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Subject # Linear control System.

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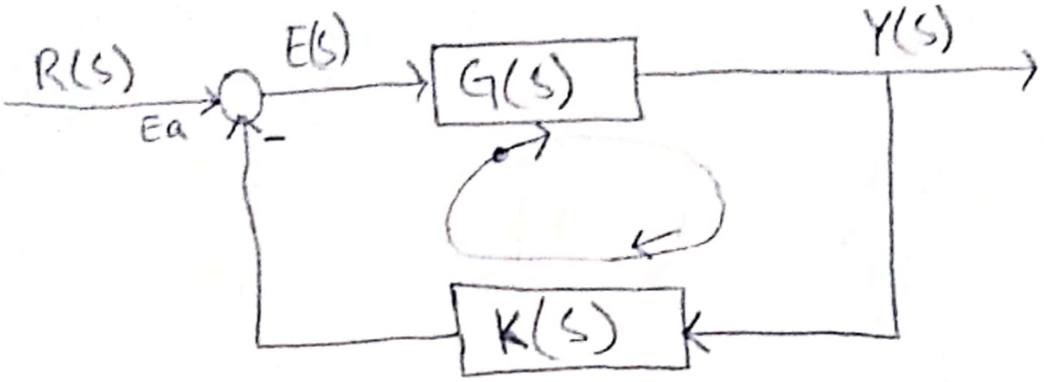
Question # 1
Part (a).

*) Find the Transfer function by using feedback loop.

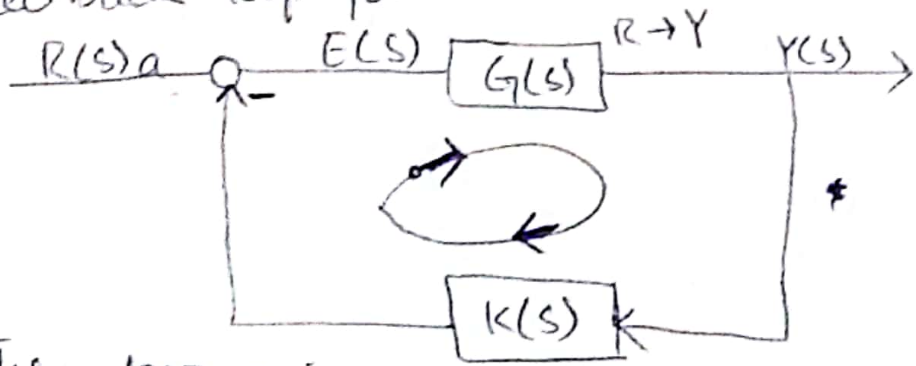


Ans:- When we compute Transfers functions from outside to inside the feedback

$$E(s) = R(s) - K(s)G(s)E(s) \Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+G(s)K(s)}$$

$$Y(s) = G(s)E(s) \Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)K(s)}$$

Feedback loop formula, TF

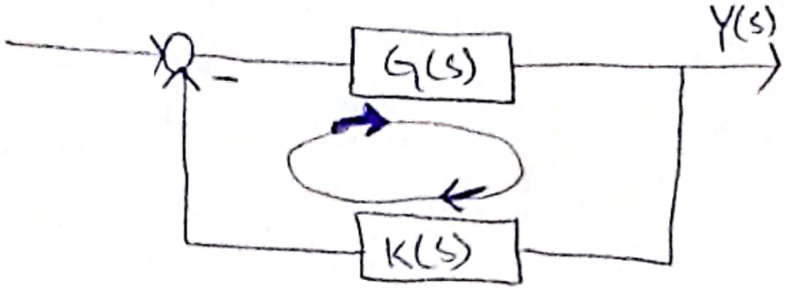


* The loop gain is the product of all T.F that form the loop.

Fg : Forward gain from R(s) to Y(s) ⇒ G(s)

Lg : loop gain : G(s) ⇒ K(s)(-1)

$$R(s) \longrightarrow \boxed{\frac{F(s)}{1-L_g}} \longrightarrow Y(s) \quad \Rightarrow \quad \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)K(s)}$$



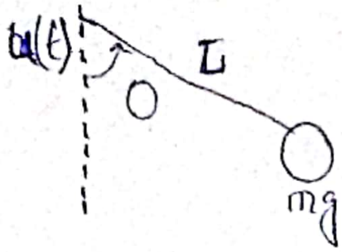
F_g : Forward gain from $R(s)$ to $E(s) \Rightarrow 1$

L_g : Loop gain : $G(s) \Rightarrow K(s)(-1)$

$$R_s \longrightarrow \boxed{\frac{F_g}{1-L_g}} \longrightarrow E(s) \quad \Rightarrow \quad \frac{E(s)}{R(s)} = \frac{1}{1+G(s)K(s)}$$

Question #2
Part (A)

Answer:-



$$mL^2\ddot{\theta}(t) + mgL\sin\theta(t) = u(t)$$

$$\ddot{\theta}(t) + \underbrace{\frac{g\sin\theta(t)}{L} - \frac{u(t)}{mL^2}}_{f(\theta,u)} = 0$$

* Linearize it at $\theta_0 = \pi$

* $u_0 = ?$

$$\pi + \frac{g\sin\pi}{L} - \frac{u_0}{mL^2} = 0$$

$$u_0 = 0$$

* New coordinates:-

$$\theta = \theta_0 + \delta\theta = \pi + \delta\theta$$

$$u = u_0 + \delta u = 0 + \delta u$$

Taylor series expansion of : $f(\theta,u)$ at $\theta=\pi$
$u,=0$

$$\frac{\partial f(\theta,u)}{\partial\theta}\bigg|_{\substack{\theta=\pi\\u=0}} = \frac{g\cos\theta}{L}\bigg|_{\theta=\pi} = -\frac{g}{L}$$

$$\frac{\partial f(\theta,u)}{\partial u}\bigg|_{\substack{\theta=\pi\\u=0}} = -\frac{1}{mL^2}$$

$$\delta\ddot{\theta} + \frac{\partial f(\theta,u)}{\partial\theta}\bigg|_{\substack{\theta=\pi\\u=0}}\delta\theta + \frac{\partial f(\theta,u)}{\partial u}\bigg|_{\substack{\theta=\pi\\u=0}}\delta u=0$$

$$\delta\ddot{\theta} - \frac{g}{L}\delta\theta - \frac{1}{mL^2}\delta u = 0$$

Question # 2
Part (b)

Answer:- Linear System:- A System having principle of Superposition.

$$u(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

$$\left.\begin{array}{l} u_1(t) \to y_1(t) \\ u_2(t) \to y_2(t) \end{array}\right\} \Rightarrow \alpha_1 u_1(t) + \alpha_2 u_2(t) \to \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

$$\forall \alpha_1, \alpha_2 \in R$$

- A non linear system does not satisfy the principle of
- Superposition.
- Easy to understand & obtain solutions.
- Linear ordinary differential equations (ODEs)
- Add many simple solutions to get more complex ones
- Easy to check the stability of stationary states

* **Why do we linearization:-**

- Real systems are inherently nonlinear (Linear system do not exist)
- TF models are only for Linear time-invarient (LTI) system.
- Many control analysis / design techniques are available for linear System.
- Nonlinear systems are difficult to deal with mathematically.
- often we linearize nonlinear systems before analysis and design.

Question # 3.

Part (A) :

Ans) → $Q(s) = s^4 + s^3 + 3s^2 + 2s + 2$

s^4	1	3	2	
s^3	1	2	0	
s^2	$\frac{1\cdot3 - 1\cdot2}{1}$	$\frac{1\cdot2 - 1\cdot10}{1}$	0	
s^1	$\frac{1\cdot2 - 1\cdot2}{1}$	$\frac{1\cdot0 - 1\cdot0}{1}$	0	
s^0	0	0	0	

s^4	1	3	2
s^3	1	2	0
s^2	1	2	0
s^1	0	0	0
s^0	0	0	0