NAME # SHAHKAR SALEEM

ID # 7943

SECTION # "B"

SUBJECT # CALCULUS

SEMESTER # 4th

DATE # 25- Sep- 2020.

Q1:- Find PQ where P is the point in three- dimensional space with Coordinates (4, 1, 3) and the point Q with Coordinates (1, 2, 4). Find the distance b/w P & Q Futher, find the position vector of The point dividing PQ in the ratio 1 : 3

Sol Coordinate of P = (4, 1, 3)

$$OP = 4i + 1j + 3K$$

$$\text{or} \quad OQ = \vec{OQ} - \vec{OP}$$

$$= (i + 2j + 4K) - (4i + 1j + 3K)$$

$$= -3i + 1j + 1K \quad \text{— (1)}$$

Now distance between P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \quad \text{— (2)}.$$

Let M be the point which divided PQ in ratio 1:3 Then by the ratio theorm position vector of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3K) + (1)(i + 2j + 4K)}{1+3}$$

$$= \frac{12i + 3j + 9K + i + 2j + 4K}{4}$$

$$= \frac{13i + 5j + 13K}{4} \longrightarrow (3)$$

Hence eq 1, 2, and 3 are required Solution.

Q2:-

Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Sol

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

$$\begin{array}{r|l} & \quad 2x-1 \\ \hline 2x^2+x & \cancel{4x^3}+10x+4 \\ & \underline{\pm\cancel{4x^3} \qquad \pm 2x^2} \\ & -\cancel{2x^2}+10x+4 \\ & \underline{\mp \cancel{2x^2} \mp x} \\ & 11x+4 \end{array}$$

So $\quad 2x-1+\dfrac{11x+4}{2x^2+x} = \dfrac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int \frac{11x+4}{2x^2+x} \longrightarrow (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+2}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow (2)$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow (A)$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1)+Bx \longrightarrow (3)$$

put $x = 0$ in (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in (3)

$$11(-1/2)+4 = B(-1/2)$$

$$-\frac{11}{2}+4 = -B/2$$

$$-\frac{11+8}{\cancel{2}} = -B/\cancel{2}$$

$$-3 = -B \Rightarrow \boxed{B = 3}$$

putting the value of A and B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$4\ln|x| + 3/2 \ln|2x+1|$$

putting these values in (2)

$$= x^2 - x + 4\ln|x| + 3/2 \ln(2x+1)$$

Now put these values in (1)

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\,dx = x^2 - x + 4\ln|x| + 3/2 \ln|2x+1| + C.$$

Q3 (A):- $\int_0^2 x^2 e^x dx$.

Solution $\int_0^2 x^2 e^x dx$

Now first find integration

$$= \int x^2 e^x dx$$

$$= x^2 \int e^x dx - \int \left(\left(\int e^x dx \frac{d}{dx} x^2\right) \right) dx$$

$$= x^2 e^x - \int e^x (2x) dx$$

$$= x^2 e^x - 2\int x e^x dx$$

$$= x^2 e^x - 2\left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} \right) dx\right]$$

$$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

Now put limits

$$= \left| x^2e^x - 2xe^x + 2e^x \right|_0^2$$

$$= \left(2^2e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$$

$$= \left(\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2\right)$$

$$= 2e^2 - 2 \quad \text{ANSWER.}$$

Q3 (B):- $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx.$

Sol $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx.$

First find integration.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \longrightarrow (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2dy = \frac{1}{\sqrt{x}}\, dx} \quad \text{put in (1)}$$

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

put Limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2\left(\cos\sqrt{2} - \cos 1\right)$$

$$= -2\cos\sqrt{2} + 2\cos(1)$$ ANSWER.

Q(4):- Verify that

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three-dimensional Laplace's equation.

Sol: The laplace equation in 3-d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad — \quad (A)$$

Scanned with CamScanner

so $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x\,(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/x)(x^2+y^2+z^2)^{-5/2}(2x)+(x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \quad -(1)$$

<u>Now</u>

$$\frac{\partial u}{\partial y} = -1/\cancel{2}\,(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y\,(x^2+y^2+z^2)^{-3/2}$$

$$\partial^2 u/\partial y^2 = -\left[y(-3/\cancel{2})(x^2+y^2+z^2)^{-5/2}\,(\cancel{2}y)+(x^2+y^2+z^2)^{-3/2}\right.$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2 (x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \rightarrow ②$$

$$\frac{\partial u}{\partial z} = -1/\cancel{2} (x^2+y^2+z^2)^{-3/2} (\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z \ (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} - ③$$

putting ① ② and ③ in Ⓐ

$$3x^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}.$$

$$= (x^2+y^2+z^2)^{-5/2} \left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2 \right]$$

$$(x^2+y^2+z^2)^{-5/2} \left[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - z^2 + 3y^2 - x^2 - \cancel{y^2} - z^2 + 3z^2 - \cancel{x^2} - \cancel{y^2} - z^2 \right]$$

$$= (x^2+y^2+z^2)^{-5/2} \ (0) = 0$$

So the given $u(x,y,z)$ is solution of laplace equation.