Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

Name:	RAFI UDDIN	**Student ID:**	12401

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{2, \underset{\uparrow}{\tfrac{1}{}}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{\tfrac{3}{}}, 1, 2, 1, 4\right\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

QNO1

a) Consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

i) Determine the minimum sampling rate required to avoid aliasing.

ii) Suppose the signal is sampled at the rate $F_s = 100Hz$ what is the discrete time signal obtain after sampling? Also explain the effects of this sampling rate on the newly generated discrete time signal

iii) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Sol:- As we know that

$$f_s \geq 2 f_{max} \qquad f = \frac{\omega}{2\pi}$$

So $f_1 = \frac{100\pi}{2\pi}$

$f_1 = 50 Hz$

$f_2 = \frac{200\pi}{2\pi}$

$f_2 = 100 Hz$

So f_2 is greater then f_1

$f_s \geq 2 \times 100Hz$ is sampled frequncy ~~then~~ to avoid aliasing.

ii) $F_s = 100 Hz$

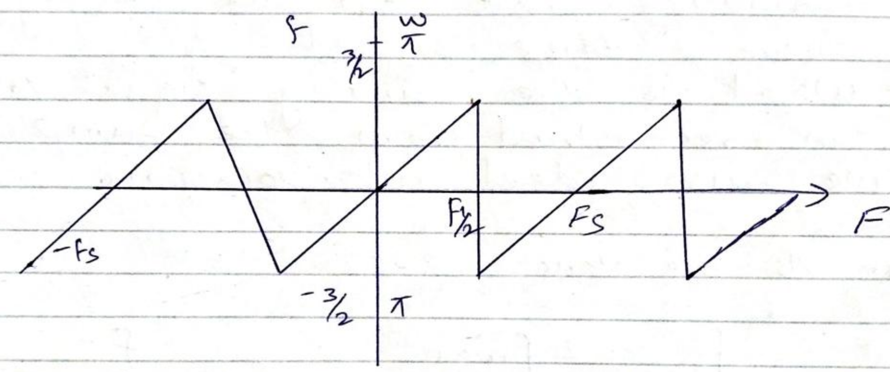
$$f = \frac{100}{2} = 50 Hz$$

This is the maximum frequeny that can be represented unequaly by the sampled signal

As

$$x_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$x_a[n] = 3\cos\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



The effect of sampling rate on the newly generated discrete time signal is that there will ~~be~~ not present unwanted component in Reconstruction of the signal.

iii) As we know that the folding frequeny

$$= \frac{f_1}{2} = \frac{100}{2}$$

$$= 50 Hz$$

$f_1 = 50 Hz$, $f_2 = 100 Hz$

• Both The frequencies are either equal are greater than the folding frequency.

Hence so ideal inter polatation we can construct the orignal signal.

$$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t.$$

Since only the frequny componet that at 100Hz are present on the sampled signal the analog we can remove ~~are~~ or reconstruct

$$y_a(t) = 3\cos 100\pi t \quad \text{Ans.}$$

QNo1

b) Consider a discrete time signal which is givm by.

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This signal is sampled at the Rate

$F_s = 2Hz$

i) Draw the sampled signal

ii) The samples of the signals are intended to carry 3 bits per sample Determing the quantization resolution to quantized the sampled signal achieved in part i.

iii) Perform the process of trunction and Rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data Express the Answer in tabular form.

Ans)

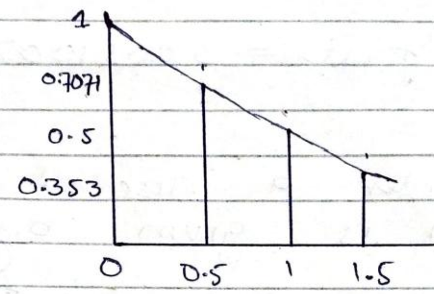
1) The sampled signal

$$f_s = \frac{1}{T}$$

$$T = 1/f_s$$

$$= 1/2 = 0.5 \text{ sec.}$$

x_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.303



ii) $L = 2n$

$n = \text{bits} = 3$

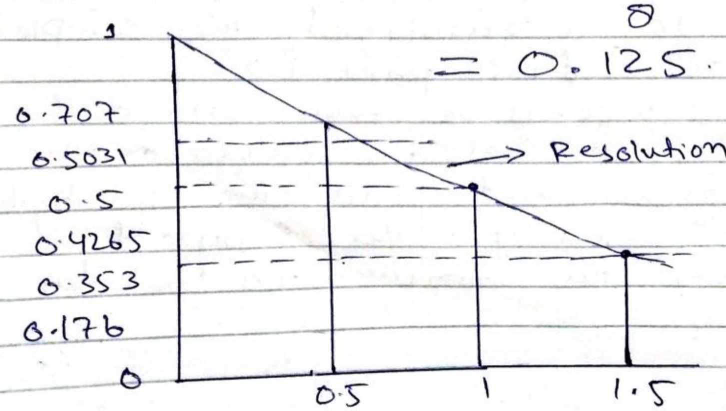
$L = 2^n$

$L = 2^3 = 8$ levels

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1-0}{8}$$

$$= 0.125.$$



iii)

	Discrete signal	trundion	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.1	-0.1

QNo2

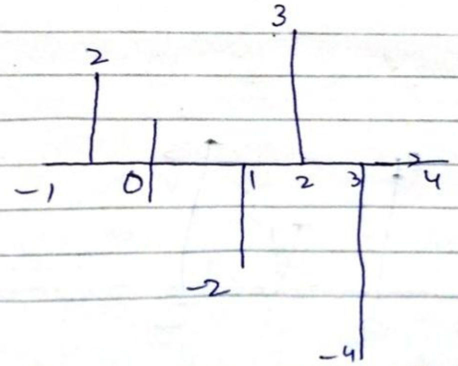
a) Determine the response of the system to the following input signal with givn imple response

$$x[n] = \{2, \underset{\uparrow}{1}\ -2, 3, -4\}$$

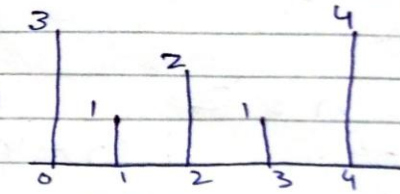
$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution:-

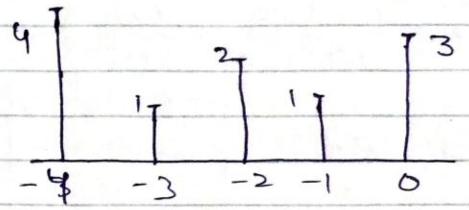
$$y[n] = \sum_{-\infty}^{\infty} x[K]h(n-K)$$



h(K)



$h[-k]$ = folded signal



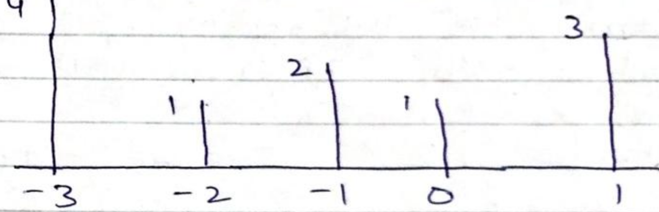
$$X[0] = \sum_{k=1}^{0} x[-1]\, h(-1] + 1(0)\, h(0)$$

$= 2 \times 1 + 1(3)$

$= 5$

for $n = 1$

$h(1-k]$



$$Y[1) = \sum_{k=1}^{1} x[h)\, [h(1-k)]$$

$= x(-2)\, h(-1) + x(0)\, h(0) +$

$x(1)\, h(1)$

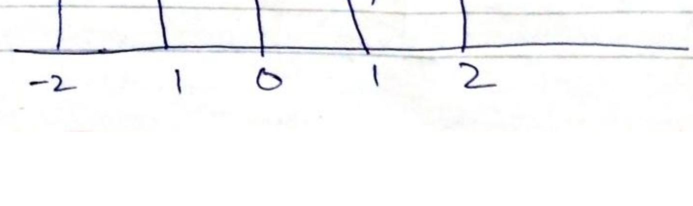
$= 2 \times 2 + 1 \times 1 + 3 \times (-2)$

$= 4 + 1 - 6$

$= -1$

$n = 2$

$h(2-k)$



$Y(2) = \sum_{k=1}^{1} x(n)\ h(2-k)$

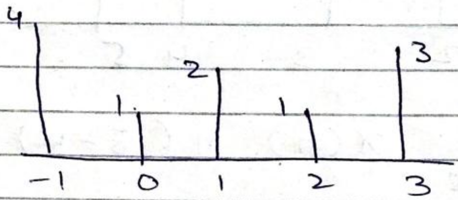
$x(-3)\ h(-1) + 2(0) + h(0) + x(1)\ h(1)$

$= 2\times 1 + 1\times 2 + (-2)(1) + 3\times 3$

$= 2+2-2+9$

$= 11$

$n = 3$



$n[3-k]$

$Y[3] = \sum_{k=2}^{3} x(n)\ n(3-k)$

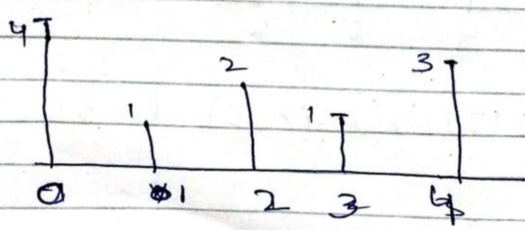
$= x(-1)\,h(-1) + x(0)\ h(0) + x(1)\ h(1) + x(2)\,h(2)$

$= 2\times 4 + 1\times 1 + (-2)(2) + 3\times 1 + (-4)(3)$

$= 4+1-4+3-12$

$= -8$

$n = 4$



$n(4-k]$

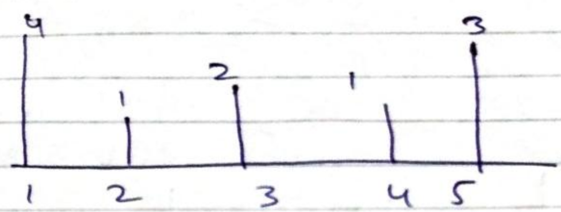
for 4 $\sum_{k=0}^{4} x(n)\ h(4-k)$

$= x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2) + x(3)\,h(3)$

$= 4 - 2 + 6 - 4$

$= 4$

For $n = 5$



$y(5) = \sum_{k=5}^{5} x(h)\; n(5-k)$

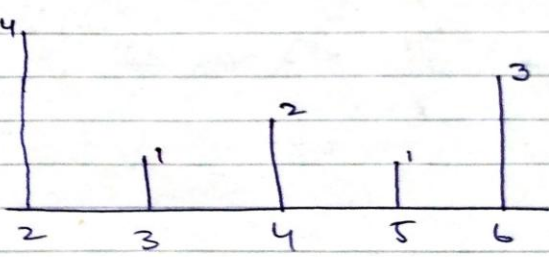
$= x(1)\,h(1) + x(2)\,h(2) + x(3)\,h(3)$

$= (-2)(4) + 3(1) + (-4)(2)$

$= -8 + 3 - 8$

$= 13$

$n = 6$



$y(6) = \sum_{k=6}^{6} x(2)\,h(2) + x(3)\,h(3)$

$= (3)(4) + (1)(-4)$

$= 8$

Q No 2

b) Compute the convolution $y\{n\}$ of the following signal

$$x(n) = \begin{cases} a^{n+1}, & -3 \le n \le 5 \\ 0, & \text{else where} \end{cases}$$

$$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0, & \text{else where} \end{cases}$$

Solution:

We have

$$x(n) = x(k) = \{a^{-2}, a^{-1}, 1, \underset{\uparrow}{\alpha}, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, \text{----}\} \longrightarrow (1)$$

2^n exults Bits.

$$h(n) = h(k) = \{0, \underset{\uparrow}{1}, 2\ 4, 8, 16, 0 \text{---}\}$$

To Find

$y(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

For $n=0$ first to find $h(n-k) = h(0-k)$

So by inverting $h(k)$ we get $h(-k)$

$$\Rightarrow h(-k) = \{16, 8, 4, 2, \underset{\uparrow}{1}\} \longrightarrow (2)$$

$$\text{So } y(0) = \sum_{k=-8}^{\infty} x(k)\ \cancel{x}\ h(-k)$$

$\cancel{y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times \alpha}$

$y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times 4) + (1 \times 2) + (\alpha \times 1)$

$y(0) = 8\alpha^{-2} + \alpha^{-1} + \alpha + 2$

for $n=1$ $h(1-k) = \{16, 8, 4, \underset{\uparrow}{2}, 1\}$

So $y(1) = (\alpha^{-2} \times 16) + (\alpha^{-1} \times 8) + (1 \times 4) + \alpha \times 2) + (\alpha^{2} \times 1)$

$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^{2}$

for $n = 2$

$h(2-k) = \{16 \quad 8 \quad \underset{\uparrow}{4} \quad 2 \quad 1\}$

$y(2) = (\alpha^{-1} \times 16) + (1 \times 8) + (\alpha \times 4) + (\alpha^{2} \times 2) + (\alpha^{3} \times 1)$

$= 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$

for $n = 3$

$h(3-k) = \{16 \quad \underset{\uparrow}{8}, 4, 2, 1\}$

$y(3) = (1 \times 16) + (\alpha \times 8) + (\alpha^{2} \times 4) + (\alpha^{3} \times 2) + (\alpha^{2} \times 1)$

$= 16 + 8\alpha + 4\alpha^{2} + 2\alpha^{3} + \alpha^{4}$

Now for $n=4$

$h(4-k) = \{\underset{\uparrow}{16}, 8, 4, 2, 1\}$

$= (\alpha \times 16) + (\alpha^2 \times 8) + (\alpha^3 \times 4) + (\alpha^4 \times 2) + (\alpha^5 \times 1)$

$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$

$n = 5$

$h(5-k) = \{\underset{\uparrow}{0}, 16, 8, 4, 2, 1\}$

$y(5) = (\alpha \times 0) + (\alpha^2 \times 16) + (\alpha^3 \times 8) + (\alpha^4 \times 4) + (\alpha^5 \times 2) + (\alpha^6 \times 1)$

$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$

Similarly if we calculate for rest of the value of n uptill there are any comman values we get.

$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

$y(7) = 0 + 0 + 0 + 16\alpha^5 + 8\alpha^6$

$y(8) = 0 + 0 + 0 + 0 + 16\alpha^5 + 8\alpha^6$

$y(9) = 0 + 0 + 0 + 0 + 0 + 16\alpha^6$

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QNo3:- Determine the z-transform of the following signals and also sketch its region of convergence (ROC)

i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n & , n \geq 0 \\ \left(\frac{1}{3}\right)^{-3} & n < 0 \end{cases}$

ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

Solution:-

As we know that the z transform

$$X(z) = \sum_{n=0}^{\infty} \left(\tfrac{1}{4}\right)^n z + \sum_{n=-\infty}^{0} \left(\tfrac{1}{3}\right)^n z^n - 1$$

Using geomatric series

$$= \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{1}{3}} - 1$$

$$= \frac{1 - \frac{1}{4} z^{-1} + 1 - \frac{1}{3} - 1}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z^{-3}\right)^{-1}}$$

$$= \frac{1 - \frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - \left(1 - \frac{1}{4}z^{-2}\right)\left(1 - \frac{1}{3}z\right)}{\left(1 - \frac{1}{4}z^{-3}\right)\quad\left(1 - \frac{1}{3}z\right)}$$

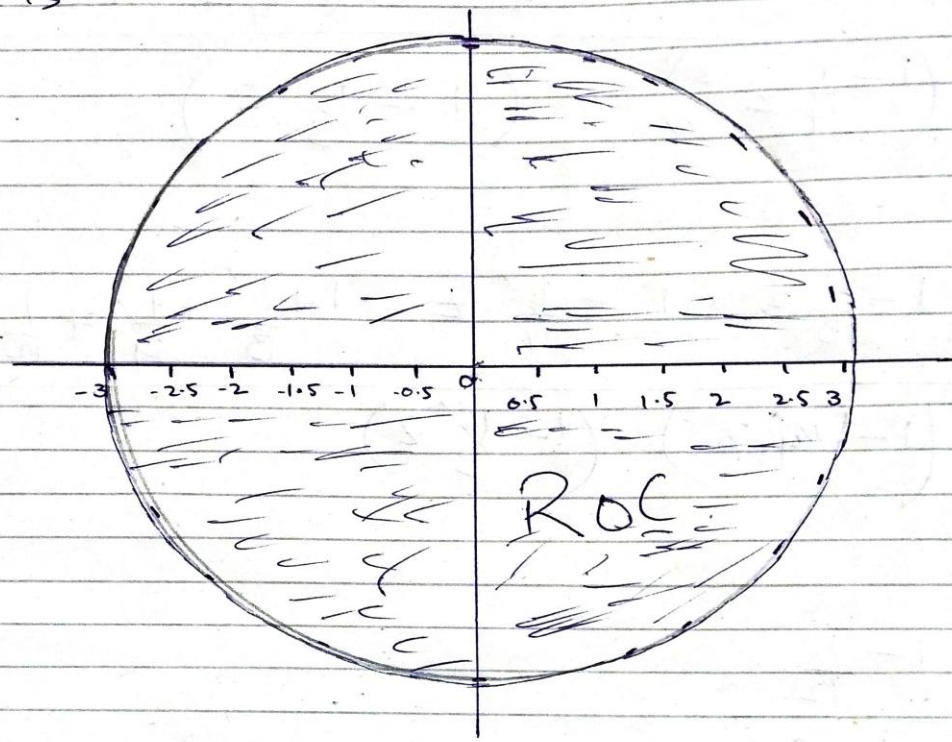
$$= \frac{1 - \cancel{\frac{1}{3}}z + 1 - \cancel{\frac{1}{4}}z - 1 + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \frac{1}{12}}{\left(1 - \cancel{\frac{1}{4}}z^{-3}\right)\left(1 - \cancel{\frac{1}{3}}z\right)}$$

$$= \frac{1 - \frac{1}{12}}{1 - \frac{1}{4}z^{-2}\left(1 - \frac{1}{3}z\right.}$$

$$= \frac{13/12}{1 - \frac{1}{4}z^{-1}\left(1 - \frac{1}{3}z\right)}$$

Hence the ROC is $\frac{1}{4} < |z| < 3$

So the Region of convergence is



ii) By using z - transform

i.e $x(n) = \alpha^n u(n) \longleftrightarrow x(z) = \frac{1}{1-\alpha z^{-1}}$

By putting the value.

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) z^{n-n} - \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$= \frac{-5/2 \, z^{-1}}{\left(1-\frac{1}{2}z^{1}\right)\left(1-2z^{-1}\right)}$$

As seen the ROC use $|z| > 2$