from destination J, that is minimum number of 150 and 140.

In the next pass, column 1 is ignored and the vogel number of the column 2 and 3 remain unchanged.

Row :

1 : $100(C_{13}) - 100(C_{12}) = 0$

2 : $300(C_{22}) - 200(C_{23}) = 100$

3 : $300(C_{33}) - 200(C_{32}) = 100$

Column :

2) $= 100$

3) $= 100$

Arbitrarly selecting the first 100 in the List (Row 2) results in an assignment of 80 trucks from origin 2 to destination 3 ($X_{23} = 80$).
Since all but one of the column constraints have now been satisfied, all the remaing trucks at origins 1, 2, 3 must be assigned to destination and the initial solution is;

$x_{12} = 110$

$x_{22} = 80$

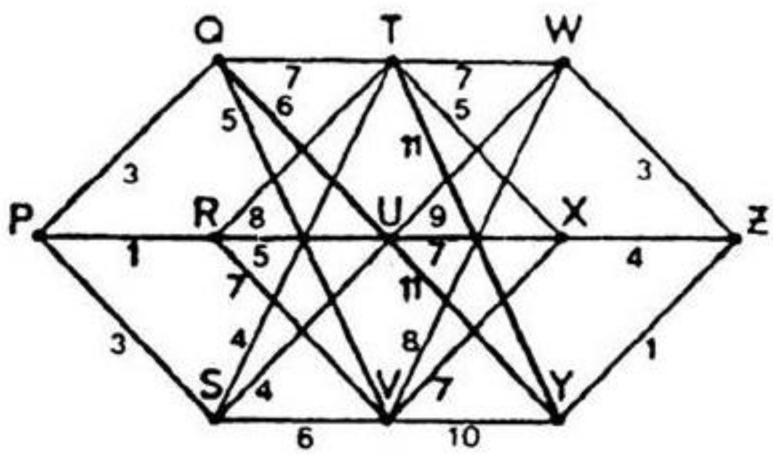
$x_{23} = 80$

$x_{31} = 140$

$x_{32} = 10$

$Z = \$67,000$

Q 3: For the figure given below, use dynamic programming approach to find out the shortest possible path?



Answer:

Starting from P, consider the problem in stages:

First: Q, R or S?

Second: T, U or V?

Third: W, X or Y?

Final: Best route to Z?

First stage: It is not yet known whether the quickest route lies through Q, but if it does the quickest route from P to Q is obviously PQ. Similar statements about getting to R and to S are equally obvious. Thus P to Q = 3 and P to R = 1 and P to S = 3 by the shortest and only rout

Second stage: It is not yet known whether the quickest route lies through T, but if it does, would it have gone through Q, R or S? Now the route PQT takes 3 + 7 = 10, PRT takes 1 + 8 = 9 and PST takes 3 + 4 = 7. Therefore the quickest way to T is through S, though T is completely inaccessible, once the overall quickest route is known.

It is not yet known whether the quickest route lies through U rather than T. But if it does, would it have gone through Q, R or S? Now the route PQU takes 3 + 6 = 9, PRU takes 1 + 5 = 6 and PSU takes 3 + 4 = 7.

Therefore the quickest way to U is through R, though U is completely inaccessible, once the overall quickest route is known. To finish the second stage, it is not yet known whether the quickest route lies through V rather than T or U. But if it does, would it have gone through Q, R or S? Now the route PQV takes 3 + 5 = 8, PRV takes

1 + 7 = 8 also, and PSV takes 3 + 6 = 9. Therefore the quickest way to V is through either Q or R, though V is still inaccessible, once the overall quickest route is known.

Thus P to T = 7 and P to U = 6 and P to V = 8 by the quickest route

Third stage: It is not yet known whether the quickest route lies through W, but if it does, would it have gone through T, U or V? Now the route PTW takes 7 + 7 = 14 if T is reached by the quickest route; PUW takes 6 + 9 = 15 if U is reached by the quickest route; PVW takes 8 + 8 = 16 if V is reached by the quickest route. Therefore the quickest way to W is through T, provided that T itself was reached in the quickest way (through S). W, T or S are completely inaccessible, once the overall quickest route is known.

It is not yet known whether the best route lies through X rather than W. But if it does, would it have gone through T, U or V? Now the route PTX takes 7 + 5 = 12 if T is reached by the quickest route; PUX takes 6 + 7 = 13 if U is reached by the quickest route; PVX takes 8 + 7 = 15 if V is reached by the quickest route. Therefore the quickest way to X is through T, provided that T itself was reached in the quickest way (through). X, T or S are still not accessible once the overall quickest route is known.

To finish the third stage, it is not yet known whether the best route lies through Y rather than W or X. But if it does, would it have gone through T, U or V? Now the route PTY takes 7 + 11 = 18 if T is reached by the quickest route; PUY takes 6 + 11 = 17 if U is reached by the quickest route; PVY takes 8 + 10 = 18 if V is reached the quickest route. Therefore the quickest way to Y is through U, provided that U itself was reached in the quickest way (through R). Y, U, or R are still not accessible once the overall quickest route is known.

Thus

P to W = 14 and P to X = 12 and P to Y = 17 by the quickest route.

Final stage: The quickest route from P to Z can now be calculated, P to W by the quickest route and on to Z takes 14 + 3 = 17; P to X by the quickest route and on to Z takes 12 + 4 = 16; P to Y by the quickest route and on to Z takes 17 + 1 = 18. Therefore the quickest route from P to Z takes 16.

Trace the steps back through the network and determine which intermediate cities lie on the quickest route from P to Z. The final stage of the calculation indicates that X does. This implies (from the third stage) that T also does, since the quickest route from P to X is through T. This in turn implies (from the second stage) that S also is on the quickest route from P to Z, since the quickest route from P to T is through S.

The quickest route, therefore, runs from P through S, T and X to Z, taking a total time of 16. This is also the quickest route from Z to P.

Q 4: A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.

The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximize the combined sum of the units of X and the units of Y in stock at the end of the week.

Formulate the problem of deciding how much of each product to make in the current week as a linear program.

10 Marks

Q4) ANSWER :-

let

* x be the number of units of X produced in the current week
* y be the number of units of Y produced in the current week

→ Constraints are

→ $50x + 24y <= (60)$ machine A time

→ $30x + 33y <= (60)$ machine B time

→ $x => 75 - 30$

i.e

$x => 45$ so production of $X >= $ demand (75) - initial stock (30), which ensure we meet demands

→ $y >= 95 - 90$

i.e

$y >= 5$ so production of $Y >= $ demand (95) - intial stock (90), which ensure we meet demand

The objetive is maximise $(x + 30 - 75) + (y + 90 - 95)$

$= (x + y - 50)$

i.e

The maximise number of units left in stock at the end of the week

Q 5: The ICARE Company has three plants located throughout a state with production capacity 50, 75 and 25 gallons. Each day the firm must furnish its four retail shops R1, R2, R3, & R4 with at least 20, 20, 50, and 60 gallons respectively. The transportation costs (in Rs.) are given below.

Company	Retail				Supply
	R1	R2	R3	R4	
P1	3	5	7	6	50
P2	2	5	8	2	75
P3	3	6	9	2	25
Demand	20	20	50	60	

The economic problem is to distribute the available product to different retail shops in such a way so that the total transportation cost is minimum?

Q5) Solution :-

Starting from the north west corner, we allocate min (50, 20) to P_1R_1, i.e., 20 units to cell P_1R_1. The demand for the first column is satisfied. The allocation is shown in the following table.

Company	Retail R1	R2	R3	R4	Supply
P1	3 (20)	5 (20)	7 (10)	6	50
P2	2	5	8	2 (35)	75
P3	3	6	9	2 (25)	25
Demand	20	20	50	60	

Now we move horizontally to the second column in the first row and allocate 20 units to cell P_1R_2. The demand of second column is also satisfied. proceeding in this way, we observe that $P_1R_2 = 10$, $P_2R_3 = 40$, $P_2R_4 = 35$, $P_3R_4 = 25$. the resulting feasible solution is shown in the following table

Here, number of retail shops (n) = 4, and Number of plants (m) = 3. Number of variables = $m+n-1$
$= 3+4-1 \Rightarrow 6$.

→ The Initial basic feasible Solution is $x_{11} = 20, x_{12} = 5$, $x_{13} = 20$, $x_{23} = 40$, $x_{24} = 25$ and minimum cost of transportation $= 20\times3 + 20\times5 + 10\times7 + 40\times8 + 35\times2 + 25\times2$
$= 670$