(1)

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Section # "C"

Semester # 8th

Qno2:

Given DATA:

$E = 29000$ Ksi;
$I = 150$ in^4
$\delta_{st} = .7716$ lb static load
$L = 10'$

Solution:

The system is undamped ($C = 0$) and undergoin free vibration ($P(t) = 0$)
Hence General E.O.M. becomes

$$ku + m\ddot{u} = 0$$

$$k = \frac{3EI}{L^3} = \frac{3 \times 29000 \frac{k}{in^2} \times 150\ in^4}{(10 \times 12)^3}$$

$k = 7.55$ k/in $= 90625$ lb/ft

$$m = \frac{7716\ lb\,sec^2}{32.2 ft} = 239.62 \text{ slug}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{96035}{239.62}}$$

$\omega_n = 20.01$ rad/sec

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{20.01} = 0.31\ sec$$

Substituting corresponding values in eq (1)

$$90625\, u + 20.01\, \ddot{u} = 0$$

Where k is in lb/ft and m is in lb sec/ft²

So

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24}', \quad \text{and} \quad \dot{u}(0) = 0$$

$$u(t) = \frac{1}{24} \times \cos(20.01t) + 0$$

$$f_s(t) = k\,u(t) = 90625 \times \cos(20.01t)$$

$$f_s(t) = 3776.04\left(\cos(20.01t)\right)$$

Amplitude of dynamic displacement u_0 for undamped free vibration

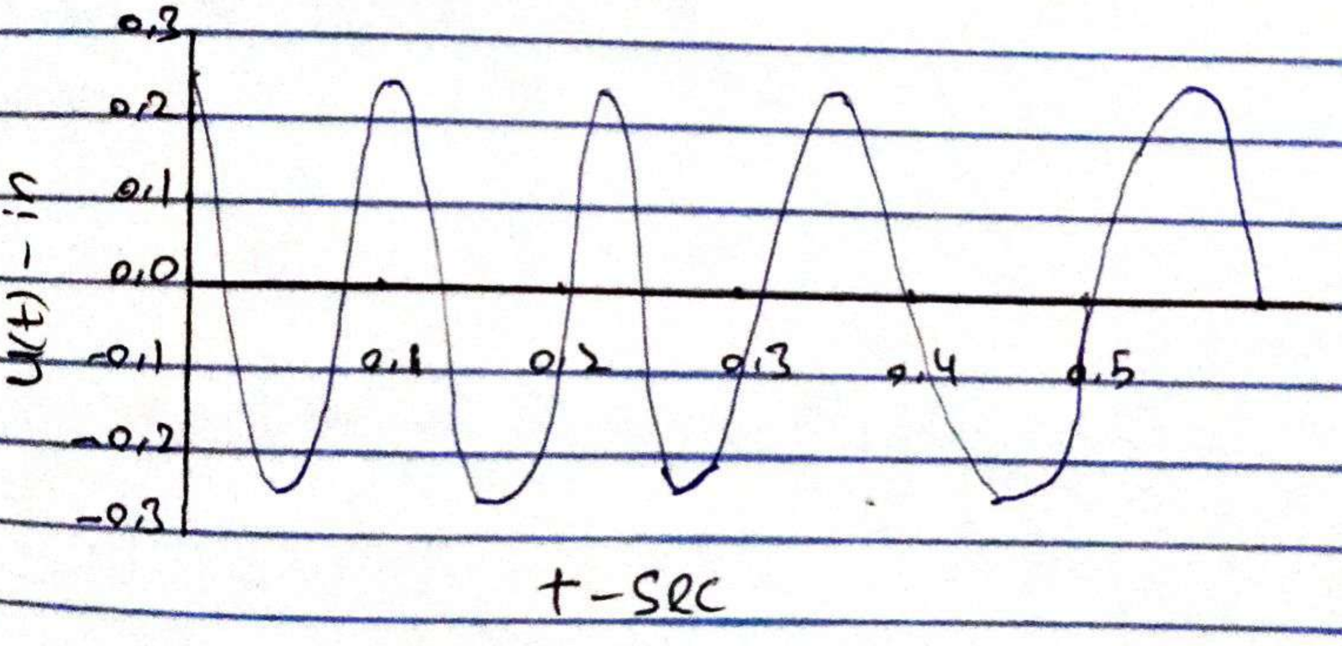
is $u_0 = \sqrt{[u(0)]^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$

$$u_0 = \sqrt{\left(\frac{1}{24}\right)^2} = \frac{1}{24}'$$

Amplitud of equivalent static force

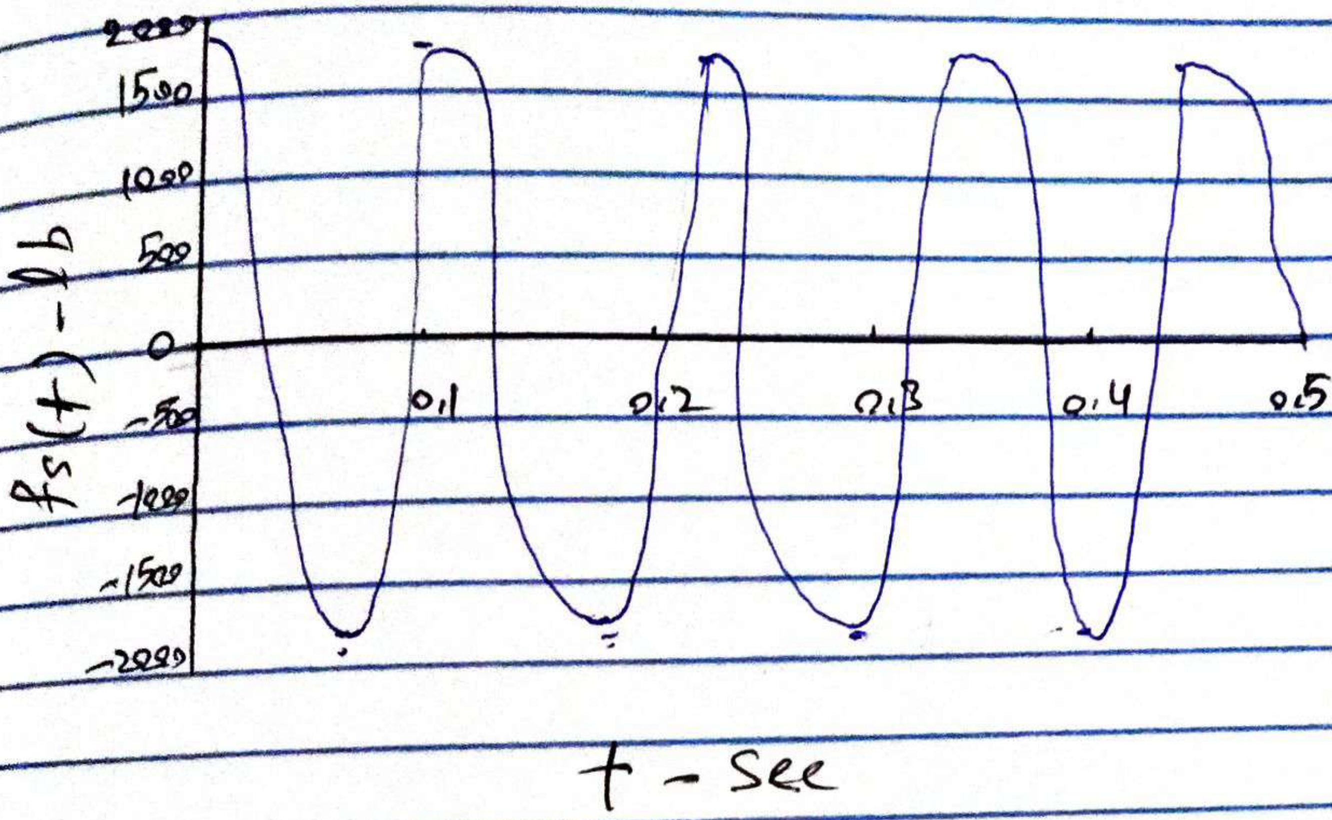
$f_s(0)$

$ku(0) = 90625 \times \frac{1}{24'} = 3776.041 lb$



Variation of displacement

with time.



Variation of Equivalent static forces with Time.

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Q no 2:

Solution: Damping ratio for RCC Ranges from 2% to 5% so here i take 2.5%.

E.O.M for damped free vibration is

$k u + c \dot{u} + m \ddot{u} = 0$ ——— (1)

It is known from problem no (1) that $k = 90625$ lb/ft and $m = 239.62$ $lb\,sec^2/ft$

Now

$c = \zeta \times 2 m \omega_n = 2 \times 239.62 \times 20.01 \times 0.025$

$c = 239.73$ lb sec/ft

By substituting values of k, c and m in equation (1) we get

$90625 u + 239.73 \dot{u} + 239.62 \ddot{u} = 0$

Solution to the E.O.M for damped free vibration is

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\,\zeta\omega_n\right]\sin(\omega_D t)\right]$$

$\omega_D = 20.01$ rad/sec

$$u(t) = e^{-0.025\times 20.01t}\left[\frac{1}{24}\times\cos(20.01t) + \frac{1}{20.01}\times\left[0+\frac{1}{24}\times 0.025\times 20.01\times\sin(20.01t)\right]\right]$$

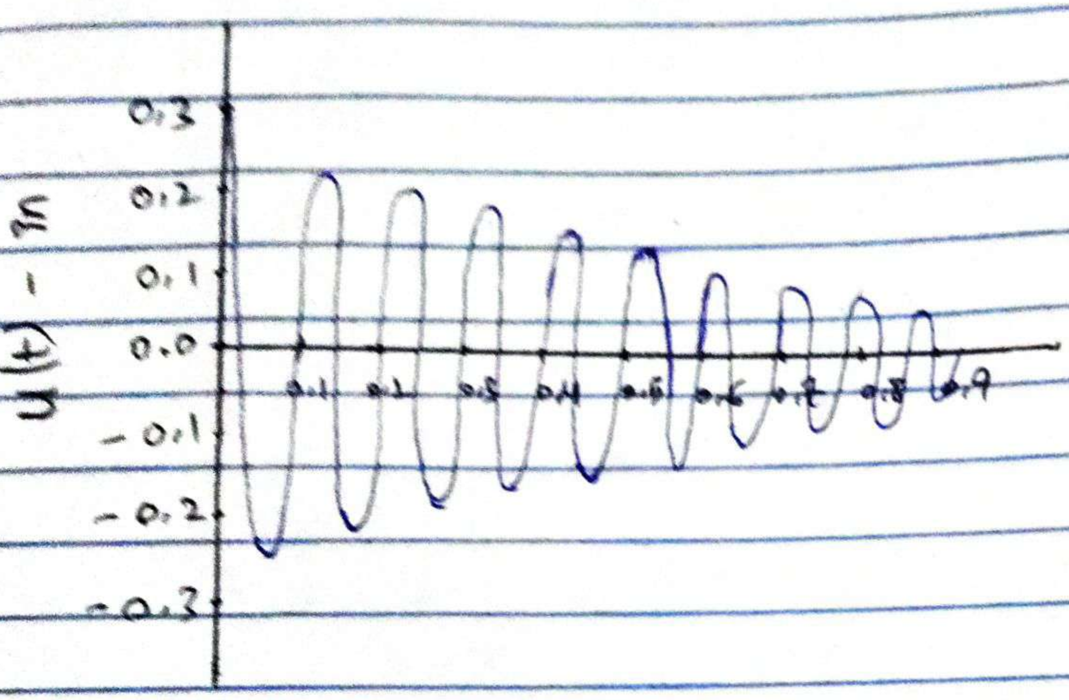
$$u(t) = e^{-0.5002}\left[0.041\cos(20.01t) + 0.0499\left(0.0208\times\sin(20.01t)\right)\right]$$

$$u(t) = e^{-0.5002}\left[0.041\times\cos(20.01t) + 0.000104\times\sin(20.01t)\right]$$

$$f_s(t) = K.\,u(t) = 90625\times u(t)$$

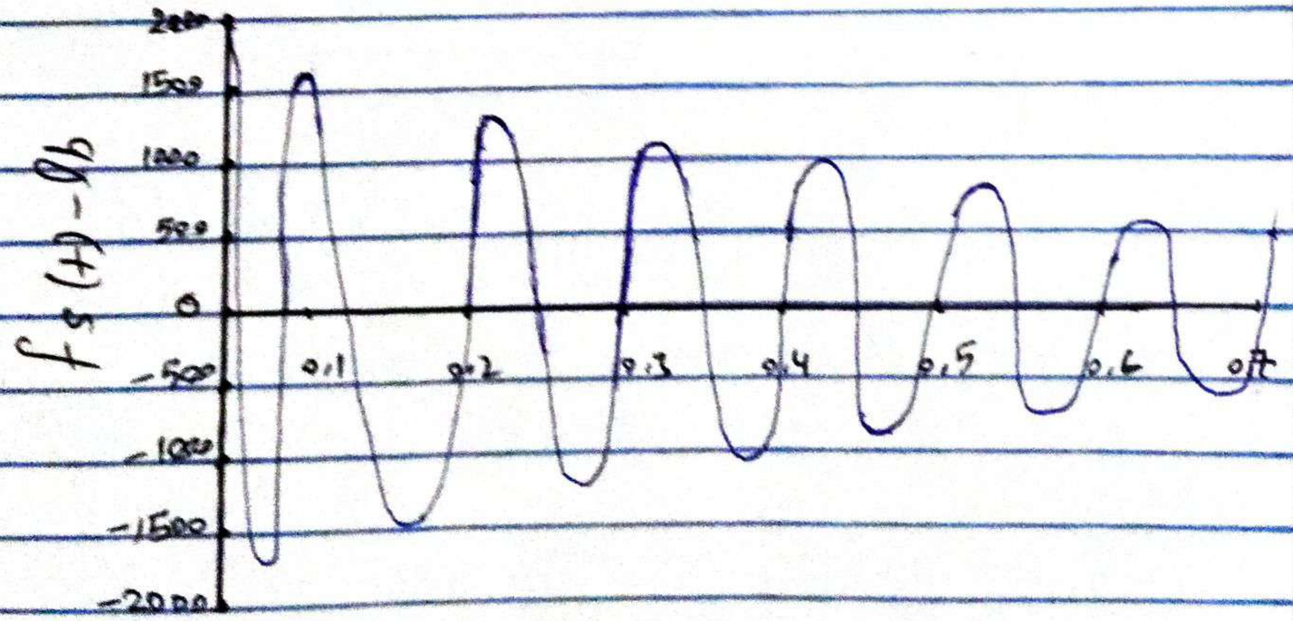
$$f_s(t) = e^{-0.5002}\left[3715.625\times\cos(20.01t) + 9.42\times\sin(20.01t)\right]$$

Damped free vibration:



t - sec

(variation of displacement with time.)



t - sec

Variation of Equivalent static forces with Time

Q no 3:

Solution:

$u_1 = \frac{7716}{1000} = 7.716 \text{ in}$

After $j = 7$, $u_{j+1} = u_6 = 2.28 \text{ cm} = 0.89''$

(a) ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left(\frac{7.716}{0.89}\right)$$

$$\zeta = 0.049 \times 100 = 4.91\%$$

$$\Rightarrow \boxed{\zeta = 4.91\%}$$

(b) $T_n = ?$

7 cycle of vibrations are completed in 3.57 sec.

Time required to complete one cycle $= 3.57/7 = T_D$

$$\boxed{T_D = 1.96 \text{ sec}}$$

New

$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$\frac{2\pi}{\omega_D} = 2\pi / \omega_n \sqrt{(1-\zeta^2)}$

$T_D = \frac{T_n}{\sqrt{1-\zeta^2}} \Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$

$T_n = 0.51 \times \sqrt{1-(0.049)^2}$

$T_n = 0.5098 = 0.51 \text{ sec}$

(c) $k = ?$

$k = \frac{60 \times \cos 60^\circ}{2} = 15 \text{ k/in}$

$K = 18000 \text{ lb/ft}$

(d) weight of the tank $= W = ?$

$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\left(\frac{W}{g}\right)}} = \sqrt{\frac{K \times g}{W}}$

$\omega_n^2 = K \times g$

$W = kxg / \omega_n^2$

Also $\omega_n = 2\pi / T_n$

$$W = \frac{Kg}{\left(\frac{4\pi^2}{T_n^2}\right)} = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{\frac{180000\,lb}{ft} \times \frac{32.2\,ft}{sec^2} \times (0.51\,sec)^2}{4\pi^2}$$

$$W = 74875\,lb = 74.9\,K$$

(e) $c = ?$

It is known that $\zeta = \frac{c}{2m\omega_n}$

$$c = \zeta \times 2m\omega_n = \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{0.0494 \times 4 \times \pi \times \left(\frac{74875}{32.2}\right)}{0.51}$$

$$c = 2807.4\ lb\ sec/ft$$

(f) No of cycles to reduce displacement amplitude from 7.716 in to 0.5 in.

$j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$j = \frac{1}{2 \times \pi \times 0.049} \ln\left(\frac{7.716}{0.5}\right)$$

$$\boxed{j = 8.88 \text{ or } 9 \text{ cycles}}$$

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