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Section A
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Q1) Find PQ where P is the point ....
.... vector of the point dividing PQ in the ration.

Soln

Cordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3K$

or

$$OQ = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \rightarrow (1)$$

Now distance between P & Q $= |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \rightarrow (2)$$

Let M be the point which divided PQ in ratio 1:3, then by ratio theorem position vectors of $M = \overrightarrow{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4}$$

Hence eq (1), (2) & (3) are the required solution.

Q2) Evaluate

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}$$

By division we get $11x + 4$

So $2x - 1 + \frac{11x + 4}{2x^2 + x} = \frac{4x^3 + 10x + 4}{2x^2 + x}$

$$\Rightarrow \int \frac{4x^3 + 10x - 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \rightarrow (a)$$

$$2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$\frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \rightarrow (2)$$

Now

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \rightarrow (A)$$

$$\frac{11x+4}{x(\cancel{2x+1})} = \frac{A(2x+1)+Bx}{x(\cancel{2x+1})}$$

$$11x+4 = A(2x+1) + Bx \quad - (3)$$

put $x = 0 \rightarrow (3)$

$4 = A$

put $x = -\frac{1}{2} \rightarrow (3)$

$B = 3$

put value of (A) & (B)

in eq (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral

$$\int \frac{11x+4}{x(2x+1)} = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$4\ln(x) + \frac{3}{2}\ln(2x+1)$$

put in eq 2

$$x^2 - x + 4\ln x + \frac{3}{2}\ln(2x+1)$$

Q3a) Evaluate

(a) $\int_0^2 x^2 e^x \, dx$

Integration by parts

$\int uv = uv - \int u'v$

$u = x^2 \qquad v' = e^x$

$u' = 2x \qquad v = e^x$

$= x^2 e^x - \int 2xe^x \, dx$

$= x^2 e^x - 2\int xe^x \, dx.$

Now solving xe^x

Integration by parts

$u = x \qquad v' = e^x$

$u' = 1 \qquad v = e^x$

$= x^2 e^x - 2\left[xe^x - \int e^x \, dx\right]$

$= x^2 e^x - 2\left(xe^x - e^x\right)\Big|_0^2$

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$$x^2 e^x - 2xe^x + 2e^x \Big|_0^2$$

$$= \left((2)^2 e^2 - 2(2)e^2 + 2e^2\right) - (2e^0)$$

$$= 4e^2 - 4e^2 + 2e^2 - 2$$

$$\boxed{= 2e^2 - 2} \text{ An.}$$

3b) $$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Substitute $u = \sqrt{x}$ $\quad \frac{dx}{dx} = \frac{1}{2\sqrt{x}}$

So $dx = 2\sqrt{x}\, dx$

$$= \int_1^2 2\sin u \; dx.$$

$$= 2\int_1^2 \sin x \, dx$$

$$= -2\cos u \Big|_1^2$$

$u = \sqrt{x}$

$$= -2\cos\sqrt{x} \Big|_1^2$$

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$= -2\cos\sqrt{2} + 2\cos\sqrt{1}$

$= \boxed{2[\cos\sqrt{1} - \cos\sqrt{2}]}$ Ans

Q4) Verify that

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three dimensional Laplace's equation.

Soln

Q4). Solution

$$U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \rightarrow (1)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Diff (1) w.r.t x.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}$$

$$= -\frac{1}{2}(x^2+y^2+z^2)^{-1/2} \frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$= -x(x^2+y^2+z^2)^{-3/2}$$

Diff Again

$$\frac{\partial^2 u}{\partial x^2} = -\frac{\partial}{\partial x}\left[x(x^2+y^2+z^2)^{-1/2}\right]$$

$$= -\left[x\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-3/2-1}(x^2+y^2+z^2) + (x^2+y^2+z^2)^{-3/2} \cdot 1\right]$$

$$= -\left[\frac{-3x}{2}(x^2+y^2+z^2)(2x) + (x^2+y^2+z^2)\right]$$

Taking $(x^2+y^2+z^2)^{-3/2}$ common

$$= -\frac{1}{(x^2+y^2+z^2)^{3/2}}\left[\frac{-3x^2}{x^2+y^2+z^2}+1\right]$$

$$\frac{\partial u}{\partial x^2} = -\frac{1}{(x^2+y^2+z^2)^{3/2+1}}\left[y^2+z^2-2x^2\right] \quad (A)$$

Now Diff w.r.t y

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^2+y^2+z^2)^{-1/2}$$

$$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}\,\cancel{2}y$$

$$\frac{\partial^2 u}{\partial y^2} = \left[y(-3/2)(x^2+y^2+z^2)^{-3/2-1}\right.$$
$$\left.\frac{\partial}{\partial y}(x^2+y^2+z^2 + (x^2+y^2+z^2)^{-3/2}\right]$$

$$= -\left[-3y^2(x^2+y^2+z^2)^{-3/2-1} + x^2+y^2+z^2)^{-3/2}\right]$$

$$= \left[-3y^2(x^2+y^2+z^2)^{-3/2-1} + (x^2+y^2+z^2)^{-3/2}\right]$$

Taking $(x^2+y^2+z^2)^{-3/2}$ common.

$$\frac{-1}{(x^2+y^2+z^2)^{3/2}}\left[\frac{3y^2}{x^2+y^2+z^2}-1\right]$$

$$\frac{\partial^2 u}{\partial y^2} - \frac{-1}{(x^2+y^2+z^2)^{3/2+1}} \quad x^2+2y^2+z^2$$

Now w.r.t z

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(x^2+y^2+z^2\right)^{-1/2}$$

$$= -\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}\left(\cancel{2}z\right)$$

Diff again

$$\frac{\partial^2 y}{\partial z^2} = \frac{\partial}{\partial z}\left[z\cdot\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$= -\left[z\left(3/2\right)\left(x^2+y^2+z^2\right)^{-1/2-1}\cdot 2z + \left(x^2+y^2+z^2\right)^{-3/2}\right.$$

Taking $(x^2+y^2+z^2)$

$$\frac{-1}{(x^2+y^2+z^2)^{-1/2-1}}\left[\frac{3z^2}{x^2+y^2+z^2}+1\right]$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{-1}{(x^2+y^2+z^2)^{5/2}}\left(x^2+y^2+2z^2\right) - (C)$$

Adding (A) & (B) & (C)

$$= \frac{-1}{(x^2+y^2+z^2)^{5/2}}\left(y^2+z^2-3x^2\right) - \frac{1}{x^2+y^2+z^2}$$

$$\left(x^2+z^2-2y^2\right) - \frac{1}{(x^2+y^2+z^2)^{1/2}}$$

$$\left(x^2+y^2-2z^2\right)$$

Taking common

$$= \frac{-1}{(x^2+y^2+z^2)^{5/2}}\left[-y^2+z^2 - \cancel{2x^2} + x^2 - \cancel{2y^2} + z^2 + x^2 + y^2 + z^2\right]$$

$$= \frac{-0}{(x^2+y^2+z^2)^{5/2}}$$

(proved)

(=)