# Discrete Structure

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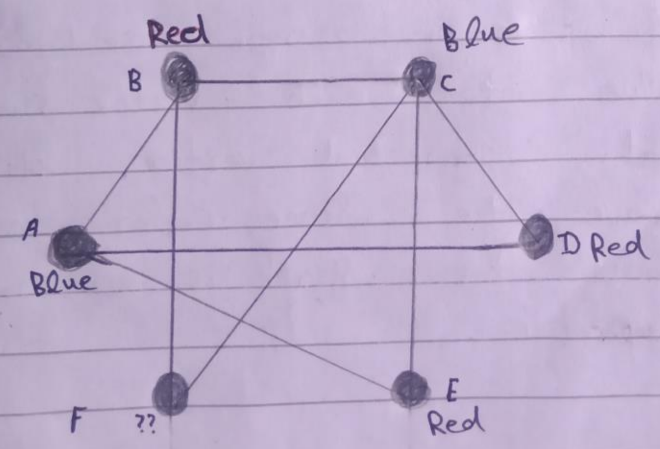
Final Exam

Question-1: Answer

(i) Solution:
Let us assign red or blue to each vertex.
If two vertices are connected, then they should not have the same color.



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we start by assigning "Blue" to a.
Since b is Connected to blue a, we assign "Red" to b.
Since c is connected to the red b, we assign "Blue" to c.
we then note that f is Connected to the red b and the blue c, which means that we Can not assign a color to f such that it differs from the colors of the Connected vertices (as there are only two colors: red and blue).

Thus it is not possible to ~~assign~~ assign red or blue to each vertex such that Connected vertices do not have the same color and thus the graph is ~~not~~ not bipartite.

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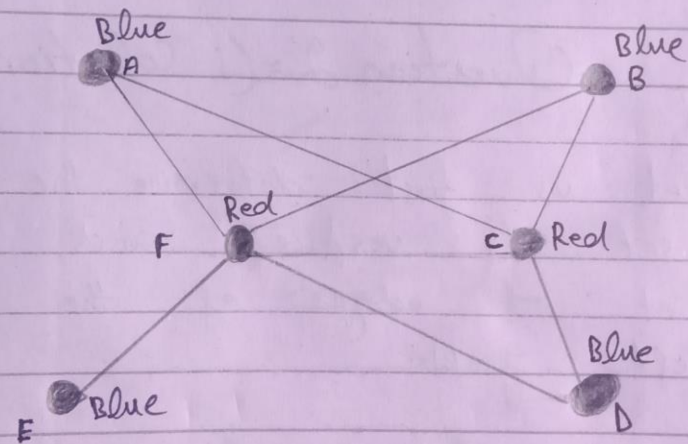
## Question 1 = (ii solution)

let us assign red or blue to each vertex.
If two vertices are connected, then they should not have the same color.




we then note that it is possible to assign red or blue to each vertex such that connected vertices donot have the same color and thus the graph is bipartite.

Moreover, the partitioning of the vertices are the set with the blue vertices and the set with the red vertices.

$V_1 = \{a, b, d, e\}$

$V_2 = \{e, f\}$

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## Question 2 = (i Solution)

Let us first determine the set of vertices and set of edges of the left graph:

$V_1 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$

$E_1 = \{(u_1, u_2), (u_1, u_4), (u_1, u_6), (u_2, u_3), (u_2, u_6), (u_3, u_4), (u_3, u_5), (u_4, u_5), (u_5, u_6)\}$

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Let us first determine the set of vertices and set of edges of the right graph:

$V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E_2 = \{(v_5, v_2), (v_5, v_6), (v_5, v_1), (v_2, v_3),$

$(v_2, v_1), (v_3, v_6), (v_3, v_4), (v_6, v_4),$

$(v_4, v_1)\}$

By Comparing the two sets of edges, we can define the following one-to-one and onto function f from $V_1$ to $V_2$.

Note: You could also use the ~~degrees~~ degrees of the vertices because the vertex and their image need to have the same degree.

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$$f(u_1) = v_5$$
$$f(u_2) = v_2$$
$$f(u_3) = v_3$$
$$f(u_4) = v_6$$
$$f(u_5) = v_4$$
$$f(u_6) = v_1$$

$f$ is then a function that makes the two graphs ~~isomor~~ isomorphic since.

## Edge in left graph.

- $u_1$ and $u_2$ are adjacent
- $u_1$ and $u_4$ are adjacent
- $u_1$ and $u_6$ are adjacent
- $u_2$ and $u_3$ are adjacent
- $u_2$ and $u_6$ are adjacent
- $u_3$ and $u_4$ are adjacent
- $u_3$ and $u_5$ are adjacent
- $u_4$ and $u_5$ are adjacent
- $u_5$ and $u_6$ are adjacent.

# Question 2: (ii Solution)

let us determine the degree of every vertex in the left graph.

$\deg(u_1) = 3$
$\deg(u_2) = 3$
$\deg(u_3) = 2$
$\deg(u_4) = 2$
$\deg(u_5) = 3$
$\deg(u_6) = 3$

Degree Sequence = 3, 3, 3, 3, 2, 2

Let us next determine the degree of every vertex in the right graph.

$\deg(v_1) = 2$
$\deg(v_2) = 3$
$\deg(v_3) = 3$
$\deg(v_4) = 2$
$\deg(v_5) = 2$
$\deg(v_6) = 4$

Degree Sequence = 4, 3, 3, 2, 2, 2

Isomorphic graphs need to have the same number of vertices, the same number of edges and the same degree sequences.

we then note that the two graphs donot have the same degree sequence and thus the two graphs are not isomorphic.

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# Question 3= (solution i)

(i) $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Let us add all elements in the two matrices, which will represent the number of connections of a vertex to another vertex (which is double the number of edges).

A contains 8 ones and thus the simple graph corresponding to A contains 8 connections.

B contains 10 ones and thus the simple graph corresponding to A contains 10 connections.

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Since the number of Connections of the two graphs are not the same and then the graphs are not isomorphic.

Question 3 = (solution(ii))

(ii) Given

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

let us add all elements in the two matrices, which will represent the number of Connections of a vertex to another vertex (which is double the number of edges)

A Contains 8 ones and thus the Simple graph Corresponding to A Contains 8 Connections.

B Contains 6 ones and thus the Simple graph Corresponding to A Contains 6 Connections.

Since the number of Connections of the two graphs are not the Same the number of edges in the graphs are not the same and then the graphs are not isomorphic.

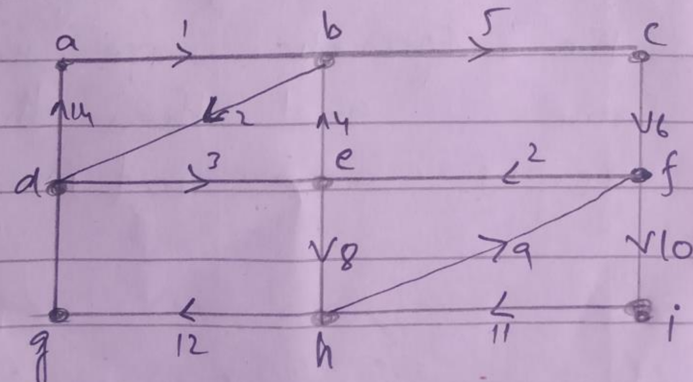
# Question 4: (i Solution)

(i):-

deg (a) = 2
deg (b) = 4
deg (e) = 2
deg (d) = 4
deg (e) = 4
deg (f) = 4
deg (h) = 4
deg (i) = 2

(~~all deges~~
all deges are even.
we have Euler circuit)

And Euler circuit is shown below



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## Question 4 (ii Solution)

(ii) By theorem 1, this graph does not have an Euler circuit because we have two vertices with odd degrees (a and d). This graph does have an Euler path. by Theorem 2. The path is as follows: a; e; c; e; b; e; d; b; a; c; d.

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## Question 5 (Answer (i))

(i) There is no Hamilton circuit because there are vertices of degree 1 (Pendants) in the graph.

## Question 5 (Answer (ii))

(ii) This graph has a Hamilton circuit a; b; c; d; e; a is a circuit.