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Section = A

Department = Civil Engineering

~~Sto~~ subject = Applied Calculus

Q1

Given the
Coordinate.

$$P = (4, 1, 3)$$

$$Q = (1, 2, 4)$$

Sol:

We know that

$$OP = 4i + 1j + 3k$$

$$OQ = 1i + 2j + 4k$$

using distance formula.

$$|PQ| = \sqrt{(z_2 - z_1)^2 + (y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$|PQ| = \sqrt{(4-3)^2 + (2-1)^2 + (1-4)^2}$$

$$PQ = \sqrt{1^2 + 1^2 + (-3)^2}$$

$$PQ = \sqrt{1+1+9}$$

$$PQ = \sqrt{11} \quad \text{--- (1) Ans (1)}$$

M the point which divided PQ
in ratio 1:3 then by ratio theorem

position $m = OM$

$$M = \overline{OM}$$
$$= \frac{3(4i + 1j + 3k) + 1(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$\boxed{= \frac{13i + 5j + 13k}{4}}$$

Soludiu (2)

p-2

Q#2

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}$$

Solution

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}$$

$$= \int \frac{2(2x^3 + 5x + 2)}{x(2x+1)} dx$$

$$= 2 \int \frac{2x^3 + 5x + 2}{x(2x+1)} dx$$

perform polynomial long division

$$\int \left(\frac{11x+4}{2x(2x+1)} + \frac{2x+1}{2} \right) dx$$

$$= \frac{1}{2} \int \frac{11x+4}{x(2x+1)} dx + \int x dx - \frac{1}{2} \int 1 dx$$

now solving

$$\int \frac{11x+4}{x(2x+1)} dx$$

$$\int \frac{2(2x^3 + 5x + 2) \ln x}{x(2x+1)} dx$$

$$= \frac{-3 \ln(2x+1) + 4 \ln(x) + x^2 - x + C}{2}$$

$$= \frac{-3 \ln(2x+1) + 4 \ln(x) + (x-1)x + C}{2}$$

$$= \ln(x)$$

plus in solved integral.

$$= 3 \int \frac{1}{2x+1} dx + 4 \int \frac{1}{x} dx$$

$$= \frac{3 \ln(2x+1)}{2} + 4 \ln(x)$$

now

$$\int x dx$$

Apply power Rule.

$$= \int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{with } n=1$$

$$= \frac{x^2}{2}$$

$$= \int 1 dx$$

$$= x$$

plus solved integral.

$$= \frac{1}{2} \int \frac{11x+4}{x(2x+1)} dx + \int x dx - \frac{1}{2} \int 1 dx$$

$$= \frac{3 \ln(2x+1)}{4} + 2 \ln(x) + \frac{x^2}{2} - \frac{x}{2}$$

$$= 2 \int \frac{2x^3 + 5x + 2}{x(2x+1)} dx$$

$$= \frac{3 \ln(2x+1)}{2} + 4 \ln(x) + x^2 - x$$

p-5

by partial fraction

$$= \int \left(\frac{3}{2x+1} + \frac{4}{x} \right) dx$$

$$= 3 \int \frac{1}{2x+1} dx + 4 \int \frac{1}{x} dx$$

$$= \int \frac{1}{2x+1} dx$$

Substitute \Rightarrow $u = 2x+1 - \frac{du}{dx}$

$$du = \frac{1}{2} dx$$

$$= \frac{1}{2} \int \frac{1}{2} dx$$

now solving

$$\int \frac{1}{u} du$$

$$= \ln(u) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{\ln(u)}{2}$$

now $u = 2x+1$

$$= \frac{\ln(2x+1)}{2}$$

solving by x

$$\int \frac{1}{x} dx$$

p.6

Q.1 part 1 (1)

$$\int_0^2 x^2 e^x dx$$

using integration by part:

Solution

$$\begin{aligned}\int_0^2 x^2 e^x dx &= x^2 \int e^x dx - \left(\frac{dx}{dx} \int e^x dx \right) \Big|_0^2 \\ &= x^2 e^x - 2x e^x \Big|_0^2 \\ &= x^2 e^x \Big|_0^2 - 2x e^x \Big|_0^2\end{aligned}$$

put the value

$$= (2)^2 (2.71)^2 - 2(2)(2.71)^2$$

$$= (4)(7.34) - 1 - (4 \times 7.34) - 1$$

$$= e^0 = 1$$

$$= 29/36 - 1 - 29/36 - 1$$

$$\boxed{= -2} \text{ Ans}$$

p. 7

Q3

part II

$$\int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

Sol

$$\text{Let } \sqrt{x} = y \quad \text{As } x = 2$$

$$x = y^2$$

$$\text{Then } y = \sqrt{x}$$

$$x = 1$$

Diff:

$$y = 1$$

$$1 = 2y \Rightarrow dx = 2y dy$$

$$= \int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx = \int_1^{\sqrt{2}} \frac{\sin y}{y} \cdot 2y dy$$

$$= 2 \int_1^{\sqrt{2}} \sin y dy = -2 (\cos y) \Big|_1^{\sqrt{2}}$$

$$= -2 (\cos \sqrt{2} - \cos 1)$$

$$= -2 (0.9996 - 0.99984)$$

$$= -2 (0.00024) =$$

$$= 1.0008 \text{ Ans}$$

p-8

$$u(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$$

Sol

Solved by Laplace three dimension.

$$u(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$$

Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{--- (A)}$$

$$= u(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} (2x)$$

$$\frac{\partial u}{\partial x} = -x (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 u}{\partial x^2} = - \left[x \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}} (2x) + (x^2 + y^2 + z^2)^{-\frac{3}{2}} \right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2 (x^2 + y^2 + z^2)^{-\frac{5}{2}} - (x^2 + y^2 + z^2)^{-\frac{3}{2}} \quad \text{--- (B)}$$

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$$\text{Now } \frac{du}{dy} = ?$$

$$\frac{dy}{dy} = \frac{1}{2} (x^2 + y^2 + z^2)^{\frac{3}{2}} \cdot (2y) \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} = -y (x^2 + y^2 + z^2)^{\frac{3}{2}} \quad \text{--- (2)}$$

$$\frac{d^2y}{dx^2} = - \left\{ y \left(\frac{3}{2} \right) (x^2 + y^2 + z^2)^{-\frac{5}{2}} (2y) + (x^2 + y^2 + z^2)^{\frac{3}{2}} \right\} \quad \text{--- (2)}$$

$$\frac{d^2y}{dx^2} = 3y^2 (x^2 + y^2 + z^2)^{-\frac{5}{2}} - (x^2 + y^2 + z^2)^{\frac{3}{2}} \quad \text{--- (2)}$$

$$\frac{dy}{dz} = \frac{1}{2} (x^2 + y^2 + z^2)^{\frac{3}{2}} \cdot (2z) \quad \text{--- (3)}$$

$$\frac{dy}{dz} = -z (x^2 + y^2 + z^2)^{\frac{3}{2}} \quad \text{--- (4)}$$

$$\frac{d^2y}{dz^2} = 3z^2 (x^2 + y^2 + z^2)^{-\frac{5}{2}} - (x^2 + y^2 + z^2)^{\frac{3}{2}} \quad \text{--- (4)}$$

putting eq 1, 2 & 3 in eq (A)

(PTD)

P 40

$$\Rightarrow 3x^2(x^2+y^2+z^2)^{-\frac{3}{2}} - (x^2+y^2+z^2)^{\frac{3}{2}} + 3y^2(x^2+y^2+z^2)^{-\frac{3}{2}} + (x^2+y^2+z^2)^{\frac{3}{2}}$$

$$+ 3z^2(x^2+y^2+z^2)^{-\frac{3}{2}} + (x^2+y^2+z^2)^{\frac{3}{2}}$$

$$= (x^2+y^2+z^2)^{-\frac{3}{2}} \left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2) \right]$$

$$= (x^2+y^2+z^2)^{-\frac{3}{2}} \left[3x^2 - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2 \right]$$

$$= (x^2+y^2+z^2)^{-\frac{3}{2}} (0) = 0$$

= 0

So the given $u(x,y,z)$ is Laplace equation solution

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